Dynamic Demand for New and Used Durable Goods without Physical Depreciation: The Case of Japanese Video Games*

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Abstract

In product categories such as CDs/DVDs and video games, the competition from used goods markets has been viewed as a serious problem by producers. These products physically depreciate negligibly, but owners' consumption values could depreciate quickly due to satiation. Consequently, used goods that are almost identical to new goods may become available immediately after a new product release. However, the existence of used goods markets also provides consumers with a selling opportunity. If consumers are forward-looking and account for the future resale value of a product at the time of making buying decisions, the existence of used goods markets could increase the sales of new goods. Thus, whether used good markets are harmful or beneficial to new-good producers is an empirical question. To tackle this question, we assemble a new data set from the Japanese video game market. This unique data set includes not only the sales and prices of new and used goods, but also the resale value of used copies, the quantity of used copies retailers purchased from consumers, and the inventory level of used copies at retailers. We then develop a structural model of forward-looking consumers' buying and selling decisions that utilizes the new data, and estimate it by recently proposed Bayesian Markov chain Monte Carlo algorithm for non-stationary dynamic programming models. The estimation results suggest that consumers are forward-looking in the Japanese video game market and the substitutability between new and used games is low. Using the estimates, we quantify the impact of eliminating used game markets on video game publishers' profits. We find that the elimination of used game markets reduces the average profits per game by 7% if publishers do not optimally adjust their prices. However, if they do, the elimination is beneficial for most of the games and increases the average profits per game by 11%.

Keywords: Used Goods Market, Dynamic Consumer Buying and Selling Decisions, Consumer Satiation, Discount Factor, Bayesian Estimation of Dynamic Discrete Choice Models

1 Introduction

The existence of used goods markets has been viewed as a serious problem by producers in categories such as books, CDs/DVDs, and video games. Producers and their supporters argue that used goods retailing significantly lowers their profits and limits the development of new products. For instance, book publishers and authors expressed their annoyance to Amazon over used books sold on its websites (Tedeschi 2004). Video game publishers in Japan attempted to kill off used video game retailing by suing used video game retailers (Hirayama 2006). Their main argument is that products like books and video games physically depreciate negligibly, but owners' consumption values can decline very quickly due to satiation. As a result, unlike products that physically depreciate more considerably (such as cars), producers of books, video games, and other digital products may face competition from used goods that are almost identical to new goods immediately after a new product release.

However, their argument focuses only on one aspect of used goods markets (substitution effect). The existence of used goods markets provides consumers with a selling opportunity. If consumers are forward-looking and account for the future resale value at the time of making a buying decision, the effective price consumers pay for a product is lower than the price of the product (resale effect). This feature implies that the existence of used goods markets could increase the sales of new goods. Thus, whether the existence of used goods markets hurts or benefits new-good producers is an empirical question, and the answer depends on which effect, substitution or resale effect, dominates.

To tackle this question, we extend the previous literature in two dimensions. First, we assemble a new data set from the Japanese video game market, which includes weekly aggregate level data for 20 video game titles released in Japan between 2004 and 2008. The novel aspect of this data set is that in addition to the sales and price of new and used video game copies, it also includes the resale value of used copies,[1] the quantity of used copies retailers purchased from consumers, and the inventory level of used copies at retailers. Previous studies on new and used durable goods have not exploited these new dimensions before.

[1] In this paper, resale value is defined as the amount of money consumers receive when they sell their used video games to retailers.

This new data set allows us to advance our knowledge about used goods market trading. In particular, we highlight two empirical regularities new in the literature. First, we observe that the quantity of used copies supplied by consumers is more concentrated in the earlier weeks than that purchased by consumers, and that the degree to which these two quantities exhibit different paths differs across game titles. Second, the average retail price of used copies and the average retail resale value of used copies both decline over time, but the declining rate for the resale value is slightly higher. We will utilize these variations to examine the extent to which consumers are forward-looking and to estimate the transaction costs for buying and selling used copies separately, both of which are new in the literature on new and used durable goods markets.

Second, we develop a new empirical structural framework that utilizes the new data set. In our model, the demand for new goods and the demand for and the supply of used goods are generated by a dynamic discrete choice model of forward-looking consumers that incorporates (i) new and used goods buying decisions, (ii) used goods selling decision, (iii) consumer expectations about future prices of new and used goods and resale values of used goods, and (iv) the depreciation of both owners' and potential buyers' consumption values. To the best of our knowledge, this is the first dynamic model of demand that incorporates these four important features of consumer decision making in durable goods markets. In our model, the expected present discounted value of future payoffs from buying a product is determined by the dynamic consumer selling decision problem, which depends on the depreciation rate of owners' consumption values and future resale values. Furthermore, this is the first study that uses data on resale values to identify the role of consumer expectations about future payoffs. Finally, our model allows owners' consumption values and potential buyers' consumption values to depreciate differently over time. In the previous literature, the depreciation rate is always assumed to be identical for owners and buyers (e.g., Esteban and Shum 2007, Chen et al. 2011).[2] However, this distinction is imperative in digital product categories such as CDs/DVDs and video games because for product owners, the depreciation of consumption values is mainly due to satiation (satiation-based depreciation). Thus, potential buyers who have not been satiated at all could

[2]It should be noted that the previous literature assumes consumers receive less utility from a durable good over time only because of the physical depreciation of the product.

derive a higher consumption value than owners who are satiated. But for potential buyers, the freshness of the product may diminish as a product ages, causing consumption values to depreciate (freshness-based depreciation). Our new data set allows us to measure these two concepts of depreciation separately.

To estimate the proposed dynamic model, we employ a new Bayesian Markov chain Monte Carlo (MCMC) algorithm first proposed by Imai, Jain and Ching (2009a) (IJC algorithm), and is extended by Ishihara and Ching (2012) to a non-stationary environment. In conventional approaches to estimating a finite-horizon model, value functions need to be computed at all or a subset of pre-determined grid points in all time periods by backward induction (e.g., Keane and Wolpin 1994). However, when a model has multiple continuous state variables or the number of periods in the time horizon is large, computing value functions at a large enough number of grid points per time period can be computationally very demanding. The algorithm proposed by Ishihara and Ching (2012) alleviates the computational burden by partially solving value functions at only one randomly drawn state point per time period, storing them, and then approximating expected value functions by the weighted average of those partially solved value functions evaluated at different states in past iterations. Similar to the IJC algorithm, this new algorithm solves and estimates the model simultaneously. In this paper, we extend this algorithm further by combining it with the pseudo-policy function approach (Ching 2010a; 2010b) to address potential price endogeneity problems. The method we propose augments the unobserved shocks based on the joint-likelihood of the demand-side model and the pseudo-policy functions. Unlike the GMM approach (Berry et al. 1995, Gowrisankaran and Rysman 2011), our approach does not require solving any contraction mappings to invert the market share to recover the mean utility level. Also, since we use the Bayesian data augmentation technique, unlike the simulated maximum likelihood method, we do not need to integrate out the unobserved shocks during the estimation.

We apply this new empirical structural framework to the Japanese video game market, and examine the impact of the existence of used video game markets on the profitability of video game publishers. The estimation results suggest that consumers are forward-looking in the Japanese video game market, and

that substitutability between new and used video games is relatively low. Using the estimated model, we quantify the impact of eliminating the used video game market on new-game sales and profits, holding the prices of new copies at the observed level. On average, the elimination of the used video game market reduces consumers' willingness-to-pay (WTP) for new copies of video games in the earlier part of the product lifecycle. This effect is mainly driven by the resale effects that dominate the substitution effects in the earlier part of video games' lifecycle. As a result, we find that the average profits per game declines by 7%. We then compute the optimal prices of new copies when there is no used good market, and quantify the change in profits. We find that under the optimal prices, the elimination of the used video game market increases the average profits per game by 11%.

The rest of the paper is organized as follows. Section 2 reviews the previous literature. Section 3 describes the Japanese video game data used in this paper and presents some empirical regularities that have not been explored in the previous literature. Section 4 describes the dynamic discrete choice model of consumer buying and selling decisions. Section 5 describes the estimation strategy and identification. In Section 6, we discuss the parameter estimates and the counterfactual experiments. Section 7 concludes.

2 Literature Review

Our modeling framework is built on the previous literature on the dynamic purchase decisions in consumer durable goods markets (e.g., Melnikov 2000, Song and Chintagunta 2003, Nair 2007, Gordon 2009, Goettler and Gordon 2011, Carranza 2010, Gowrisankaran and Rysman 2011). In particular, our research is closely related to Nair (2007), who studies the intertemporal price discrimination in the U.S. video game industry, and examines the role of consumer price expectation. However, during his sample period, the U.S. used video game market was very small. Thus, he did not account for the impact of the used video game market in his analysis.

Previous studies on new and used durable goods have largely focused on car and housing markets. For example, Esteban and Shum (2007) build a dynamic equilibrium model of durable goods oligopoly and study the impact of car durability and used car markets on equilibrium car manufacturers' behavior. Chen

et al. (2011) extend Esteban and Shum (2007) and allow for transaction costs. Schiraldi (2011) estimates a dynamic discrete choice model of automobile replacement decisions, and studies the impact of scrappage subsidies. Tanaka (2009) studies the market power of condominium developers in Tokyo in a dynamic durable goods oligopoly model. Purohit (1992) examines the depreciation of used cars, measured by used car prices, in response to feature changes incorporated in new model cars in primary markets. Engers et al. (2009) study how much variation in used car prices can be explained by the net flow of benefits to car owners. They provide evidence that the net flow of benefits, which is similar to owners' consumption values in our research, can explain used car prices. Recently, Shiller (2011) has also examined the role of resale markets in the U.S. video game market.

One common feature of the models in these studies is that the depreciation rate of consumption values is assumed to be common across potential buyers and product owners. This assumption is motivated by two challenges: (i) when physical depreciation is present, it is difficult to separately measure the decline of owners' consumption values due to satiation from that due to physical depreciation; (ii) even if researchers can control for physical depreciation, they still need to observe both consumer buying and selling decisions in order to separately measure the depreciation of owners' consumption values and potential buyers' consumption values. The data sets used in these studies include only time-series variation in sales and price of new and used goods. They lack crucial information on the quantities of used goods sold by consumers to retailers and the associated resale values. Time-series variation in the price of new and used goods alone is not sufficient for disentangling the depreciation rates of owners' consumption values and potential buyers' consumption values even in the absence of physical depreciation. These challenges have limited previous studies, forcing them to assume that consumption value depreciation is identical for product owners and potential buyers.[3]

Another important difference between our study and these previous studies is that we do not use the interest rate to calibrate consumers' discount factor. Instead, we will estimate consumers' discount factor. It should be highlighted that our model implies that resale values of used copies provide exclusion restrictions

[3]Shiller (2011) estimates the depreciation rate for owners in the context of U.S. video game market, but does not allow potential buyers' valuation to depreciate.

that allow us to identify the discount factor (Magnac and Thesmar 2002). The intuition is that resale values do not affect consumers' current utility, but it affects consumers' expected future payoffs. As a result, observed correlation between sales of video games and the resale values can help recover the discount factor. Our identification strategy is similar to Chevalier and Goolsbee (2009), who study whether students are forward-looking in their textbook purchase decision. However, since they do not observe when and whether students sell their textbooks, they make an assumption that all of them will sell it at the end of the semester, and the textbook resale value affects all students' utility for buying a textbook if they are forward-looking. While this assumption may be reasonable in the textbook market, the timing of selling used goods is endogenous in general. We endogenize the timing of selling used video game copies. Moreover, we observe how many owners sell their games to retailers and the corresponding resale value, which allows us to model the expected future payoff from buying a video game as a function of not only the future resale value, but also the value of keeping the video game. This feature allows us to better control for the impact of the resale value on buying decisions.

Moreover, this is the first paper that separately estimates the transaction costs for buying and selling used goods. Previous studies do not separately observe the quantities of used copies purchased and supplied by consumers and associated used-copy prices and resale values. Thus, they can estimate only one transaction cost (e.g., Chen et al. 2011, Schiraldi 2011, Shiller 2011). Our results show that the two transaction costs will play a different role in influencing dynamic consumer buying and selling decisions.

Finally, there is a large body of theoretical literature on a variety of marketing practices in new and used durable goods markets. These include leasing contracts (e.g., Desai and Purohit 1998, Desai and Purohit 1999), channel coordination (Desai et al. 2004, Shulman and Coughlan 2007), trade-ins (Rao et al. 2009), and retail versus P2P used goods markets (Yin et al. 2010). One common feature of these theoretical models is that the used goods markets clear in every period. However, our data show that this assumption is far from a good approximation for the Japanese video game market – we consistently observe excess supply of used goods. Conceivably, the aggregate inventory for a used video game indicates how difficult it is for

consumers to find this title in the used market. In our structural dynamic structural demand model, we will be able to capture this feature.

3 Data

3.1 Japanese video game industry

Since mid-80s, the Japanese video game market has grown rapidly. The size of the industry in 2009 has reached $5.5 billion on a revenue basis (including sales of hardware, software, other equipments). This is about three times larger than the theatrical movie revenue in Japan, and it has become one of the most important sectors in the Japanese entertainment industry.[4] The existence of the used video good market has been a serious issue for video game publishers since 90s. In 2009, the sales of used video games (software) alone amounts to $1.0 billion on a revenue basis. One reason for the large used video game market in Japan could be that unlike North America, video game renting by third-party companies is prohibited by law in Japan.[5] Another reason argued by Hirayama (2006) is the flat-pricing strategy commonly adopted by video game publishers - the price of new games is maintained at the initial level at least one year after the release. This may provide an opportunity for used goods market to grow to capture the segment of consumers who do not mind buying used goods. [6] However, it can also be argued that the used market has induced publishers to adopt the flat-pricing strategy. Liang (1999) uses a theoretical model to show that when used goods markets are present, durable goods monopolists may be able to credibly commit to a high price (avoiding the Coase Conjecture). While investigating the optimality of the flat-pricing strategy is interesting, this paper will focus on understanding consumers dynamic buying and selling decision problems.[7]

[4]In fact, the size of the video game industry in the US has grown to $20.2 billion, which is now higher than the theatrical movie revenues.

[5]In principle, video game publishers can run the rental business for their own video games. However, only one publisher attempted to operate it in the history and did not succeed and exited.

[6]Note that in Japan, resale price maintenance is illegal for video games while it is still legal for books, magazines, newspapers and music.

[7]Recently, Cho and Rust (2010) investigate the flat rental pricing strategy used by car rental companies, and conclude that actual rental markets are not fully competitive and firms many be behaving suboptimally.

3.2 Japanese video game data

We have collected a data set of 20 video games that were released in Japan between 2004 and 2008. The data come from several sources. For each video game, weekly aggregate sales of new copies and its manufacturer suggested retail price (MSRP) are obtained from the weekly top 30 ranking published in Weekly Famitsu Magazine, a major weekly video game magazine in Japan published by Enterbrain, Inc. The average number of weeks observed across games is 19 weeks. In Japan, the sales of new copies sharply declines after the release week (see Figure 1). In our data set, the median percentage of new game copies sold in the release week (relative to the total annual sales for the first year) is 54%, and the median percentage of new game copies sold within the first month (4 weeks) after release is 82%. Thus, the sales of new copies is highly concentrated within the first month in Japan. In addition to the data from the primary market, we collected weekly aggregate trading volumes (both buying and selling) and the associated weekly average retail prices and resale values in the used market by game title. These are collected from the Annual Video Game Industry Report published by Media Create Co., Ltd. The average number of weeks observed across games is 36 weeks. According to an annual industry report by Enterbrain, Inc., about 80% of used video game trading occur at retailers during our sample period. Thus we do not consider the possibility such as online auctions for buying and selling used copies.

We also collected various video game characteristics from Weekly Famitsu Magazine, including average critic and user rating, story-based game dummy,[8] and multi-player game dummy. Finally, the size of market for a video game is measured by the installed base of the platform in which the video game was released. The platforms of the 20 games include three consoles (PlayStation 2, PlayStation 3, Nintendo GameCube). We collected the weekly sales of all three consoles above from their release week to calculate the cumulative sales.

Table 1 shows the summary statistics. The average price of used copies across games and time is about two-thirds of the price of a new copy. Also, it can be seen that the profit-margin for used game retailers is

[8]Story-based games (e.g., role-playing games) have an ending of the story, while non story-based games (e.g., sports games) have no clear definition for ending.

large. The average relative size of the used game market to the new game market, defined as the ratio of cumulative sales of used copies to that of new copies at the end of new-copy sales sample period, is 0.11 with a maximum of 0.29 and a minimum of 0.02. These numbers might appear small, but this is mainly because of the length of our sample. We observe the sales of new copies on average for 19 weeks, and the sales of used copies typically lasts longer than that of new copies beyond 19 weeks. If we had a longer time-series data set, we would expect the average ratio to be significantly larger than 0.11. However, our data show that we have variation in the ratio. This variation is important in validating our counterfactual experiments where we eliminate the used game market. Our data set contains video games with a relatively small used game market. Thus our parameters will not be calibrated by data variation that are completely far from the counterfactual experiment situation.

3.3 Some empirical regularities

In this section, we will briefly describe three empirical regularities that have not been documented in the previous studies on used goods markets: (i) quantities demanded and supplied for used goods over time, (ii) inventory level of used goods over time, and (iii) the price of used goods and the resale value.

Figure 2 plots the average quantities demanded and supplied for used copies as well as the average inventory level of used copies over 15 weeks. The inventory level of used copies in week t for a game is defined as the difference between the cumulative quantity of used copies supplied by consumers up to week $t-1$ and the cumulative quantity of used copies purchased by consumers up to week $t-1$. First, both quantities demanded and supplied for used copies sharply increase in the first few weeks after the opening of used game markets (second week after release), reach their peaks, and gradually decrease afterwards. The initial increase is probably because it takes a few weeks for owners of a game to become satiated with their games. As the quantity supplied for used copies by owners increases, the sales of used copies also follows. Second, on average, the inventory level of used copies carried by retailers grows in the first 15 weeks. About half of the games in our data set exhibits a decline after some point during the sample period. It is clear that in the Japanese video game market, the used market does not clear in every period. Therefore, unlike

previous studies which assume the used goods market clearing in each period, we will make use of this excess supply information when estimating the dynamic demand model. Also note that although retailers accumulate used-copy inventory, they still make a positive profit due to a high profit-margin from used copies.

Figure 3 plots the average price and resale value of used goods over the first 15 weeks. They both gradually decrease over time, and the resale value decreases slightly faster. This is consistent with our assumptions that consumption values to both potential buyers and owners depreciate over time, and their depreciation rates could be different.

4 Model

In this section, we describe the dynamic discrete choice model of consumer buying and selling decisions for video games.[9] We assume that consumers make buying and selling decisions separately for each game denoted by g.[10] Let i indexes consumers, and t indexes time. To capture consumer heterogeneity, we allow discrete consumer types. At the beginning of the initial period $t = 1$ (i.e., the period in which the new game is released), no consumers own game g and used games are not available yet. Thus, consumers' decision problem is to decide whether to buy a new good or not to buy. In period $t > 1$, consumers who have not bought the game up to $t-1$ observe the prices of new and used copies, the resale value and inventory level of used copies at retailers, and decide whether to buy a new or used good, or not to buy anything. Let $j = 0, 1, 2$ denote no purchase option, new good purchase, and used good purchase, respectively. If consumers have already bought game g prior to time t and have not sold it yet, then they decide whether or not to keep the game given the resale value in period t. Let $k = 0, 1$ denote keeping and selling options, respectively. If consumers sell their game, they exit the market. Since video games will eventually be outdated, we assume a terminal period $t = T$ after which consumers can neither buy nor sell.

[9] Note that the model can be applied to other markets with similar features as well.

[10] We do not explicitly model the competition among different games since our focus in this paper is the competition between new and used copies of the same game. We control for the impact of the availability of other games on the purchase decision of game g by including the cumulative number of other newly introduced games since game g's release. Note that Nair (2007) finds evidence that the substitutability between two different video games is very low in the US market, and thus does not model the competition among differen games.

The state space of the consumer decision model consists of the following variables: (1) price of new and used goods (p_1, p_2); (2) resale value (r); (3) inventory level of used copies at retailers (Y), which controls for the impact of the availability of used copies on consumer buying decisions; (4) time since release (t), which characterizes the single-period consumption value to potential buyers; (5) time since purchase (τ), which affects the single-period consumption value to owners; (6) unobserved shocks for the demand for new and used copies (ξ_1, ξ_2); (7) unobserved shocks for the supply of used copies (ξ_s); (8) cumulative number of newly introduced games since the release of the focal game (C). As we will describe later, (1), (6), and (8) appear only in the consumer buying decision problem, (5) and (7) appear only in the consumer selling decision problem, and (2)-(4) appear in both consumer buying and selling decision problems.

We will first describe the single-period utility functions for buying and selling decisions, and then move to the description of the value functions.

4.1 Single-period utility functions

In each period, consumers derive a value from owning game g (consumption values). We assume that once consumers buy a game, they will derive the same consumption value regardless of whether it is a new copy or a used copy. This assumption is to capture the idea that the goods considered in this research have no physical depreciation. However, the decision to buy a used copy may be influenced by other factors such as the availability of used copies at retailers, psychological costs for using pre-owned goods,[11] etc. Let $v^g(t, \tau)$ be a consumer's single-period consumption value of owning game g at time t if he has owned game g for τ periods prior to time t. If he hasn't bought game g yet, he will receive $v^g(t, 0)$ if he buys it at time t. Note that we assume that all consumers share a common consumption value.

Suppose that a consumer has not bought game g up to time t. Consumer i's single-period utility for buying decisions at time t is given by:

$$u_{ijt}^g = \begin{cases} v^g(t,0) - \alpha p_{1t}^g + \xi_{1t}^g + \rho D_t^g + \epsilon_{i1t}^g & \text{if buying a new copy } (j=1) \\ v^g(t,0) - \alpha p_{2t}^g - l_Y(Y_t^g; \lambda_i) + \rho D_t^g + \xi_{2t}^g + \epsilon_{i2t}^g & \text{if buying a used copy } (j=2) \\ l_C(C_t^g) + \epsilon_{i0t}^g & \text{if no purchase } (j=0), \end{cases}$$

[11] Our assumption here is that once consumers overcome this psychological costs at the time of making a buying decision, then the consumption value they receive in subsequent periods is not affected by the psychological costs.

where p_{1t}^g (p_{2t}^g) are the prices of new (used) copies of game g at time t; ξ_{1t}^g (ξ_{2t}^g) is the unobserved demand shocks to new (used) copies; α is the price-sensitivity. As justified by the stylized facts discussed earlier, we assume that the price of new copies is constant over time, i.e., $p_{1t}^g = p_1^g$ for all t. We assume that ξ_{1t}^g (ξ_{2t}^g) is *i.i.d.* across time. We assume they are normally distributed with zero mean and the standard deviation σ_{ξ_j} for $j = 1, 2$; Y_t^g is the inventory level of used copies for game g at retailers at the beginning of period t. $l_Y(Y_t^g; \lambda_i)$ is the one-time transaction cost that consumers incur when buying a used good (search costs, psychological costs for pre-owned games, etc.). We specify $l_Y(Y_t^g; \lambda_i) = \lambda_{0i} + \lambda_1 \exp(-\lambda_2 Y_t^g)$ to capture the ideas that (i) search costs may depend on the availability of used copies by this term and (ii) consumers might be heterogeneous in psychological costs for pre-owned games. The heterogeneity in λ_{0i} is motivated by a consumer survey conducted by Enterbrain, Inc. which suggests that about 15% of consumers never intend to purchase a used copy.[12] This reduced-form specification implies that when no used copies are available at the beginning of a period, the cost is $\lambda_{0i} + \lambda_1$. As the availability of used copies increase to infinity, the cost approaches λ_{0i}. Thus, if λ_1 and λ_2 are positive, the cost decreases as Y_t^g increases; D_t^g is a vector of seasonal dummies and ρ captures the seasonal effects.[13] C_t^g is the cumulative number of newly introduced games at time t since the introduction of game g (including the games released in the same week as game g); $l_C(C_t^g)$ captures the competitive effect from other newly introduced games and is specified as $l_C(C_t^g) = \pi_0 + \pi_1 \ln(C_t^g)$.

We assume that idiosyncratic errors, ϵ_{ijt}^g, to be *i.i.d.* across consumers and time, but allow it to be correlated across options j. We model the correlation in a nested logit framework. Let $\epsilon_{ijt}^g = \zeta_{iht}^g + (1-\eta)\upsilon_{ijt}^g$ where h indexes nest and takes two possible values: $h = 1$ groups the buying options (i.e., buying a new or used copy), and $h = 0$ is the no purchase option. Thus, the consumer buying decision problem here is equivalent to a two-stage decision making where consumers first decide whether or not to buy, and if buying, then consumers choose a new or used copy. In this setup, the parameter $\eta \in [0, 1)$ measures the within-nest correlation.

[12] Famitsu Game Hakusho 2006.

[13] We include this variable only to capture the seasonal variation in sales and do not intend to study its impact on consumers' dynamic decision making. Thus, it is not included as a state variable.

Next, consider consumers' selling decisions. Suppose that a consumer has bought game g and kept it for τ periods prior to time t. Consumer i's single-period utility for selling decisions at time t is given by

$$w_{ikt}^{g}(\tau) = \begin{cases} \alpha r_t^g - \mu_i + \xi_{st}^g + e_{i1t}^g & \text{if selling to a retailer } (k=1) \\ v^g(t,\tau) + e_{i0t}^g & \text{if keeping the game } (k=0) \end{cases}$$

where r_t^g is the resale value of game g at time t; μ_i captures any additional cost of selling (transaction costs, endowment effects, etc.) and is allowed to depend on consumer type. This is again motivated by the same consumer survey by Enterbrain, Inc. which indicates that not all consumers sell their games; ξ_{st}^g is an *i.i.d.* unobserved shock to owners for selling decisions at time t.[14] We assume it is normally distributed with zero mean and the standard deviation, σ_{ξ_s}; e_{ikt}^g is an idiosyncratic error, and we assume it is *i.i.d.* extreme value distributed across consumers and time.

For the single-period consumption value, $v^g(t,\tau)$, we will assume the following evolution over time. In the release period, we set $v^g(1,0) = \gamma_g$, where γ_g is the game-specific constant. To capture the depreciation of potential buyers' consumption values due to the aging of a game (freshness-based depreciation), we allow $v^g(t,0)$ to decay as a function of t.[15] Specifically, we model the depreciation rate as: $v^g(t+1,0) = (1-\varphi(t))v^g(t,0)$, where $\varphi(t)$ is a function of time trend. Next, we capture the depreciation of owners' consumption values due to satiation by modeling the depreciation rate as a function of product characteristics and the duration of ownership: $v^g(t+1,\tau+1) = (1-\kappa(X_{g\tau}))v^g(t,\tau)$ where $\kappa(X_{g\tau})$ is a function of observed product characteristics and the duration of ownership. In Section 6, we will discuss the specific functional forms of $\varphi(t)$ and $\kappa(X_{g\tau})$ in our empirical application.

4.2 Value functions

Since the dynamic consumer selling decision is nested within the dynamic consumer buying decision through the expected future payoff, we start off by describing the dynamic consumer selling decision, and then describe the dynamic buying decision. To simplify the notation, we will drop g superscript and t subscript. Also, let $\xi_d = (\xi_1, \xi_2)$ be the unobserved demand shocks (as opposed to ξ_s, the unobserved shocks for selling

[14] The unobserved shock to owners may capture variations in the economic situation, sales of newly introduced games and their related products, etc.

[15] We do not allow for the appreciation of consumption values, given that the length of our sample is at most one year.

decisions), and β be the discount factor common across consumers.

Let $W_i(r, Y, \xi_s, t, \tau)$ be the value function of the consumer selling decision problem for consumer i. Note that other state variables (p_1, p_2, C, ξ_d) will not enter here. The inventory level, Y, is included since it could affect the distribution of the future resale value. The Bellman equation is given by

$$W_i(r, Y, \xi_s, t, \tau) = E_e \max_{k \in \{0,1\}} \{W_{ik}(r, Y, \xi_s, t, \tau) + e_{ikt}\},$$

where W_{ik}'s are consumer i's alternative-specific value functions given by

$$W_{ik}(r, Y, \xi_s, t, \tau) = \begin{cases} \alpha r - \mu_i + \xi_s & \text{if selling } (k = 1), \\ v(t, \tau) + \beta E[W_i(r', Y', \xi'_s, t + 1, \tau + 1)|(r, Y, \xi_s, t, \tau)] & \text{if keeping } (k = 0). \end{cases}$$

The expectation in $E[W_i(r', Y', \xi'_s, t + 1, \tau + 1)|(r, Y, t, \tau)]$ is taken with respect to the future resale value (r'), inventory level (Y'), and unobserved shock for selling decision (ξ'_s).

The probability of selling the game by consumer i at (r, Y, ξ_s, t, τ) is given by

$$\Pr(k = 1|r, Y, \xi_s, t, \tau; i) = \frac{\exp(W_{i1}(r, Y, \xi_s, t, \tau))}{\sum_{k'=0}^{1} \exp(W_{ik'}(r, Y, \xi_s, t, \tau))}.$$

Next, consider the dynamic consumer buying decision. Let $V_i(p_1, p_2, r, Y, C, \xi_d, t)$ be the value function for consumer i who has not bought the game prior to time t. The Bellman equation is given by

$$V_i(p_1, p_2, r, Y, C, \xi_d, t) = E_\epsilon \max_{j \in \{0,1,2\}} \{V_{ij}(p_1, p_2, r, Y, C, \xi_d, t) + \epsilon_{ijt}\},$$

where V_{ij}'s are consumer i's alternative-specific value functions given by

$$\begin{aligned} &V_{ij}(p_1, p_2, r, Y, C, \xi_d, t) \\ &\quad = \begin{cases} v(t, 0) - \alpha p_1 + \xi_1 + \beta E[W_i(r', Y', \xi'_s, t + 1, 1)|(r, Y, \xi_s, t, 0)] & \text{new copy } (j = 1), \\ v(t, 0) - \alpha p_2 + \xi_2 - l_Y(Y; \lambda_i) + \beta E[W_i(r', Y', \xi'_s, t + 1, 1)|(r, Y, \xi_s, t, 0)] & \text{used copy } (j = 2), \\ l_C(C) + \beta E[V_i(p'_1, p'_2, r', Y', C', \xi'_d, t + 1)|(p_1, p_2, r, Y, C, \xi_d, t)] & \text{no purchase } (j = 0). \end{cases} \end{aligned}$$

The expectation in $E[V_i(p'_1, p'_2, r', Y', C', \xi'_d, t + 1)|(p_1, p_2, r, Y, C, \xi_d, t)]$ is taken with respect to the future prices of new and used copies (p'_1, p'_2), resale value (r'), inventory level (Y'), cumulative number of competing games (C'), and the unobserved shocks for buying decisions (ξ'_d).

The choice probability by consumer i at $(p_1, p_2, r, Y, C, \xi_d, t)$ is given by

$$\Pr(j|p_1, p_2, r, Y, C, \xi_d, t; i) = \Pr(h = 1|p_1, p_2, r, Y, C, \xi_d, t; i) \cdot \Pr(j|h = 1, p_1, p_2, r, Y, C, \xi_d, t; i),$$

where

$$\Pr(h=1|p_1,p_2,r,Y,C,\xi_d,t;i) = \frac{\left[\sum_{j'=1}^{2}\exp\left(\frac{V_{ij'}}{1-\eta}\right)\right]^{1-\eta}}{\exp(V_{i0})+\left[\sum_{j'=1}^{2}\exp\left(\frac{V_{ij'}}{1-\eta}\right)\right]^{1-\eta}},$$

$$\Pr(j|h=1,p_1,p_2,r,Y,C,\xi_d,t;i) = \frac{\exp\left(\frac{V_{ij}}{1-\eta}\right)}{\sum_{j'=1}^{2}\exp\left(\frac{V_{ij'}}{1-\eta}\right)}.$$

Given a finite time horizon, the value functions for both buying and selling decisions can be computed by backward inductions from the terminal period $t = T$. We assume that at the terminal period, if a potential buyer buys or an owner keeps the game, then they will receive not only the single-period utility for the terminal period, but also the present discounted value of the future consumption values (but no selling option after the terminal period). In the empirical application, we set $T = 100$. The present discounted value of the future consumption values at the terminal period is computed based on another 100 more periods from the terminal period.

4.3 Aggregate sales

Let ψ_i be the population proportion of type-i consumers and $\sum_{i=1}^{I}\psi_i = 1$. In order to derive the aggregate demand for new and used copies, and aggregate volume of used copies sold to retailers by owners, we need to derive the evolution of the size of each consumer type. Let M_{it}^d be the size of type-i consumers who have not bought the video game. It evolves according to

$$M_{it+1}^d = M_{it}^d(1-\sum_{j=1}^{2}\Pr(j|p_1,p_2,r,Y,C,\xi_d,t;i)) + N_{it+1},$$

where N_{it+1} is the size of new type-i consumers who enter the market at time $t+1$. We assume that the proportion of new type-i consumers follows the population proportion, ψ_i.

Next, let $M_{it}^s(\tau)$ be the size of type-i consumers who have bought and owned the game for τ periods at time t. It evolves according to

$$M_{it+1}^s(\tau) = \begin{cases} M_{it}^d\sum_{j=1}^{2}\Pr(j|p_1,p_2,r,Y,\xi_d,t;i) & \text{if } \tau = 1, \\ M_{it}^s(\tau-1)\cdot\Pr(k=0|r,Y,\xi_s,t,\tau-1;i)) & \text{if } \tau > 1. \end{cases}$$

The aggregate observe sales is then

$$Q_j(p_1,p_2,r,Y,C,\xi_d,t) = \sum_{i=1}^{I} M_{it}^d\Pr(j|p_1,p_2,r,Y,C,\xi_d,t;i) + \varepsilon_{jt}, \quad (1)$$

where $j = 1$ is new copies and $j = 2$ is used copies, and ε_{jt} represents a measurement error. The aggregate observed quantity sold to retailers by consumers is given by

$$Q_s(r, Y, \xi_s, t) = \sum_{i=1}^{I} \sum_{\tau=1}^{t-1} M_{it}^s(\tau) \Pr(k = 1 | r, Y, \xi_s, t, \tau; i) + \varepsilon_{st}, \tag{2}$$

where ε_{st} represents a measurement error.

5 Estimation Strategy

The estimation of consumer preference parameters is carried out in two steps. In the first step, we recover the evolution processes of used game prices, resale values, inventory levels, and cumulative number of newly introduced games from the data.[16] These processes will then be used to form consumers' expectation about future price of used goods, resale value, inventory level, cumulative number of newly introduced games in the second step demand estimation (e.g., Hendel and Nevo 2006, Erdem et al. 2003). We model the processes of the price of used goods and the resale value to be a function of own lagged value, the lagged inventory level, and game characteristics, except for $t = 2$ where we assume that the initial price of used copies and resale value are a function of the price of new copies and game characteristics. We model the processes of the inventory level to be a function of its lagged value and game characteristics. Finally, we assume that the cumulative number of newly introduced games to be a function of its lagged value. The estimation results based on all 20 video games are reported in Tables 2 and 3.[17]

In the second step, we estimate the dynamic discrete choice model with a finite time horizon. Notice that the price of used goods and resale value may be correlated with unobserved shocks. We employ a new Bayesian MCMC algorithm for non-stationary DDP models proposed by Ishihara and Ching (2012), and combine it with the pseudo-policy function approach in Ching (2010a; 2010b) to control for the potential endogeneity problems.

[16] We assume that consumers expect the price of new copies to remain constant over time.

[17] We estimate the process for the cumulative number of newly introduced games separately for each of the 20 games. The results are not reported but available upon request.

5.1 Modified IJC algorithm

This section briefly explains the new Bayesian MCMC algorithm proposed by Ishihara and Ching (2012). Since it extends Imai et al. (2009b), we first discuss their key ideas. The original IJC algorithm uses the Metropolis-Hastings algorithm to draw a sequence of parameter vectors from their posterior distributions. During the MCMC iterations, instead of fully solving for the value functions at each draw of parameter vectors as proposed in the nested fixed point algorithm (Rust 1987), the IJC algorithm partially solves for the value functions at each draw of parameter vectors (at the minimum, apply the Bellman operator only once), stores those partially solved value functions (they call those value functions *pseudo*-value functions), and uses them to nonparametrically approximate the expected value functions at the current trial parameter vector.[18] Imai et al. (2009b) show that as the MCMC iterations and the number of past pseudo-value functions for approximating the expected value functions increase, pseudo-value functions will converge to the true value functions, and the posterior parameter draws based on the pseudo-value functions will also converge to the true posterior distribution.[19]

One issue in applying the IJC algorithm in our framework is that the dynamic programming (DP) problem is non-stationary. However, the original IJC algorithm in Imai et al. (2009a) applies to stationary DP problems. Their algorithm is essentially an extension of the contraction mapping procedure for solving the stationary DP problem. In general, the computational advantage of using IJC to estimate a finite horizon DP problem may be limited. However, if the dynamic model has multiple continuous state variables, Ishihara and Ching (2012) argue that IJC's idea for estimating DP models with continuous state variables (see Section 3.2 of Imai et al. 2009a) can be extended to help reduce the computational burden of integrating the continuation payoffs out even for finite-horizon non-stationary dynamic models. The main idea behind their algorithm for continuous state variables is to compute pseudo-value functions at one randomly drawn state in each iteration and store them. The set of past pseudo-value functions used in approximating the

[18]Ching et al. (2011) provide a step-by-step guide for implementing the IJC algorithm.

[19]Imai et al. (2009b) shows that their algorithm converges under Gaussian kernel. Norets (2009) further shows that the IJC algorithm converges under nearest neighborhood kernel.

expected future values will then be evaluated at different state points. Thus, one can simply adjust the weight given to each of the past pseudo-value function by the transition density from the current state to the state at which the past pseudo-value function is evaluated. In the new algorithm, the main differences from the original IJC algorithm are (1) pseudo-value functions are computed and stored for each time period, (2) in each MCMC iteration and in each time period, pseudo-value functions are computed only at one randomly drawn vector of continuous state variables, and (3) expected future values at time t are approximated using the set of pseudo-value functions at time $t+1$. Unlike conventional approaches, in which value functions need to be computed at all or a subset of pre-determined grid points in all periods (e.g., Rust 1997), the new algorithm computes pseudo-value functions at only one randomly drawn state point in each period and the integration of the continuation value with respect to continuous state variables can simply be done by the weighted average of past pseudo-value functions. Thus, it has the potential to reduce the computational burden. We describe the details of the estimation procedure in Appendix A.1.

5.2 Pseudo-policy function approach

To control for the potential endogeneity problems of the price and resale value of used games, we follow the pseudo-policy function (PPF) approach proposed by Ching (2010a; 2010b). To use this approach, we approximate the pricing policy functions as a polynomial of observed and unobserved state variables of the equilibrium model and jointly estimate it with demand model.[20] Compared with other existing approaches (e.g., Villas-Boas and Winer 1999, Park and Gupta 2009, Petrin and Train 2010), the PPF approach has the potential to capture the correlation between prices and unobserved shocks in a more flexible way. Thus, it does not impose an assumption on the joint distribution of the error in the pricing equation and unobserved shocks, which could be inconsistent with some of the supply-side structures. In our model, the state space of the equilibrium model includes unobserved shocks $(\xi_{1t}, \xi_{2t}, \xi_{st})$, consumption values $(v(t,\tau))$, inventory level (Y_t), cumulative number of newly introduced games (C_t), the size of potential buyers (M^d_{it}), and the size of owners for each each duration of ownership $(M^s_{it}(\tau))$. Furthermore, we include seasonal dummies for

[20]This approach can also be applied to control for the potential endogeneity of advertising/detailing (e.g., Ching and Ishihara 2010).

Golden Week (D_{1t}) and Christmas (D_{2t}).[21]

After experimenting several functional forms, instead of using high order polynomials, we decided to use the following specification for the price of used goods (for $t \geq 2$):

$$\begin{aligned}\ln p_{2t} \quad = \quad & \omega_{p0} + \omega_{p1} v(t,0) + \omega_{p2} \frac{1}{(t-1)} \sum_{\tau=1}^{t-1} v(t,\tau) + \omega_{p3}\xi_{1t} + \omega_{p4}\xi_{2t}) + \omega_{p5}\xi_{st} \\ & + \omega_{p6} \frac{1}{I} \sum_{i=1}^{I} M_{it}^{d} + \omega_{p7} \frac{1}{I(t-1)} \sum_{i=1}^{I} \sum_{\tau=1}^{t-1} M_{it}^{s}(\tau) + \omega_{p8} Y_t + \omega_{p9} C_t + \omega_{p10} D_{1t} + \omega_{p11} D_{2t} + \nu_{pt},\end{aligned} \tag{3}$$

where ν_{pt} is the prediction error. Also, the pseudo-policy function for the resale value is specified as (for $t \geq 2$):

$$\begin{aligned}\ln r_t \quad = \quad & \omega_{r0} + \omega_{r1} v(t,0) + \omega_{r2} \frac{1}{(t-1)} \sum_{\tau=1}^{t-1} v(t,\tau) + \omega_{r3}\xi_{1t} + \omega_{r4}\xi_{2t}) + \omega_{r5}\xi_{st} \\ & + \omega_{r6} \frac{1}{I} \sum_{i=1}^{I} M_{it}^{d} + \omega_{p7} \frac{1}{I(t-1)} \sum_{i=1}^{I} \sum_{\tau=1}^{t-1} M_{it}^{s}(\tau) + \omega_{r8} Y_t + \omega_{r9} C_t + \omega_{r10} D_{1t} + \omega_{r11} D_{2t} + \nu_{rt},\end{aligned} \tag{4}$$

where ν_{rt} is the prediction error.

Note that Y_t in the two equations play a role of an instrument. Y_t is the inventory level of used games at retailers at the beginning of period t. Thus, it is uncorrelated with ξ_t's. However, it is plausible that the price and resale value of used games at time t depend on the inventory level, Y_t. Similarly, C_t could play a role of an instrument. C_t is the cumulative number of newly introduced games since the release of the focal game. The introduction timing of games is rarely postponed once the release week has been reached. This is because copies of games are already manufactured before the release week. Thus, we expect C_t to be uncorrelated with ξ_t's. However, C_t would likely affect p_{2t} and r_t. On one hand, C_t could influence p_{2t} because it affects the demand for used copies of the focal game. On the other hand, C_t could affect r_t because owners of the focal game may be attracted to newly introduced games and choose to sell the focal game, which affect the supply of used copies of the focal game. Inclusion of these two exogenously observed state variables should help reduce the reliance of using functional form restrictions for identification.

Assuming measurement errors in the sales of new and used copies as well as the quantities sold by consumers to retailers, and prediction errors in the pseudo-pricing policy functions, we derive the joint like-

[21] Golden Week in Japan refers to a period in late April and early May that contains multiple public holidays.

lihood of the demand-side model and the pseudo-policy functions. In the modified IJC algorithm described in the previous subsection, the joint likelihood will be used to compute the acceptance probabilities in the Metropolis-Hastings algorithm. Appendix A.2 describes the construction of the likelihood function.

5.3 Remark

Note that if consumers observe all the state variables and understand how the equilibrium prices are generated, then we could gain efficiency by using the pseudo-policy functions to form the consumers' future price expectations as well. We decided not to take this approach for the following two reasons. First, it is unclear if consumers are aware of all the state variables of the equilibrium model. In particular, it is difficult for consumers to obtain information about the size of potential buyers and owners for each duration of ownership at the time consumers make a buying or selling decision. Under such a situation, consumers may use a simple Markov process to forecast future prices and resale value (Erdem et al. 2003, Février and Wilner 2009). Hendel and Nevo (2006) make a similar assumption and argue that a simple Markov process might be a reasonable assumption about consumers' memory and formation of expectations. Second, if one uses the pseudo-policy functions to form the consumer price expectation, one needs to specify the state space for the dynamic consumer buying and selling decision model to be that of the equilibrium model (Nair 2007). Given that there are many more continuous state variables in the equilibrium model, including them will increase the computational burden of estimating the dynamic consumer buying and selling decision model dramatically.

5.4 Identification

In this section, we provide an informal discussion of identification for our proposed model. First, consider the price sensitivity (α), the impact of used-copy inventory on the costs for buying a used copy (λ_1, λ_2), and the competitive effect (π_1). These are identified by variation in the sales of new and used copies due to the variation in the price of used copies, used-copy inventory, and the number of competing games, respectively. The within-group correlation (η) is identified by the extent to which the conditional market share of new (or used) copies is correlated with the unconditional market share of new (or used) copies (Berry 1994).

The game-specific constant (γ^g), which determines the initial consumption value in the release week, is identified partly by differences in the initial sales of new copies across games, and partly by differences in the mean-level sales of new and used copies across games and over time. This is because once the freshness-based depreciation rate (φ's), which are common across games, is identified by the average declining rate of the sales of new and used copies across games and over time, the game-specific constant will be identified by matching not only the initial predicted sales to the observed sales, but also matching the predicted sales to the observed sales in the subsequent weeks. Note that the constant term in the outside option (π_0) is identified in our model. This is because (1) the term is constant over time and common across games, and (2) the consumption value changes over time parametrically. To illustrate the intuition behind it, consider a simple static two-period model of buy/no buy, with the utility functions, $u^g_{buy,t} = (1-\varphi)^{t-1}\gamma^g$ and $u^g_{not\ buy,t} = \pi_0$, where γ^g is the game-specific constant for game g, φ is the freshness-based depreciation common across games, and π_0 is the constant in the outside option utility function. In each period, the relative market share of buying to outside option allows us to back out $V^g_1 \equiv \gamma^g - \pi_0$ ($t = 1$) and $V^g_2 \equiv (1-\varphi)\gamma^g - \pi_0$ ($t = 2$) for all g. Rewriting them gives us $V^g_2 = (1-\varphi)V^g_1 - \varphi\pi_0$. As long as we have variation in V^g_1 and V^g_2, we can identify φ by the extent to which V^g_1 and V^g_2 are correlated, and thus π_0 can be backed out. Given the game-specific constant, the satiation-based depreciation rate (δ's) is identified by the volume sold by consumers to retailers over time across games (i.e., different game characteristics).

The discount factor (β) is identified by the extent to which the sales of new and used copies are affected by the future resale value (r^g_t) through consumers' expectation processes. Since the resale value does not affect the current utility functions for buying decisions, the extent to which the future resale value affects the current sales of new and used copies determines the importance of the continuation payoffs given the values of other parameters.

Finally, consider the unobserved heterogeneity in the costs for buying and selling a used copy (λ_{0i}, μ_i) and the proportion of each consumer type (ψ_i). First, note that a model without unobserved heterogeneity in λ_{0i} implies that variation in the relative market share of used to new copies across games is determined

only by the difference between new-copy price and used-copy price, $(p_1^g - p_{2t}^g)$, and used-copy inventory, (Y_t^g). For example, consider the relative market share at $t = 2$. Since $Y_t^g = 0$ for all g at $t = 2$, variation in the relative market share at $t = 2$ across games has to be explained solely by the price differential, $(p_1^g - p_{22}^g)$.[22] Figure 4 plots the data on the relative market share and the price differential at $t = 2$ for all 20 games in our sample. It shows significant variation in the relative market share for a given level of the price differential, thus the price differential alone is not sufficient in explaining the variation in relative market shares. This variation helps us identify unobserved heterogeneity in λ_{0i} and the proportion (ψ_i). To illustrate this point, suppose that there are two types with $\lambda_{01} < \lambda_{02}$. Then, we know that at $t = 1$, type-2 consumers are more likely to buy a new copy than type-1 consumers because they have a larger cost for buying a used copy and thus have less incentive to delay purchase today and buy a used copy in the future. Now, consider two games $(g = 1, 2)$ with a similar price differential between new and used copies at $t = 2$, and game 1 has a higher relative market share of used to new copies at $t = 2$ than game 2. Since the relative choice probability of used to new copies for a given consumer type is the same for both games, the difference in the observed relative market share at $t = 2$ will identify the relative size of consumer types at $t = 2$, which in turn identify the population proportion of each consumer type (ψ_i) and the heterogeneity in the cost for buying a used copy (λ_{0i}). Once ψ_i and λ_{0i} are identified, μ_i will be identified by both time-series and cross-sectional variation in the volume of used copies sold by consumers to retailers.

6 Results

In estimating the dynamic model, we set the terminal period to be 100 (about two years). Also, we allow for two types of consumers who differ in their costs for buying and selling at used goods retailers (i.e., λ_{0i} and μ_i). We use the following functional form for the freshness-based depreciation: $\varphi(t) = \frac{\exp(\phi_1 \mathbb{I}(t=1) + (\phi_2 + \phi_3 \ln(t-1))\mathbb{I}(t>1))}{1+\exp(\phi_1 \mathbb{I}(t=1) + (\phi_2 + \phi_3 \ln(t-1))\mathbb{I}(t>1))}$; $\mathbb{I}(\cdot)$ is an indicator function. Note that we treat the depreciation from the first period to the second period differently from the rest of the periods. This is motivated by the obser-

[22]Here we use data at $t = 2$ to make the argument cleaner. However, the argument is valid even for data at $t > 2$ where we have additional variation (inventory level) to explain the relative market share. We find that even with the additional variation in inventory level, a model without unobserved heterogeneity cannot explain the relative market share well.

vations that the decline in the sales of new copies from the release week to the second week is much larger than that in the rest of the weeks in the Japanese video game market. For the satiation-based depreciation rate, we use the following functional form: $\kappa(X_{g\tau}) = \frac{\exp(X'_{g\tau}\delta)}{1+\exp(X'_{g\tau}\delta)}$ where $X_{g\tau}$ includes observed product characteristics of game g (dummies for story-based games and multi-player games, and average critic and user ratings) and the duration of ownership (τ).

6.1 Parameter Estimates

The estimation results are reported in Table 4. We estimate two versions; in one version (demand-only model), we do not control for the potential price endogeneity using the pseudo-policy function approach. In another version (full model), we control for the price endogeneity. We find that the estimates are qualitatively similar in the two versions. Below, we will mainly discuss the estimates of the full model.

We find that most of the parameter estimates are statistically significant and have the expected signs. The estimated discount factor is 0.902. Recall that the unit of periods in our application is a week. Our estimate is much lower than the discount factor translated from a weekly interest rate ($\simeq$ 0.999), which is typically assumed when a dynamic model does not have any exclusion restriction to help identify the discount factor. However, our result is consistent with previous studies in experimental/behavioral economics which also find that the discount factor is lower than the interest rate (e.g., Frederick et al. 2002) as well as recent work by Yao et al. (2011) who also find from a field data that the weekly discount factor is about 0.9. Price-sensitivity (α) is positive because it enters the utility function as a negative term. The magnitude is small mainly because price are in JPY and 1 JPY is about 0.01 US dollar.

As for the costs for buying a used copy (λ), recall that we use the following functional form: $l_Y(Y_t^g; \lambda_i) = \lambda_{0i} + \lambda_1 \exp(-\lambda_2 Y_t^g)$. Positive signs of λ_1 and λ_2 indicate that costs for buying a used copy diminish as the inventory level rises. This is intuitive as the availability of used copies increases, consumers' search costs may decrease. In particular, the cost for buying a used copy in a monetary term falls in [-1,267, 6,742] in JPY for type-1 consumers and [5,068, 13,077] in JPY for type-2 consumers, where the estimated proportion of type-1 consumers is 0.725. The minimum number represents the cost when there are an infinite number of

used copies available at retailers, and the maximum represents the cost when no used copies are available at retailers at the beginning of a period. These numbers suggest that type-1 consumers are the main purchaser of used copies. Indeed, as we will show below, type-2 consumers' cost is so high that they hardly purchase used copies. Note that a negative minimum cost for type-1 consumers (-JPY 1,267) is partly because of our reduced-form linear specification to capture the impact of used-good inventory on the utility for buying a used copy. We also find a significant difference in the cost for selling between the two types. Type-1 consumers' cost is about three times larger than that of type-2 consumers, suggesting that type-2 consumers are the main supplier of used copies.

We find that both Golden Week and Christmas seasons have a positive impact on sales, but the magnitude is larger for a Christmas season. Finally, the parameter for the competitive effect from other games (π_1) is positive, suggesting that the increasing number of new game introduction may reduce the size of potential buyers who still consider buying the focal game.

Parameters for the freshness-based depreciation rate include two intercepts (ϕ_1, ϕ_2) and a time effect (ϕ_3). The estimated depreciation rate from the first to the second week (captured via ϕ_1) is about 22%, and that from the second to third week (captured via ϕ_2) is 10%. A negative and large estimate of the time effect (ϕ_3) indicates that the depreciation rate becomes close to zero after the third week. These results are consistent with the observed pattern of the sales of new copies, which declines quickly during the first few weeks after release.

For the depreciation rate of owners' consumption values due to satiation, we include the following product characteristics in $X_{g\tau}$: an intercept, story-based game dummy, multi-player game dummy, average critic and user rating, and duration of ownership. A positive coefficient of a variable implies that the variable will increase the depreciation rate. Our estimates suggest that on average, multi-player games and games with a higher critic rating exhibit a higher depreciation rate. On the other hand, story-based games, and games with a higher user rating exhibit a lower depreciation rate. Depending on product characteristics, the weekly depreciation rate at $\tau = 1$ ranges from 76% to 90%. Finally, the coefficient for the duration of ownership

suggests that the per-period depreciation rates become lower as consumers keep the game longer.

The parameters for pseudo-policy functions are reported in Table 5. In particular, we find that both unobserved demand shocks to new and used copies have a small and non-significant impact on both price and resale value, but unobserved shocks to selling have a positive and significant impact on both price and resale value of used copies. Note that unobserved shocks to selling affects only the selling decision and resale values enter the utility function for selling positively. Thus, if resale values are positively correlated with unobserved shocks to selling, we expect that the price coefficient would be biased upwards (i.e., larger negative) if we did not control for the endogeneity problem. This upward bias is what we find from the comparison of the price coefficients between the full and demand-only models. However, the difference is very small (2.87e-04 vs. 3.02e-04), suggesting that the price endogeneity problem does not appear to be too important in the Japanese video game market.

6.2 Goodness of fit

Our estimated dynamic model provides a good fit to the data. To show the goodness-of-fit, we simulate the sales of new and used copies as well as the volume sold to retailers by consumers by drawing 1,000 sets of parameters (including unobserved shocks) from the posterior distribution, computing the predicted quantities, and averaging them over 1,000 draws. Figures 5-7 show the fit of (1) new-copy sales, (2) used-copy sales, (3) volume sold to retailers by consumers, respectively, for all 20 games. In general, our dynamic model is able to fit the data quite well even with two types of consumers. Similarly, we also simulate the predicted price and resale value of used copies by the pseudo-pricing policy functions. Figures 8-9 show the fit of (1) used-copy price and (2) resale value, respectively, for all 20 games. The pseudo-pricing policy functions specified in Equations (3) and (4) are able to capture the data trend quite well.

6.3 Roles of Heterogeneity in Transaction Costs

The above simulation on the predicted quantities also reveals that the two types of consumers exhibit quite different dynamic behaviors. In Table 6, we show the proportion of (1) new-copy sales, (2) used-copy sales, (3) volume sold to retailers by consumers, by consumer type, averaged across parameter draws and 20 games.

Recall that the population proportion of type-1 consumers is 0.725 and that each type differs in terms of the costs for buying and selling used copies. Table 6 shows that in the release week, about 61% of new-copy sales are generated by type-1 consumers. This number is lower than the population proportion of type-1 consumers (72.5%) mainly due to two reasons. First, type-1 consumers have a lower cost for buying a used copy. Thus, they have more incentives to wait until used copies become available in later weeks than type-2 consumers. Second, type-2 consumers have a lower cost for selling to retailers, implying that they have more incentives to purchase earlier so that they can sell the game at a higher resale value. Both elements imply larger incentives for type-2 consumers to purchase a new copy in the release week. After the release week, the proportion of new-copy sales by type-1 consumers decline over. This is partly because some of type-1 consumers purchase used copies.

Table 6 also shows the breakdown of used-copy sales and the volume sold to retailers by consumers. As mentioned earlier, more than 90% of used-copy sales are generated by type-1 consumers and more than 99% of used copies are supplied by type-2 consumers. These findings illustrate that our model and the data pick up two important segments of consumers that are believed to exist in the market: type-1 segment captures consumers who are willing to buy a used copy and do not bother selling at all, and type-2 segment captures consumers who purchase a new copy and consider selling. It might first appear that these two segments can also be characterized through the heterogeneity in price-sensitivity. However, this is not possible because price-sensitive consumers should be the main purchaser of used copies (type-1) but price-sensitive consumers are also likely to sell (type-2).

Finally, the last two columns of Table 6 show the relative market share of used to new copies. It reveals that the relative market share increases for both types. By the fourth week, type-1 consumers buy more used copies than new copies (the relative market share is greater than one). Since type-2 consumers have a high cost for buying a used copy, their relative market share remains well below one even by the fifteenth week.

6.4 Elasticities

This section explores elasticities based on our estimates of the full model. We report elasticities mainly for two reasons. First, a conventional way of examining the potential competition from used goods in a reduced-form framework is to examine the cross-price elasticity of demand for new goods with respect to the price of used goods (e.g. Ghose et al. 2006). Second, in addition to the price elasticities of demand, our dynamic model is able to quantify (i) the inventory elasticities of demand, and (ii) the price elasticities of used-copy supply, i.e., volume sold to retailers by consumers. Previous studies on the interaction between new and used goods markets could not quantify these two types of elasticities because they do not observe the inventory of used copies, quantities supplied of used copies and associated resale values. Our results for these elasticities might shed some lights on retailers' strategies on the inventory management in terms of its impact on used-copy sales and procuring used copies from consumers.

Table 7 shows seven types of elasticities labeled by E.1-E.7. E.1-E.4 examine the price elasticities of demand. E.1 and E.2 investigate the percentage change in the sales of new and used copies in week t in response to a 1% change in new-copy price in week t. E.1 reveals that the average own-price elasticity of demand for new copies across games starts off at -1.99 and increases in absolute term over time. The magnitude and the trend are both consistent with Nair (2007). In our model, there is no heterogeneity in price-sensitivity. What drives the increasing trend is the constant price of new copies and a shrinking demand caused by (1) a shrinking size of potential buyers, (2) a decline in potential buyers' consumption value due to the freshness-based depreciation, and (3) an increase in the utility of choosing the outside option due to an increase in the number of competing video games. E.2 presents the cross-price elasticity of used-copy demand and shows a declining trend. The initial sharp decline is partly because used-copy demand expands in the first few weeks due to more available used copies, without much declines in used-copy price. As time goes on, the outside option becomes more attractive due to more new games, causing the cross-price elasticity to be declining over time.

Similarly, E.3 and E.4 show the percentage change in the sales of new and used copies in response to a 1%

change in used-copy price. E.4 is the own-price elasticity of used-copy demand. It declines in absolute term due to a similar reason for the above cross-price elasticity of used-copy demand. E.3 shows the cross-price elasticity of new-copy demand. It initially increases due to the expansion of used-copy demand, but slowly decreases after week 6 because the outside option becomes more attractive. Overall, the cross-elasticity remains small: the sales of new copies increases on average only by 0.24%.

Based on E.3 and E.4, one interesting number to examine is, among those who switch away from used copies due to a 1% increase in used-copy price, how many percent of them actually switch to new copies rather than to no purchase. If most consumers switch to new copies, it implies that the substitutability between new and used copies is high and the elimination of used goods markets might significantly increase the sales of new copies. In Table 8, we show the proportion of switchers who buy new copies and its breakdown by consumer type over time. In the second week where used copies just become available, about 33% of those who switch away from used copies buy new copies and the rest of them switch to no purchase. As time goes on, the proportion diminishes to less than 10%. This is partly because the outside option becomes more attractive over time through an increase in the number of newly introduced competing games. However, to further understand the declining pattern, we need to examine the extent of the incentive to delay purchase by consumer type.

The breakdown by consumer type shows that type-2 consumers are more likely to switch to new copies than type-1 consumers are. For example, in the second week, about 40% of type-2 consumers who switch away from used copies buy new copies while about 33% of type-1 consumers who switch away from used copies buy new copies. The difference becomes larger as the number of weeks in release increases. Note that type-1 consumers have a larger incentive to delay purchase than type-2 consumers because type-1 consumers have a low cost for buying a used copy. Especially, in the first few weeks after release, the availability of used copies increases quickly and this increase will raise the continuation payoff from no purchase today. This pattern for type-1 consumers is reflected in the larger drops of the proportion in the first few weeks (from 0.325 to 0.187, from 0.187 to 0.121, in the first few weeks): due to the increase in the continuation

payoff, type-1 consumers who switch away from used copies are more likely to switch to no purchase option than to new copies. However, the availability of used copies has a negligible impact on type-2 consumers' continuation payoff from no purchase today, because they will still have a high cost for buying a used copy even when the availability of used copies becomes high. Moreover, type-2 consumers' continuation payoff from buying today is higher than that for type-1 because type-2 consumers have a lower cost for selling. These two effects both lead to a higher likelihood of type-2 consumers switching to new copies when they switch away from used copies due to an increase in used-copy price. However, since only less than 10% of used-copy sales are generated by type-2 consumers, type-1 consumers are mainly responsible for determining the overall proportion of switches to new copies.

In sum, the results in E.3 and E.4 of Table 7 and Table 8 imply that while the cross-price elasticity of new-copy demand increases over time, the proportion of switches from used to new copies declines. However, these results on the substitutability are based on a change in used-copy price. To examine the impact of the existence of used goods markets fully, we need to use the structural estimates and conduct a policy experiment where we shut down the used game market. For example, in the above examination, type-1 consumers who switch from used copies to no purchase might simply be delaying a used-copy purchase. If the used game market shuts down completely, more type-1 consumers who used to buy used copies might switch to new copies. We will conduct this policy experiment in the next subsection.

Before moving to the policy experiment, we discuss the other two types of elasticities: inventory elasticities of demand and price elasticities of used-copy supply. We re-emphasize that to our knowledge, this paper is the first to quantify these elasticities. While we do not study retailers' decision problem in this paper, these elasticities might provide some implications for their inventory management problem. For example, an important factor that determines the inventory level of used copies is the resale value. While a higher resale value will lower retailers' profit-margin, it will induce more consumers to sell their games and increase the inventory level of used copies. This in turn may increase the sales of used copies more than enough to cover the loss due to a high resale value. Therefore, it is of retailers' interest to examine these two types of

elasticities.

E.5 and E.6 of Table 7 show the inventory elasticities of demand, i.e., the percentage change in the sales of new and used copies due to a 1% change in the inventory of used copies. The elasticities in both E.5 and E.6 are declining over time, and this is because of the diminishing return in the cost for buying a used copy. In the first few weeks where not many used copies are available, a 1% increase in the inventory has a large effect in reducing the cost for buying a used copy. However, the effect diminishes as more inventory is accumulated. Finally, E.7 examines the elasticity of used-copy volume supplied to retailers by consumers. Note that due to a high cost for selling, type-1 consumers hardly sell. Moreover, most of type-2 consumers' new-copy purchase occur in the earlier weeks. Thus, as more type-2 consumers sell, the proportion of type-2 owners shrinks, which lowers the elasticity. This is reflected in the declining trend of the elasticity.

6.5 Elimination of Used Game Markets

Video game publishers often claim that the existence of used game markets lowers the sales of new games. The claim is often based on the conjecture that if there were no used game market, most of the consumers who used to buy a used copy would switch to a new copy. In the previous section, we examine this conjecture and find that some of type-1 consumers may indeed switch to new games but the proportion declines quickly over time. Also, the role of used game markets for consumers is not to simply offer a cheaper alternative. It also serves as a place for them to sell games that they no longer want to keep. If used game markets shut down, it is possible that consumers demand even less new copies because the future value from buying a new copy today could be lowered due to the lack of selling opportunities. The analysis on the elasticity cannot take this important factor into account. However, after estimating the dynamic structural model, we are able to quantify the overall impact of eliminating used game markets on new-copy profits.

To examine the impact of eliminating used game markets, we simulate the model by setting the parameters that capture the transition costs for buying and selling used copies (λ_{0i} and μ_i) to be large so that used game markets shut down in effect (we set $\lambda_{0i} = \mu_i = 1,000$ for $i = 1, 2$). In the first experiment, we keep the prices of new copies at the observed values even after the elimination of used game markets. In

general, the optimal price of new copies in the absence of used game markets could be very different from the observed prices in the data. Thus, in the second experiment, we compute the optimal flat-prices of new copies when there is not used game market. In computing the optimal flat-price for each game, we set the marginal cost at JPY 1,000. We maintain the flat-pricing scheme because it has been an industry practice in Japan, and there might be some obstacles for the industry to shift to the price-skimming scheme.

Based on the simulation, we compute the statistics on the average percentage change in video game publishers' profits for a game due to the elimination of used game markets. Table 9 summarizes the results. We first discuss the scenario where we maintain the price of new copies at the currently observed prices. One average across games, the profits decline by 7%, and only one game exhibits an increase. This finding suggests that the future selling opportunity plays an important role in determining consumers' buying decisions. To further investigate this conjecture, we plot the average percentage change in new-copy sales across games for the first 15 weeks in Figure 10. It shows that the sale of new copies actually declines only in the first two weeks after release. After the second week, the percentage change in sales becomes positive. However, the sales of new copies is so high in the first few weeks that an increase in the sales after the second week does not compensate the loss in the sales in the first two weeks, resulting in a reduction in profits. The initial decline is due to the lowered future value from buying for type-2 consumers: when the resale value is still high, type-2 consumers' buying decision is critically influenced by the future selling opportunity. While the percentage changes for type-2 consumers are consistently negative in the first 15 weeks, the magnitude becomes smaller over time and this is due to a decline in resale value. The increasing trend of the percentage change for the aggregate sales is also motivated by the increasing trend in the percentage change for type-1 consumers. Recall that in Table 6 we saw that the relative market share of used to new copies for type-1 was increasing over time. After the elimination of used game markets, those of type-1 consumers who used to buy a used copy in later weeks switch to new copies. Also note that since type-1 consumers seldom sell, they are not affected by the eliminated selling opportunity. Thus the percentage change for type-1 consumers is consistently positive in the first 15 weeks.

To quantify the value of having the future selling opportunity, we compute the change in continuation payoffs from buying today due to the elimination of used game markets.[23] We further monetize the change by the estimated price-coefficient. Figure 11 shows the value of having the future selling opportunity for each type. As expected, the average value for type-1 is close to zero because most of them do not sell even when there is a used game market. For type-2, the average value of having the future selling opportunity is about JPY 2,000 in the release week, and slowly declines over time.

Recall that the results above are based on the observed flat-prices. If we allow video game publishers to adjust the price of new copies optimally after used game markets shut down, the results above could reverse. Thus, we compute the optimal flat-prices and then changes in profits under the optimal flat-prices. We find that the optimal flat-prices are on average 33% lower than the observed prices. On average across games, publishers' profits for a game increases by 11% due to the elimination of used game markets.

7 Conclusion

Motivated by the controversy on the roles of used goods markets, this paper assembles a novel data set from the Japanese new and used video game markets. The new data set contains not only the sales and prices of new and used goods, but also the resale value of used goods, the quantity of used goods sold to retailers by consumers, and the inventory of used goods at retailers. We believe that this new data set allows us to advance our knowledge about used goods markets significantly.

Based on the new data set, we develop a new empirical framework for studying consumers dynamic buying and selling decisions. One critical modeling contribution is that we model buying and selling decisions separately. This feature allows us to estimate the transactions costs for buying and selling used goods separately, and we find these transactions costs, together with consumer heterogeneity, generate an interesting dynamic pattern of buying and selling decisions. Furthermore, the feature allows us to estimate the depreciation rates of potential buyers' and owners' consumption values separately. Finally, since we observe the evolution of the price and resale value of used goods separately, we are able to utilize the variation of

[23] We thank Mitchell Lovett for suggesting this experiment.

the resale value and the sales to identify the discount rate.

Like any other research, this research also has limitations. We assume that consumers expectation process is well-approximated by AR(1) processes but retailers pricing policy depends potentially on all state variables. While both processes are well-fitted and thus imply very similar process, and this assumption is evidenced by Février and Wilner (2009), it might generate some biases if consumers fully understood how retailers set prices and resale values. Second, we do not explicitly model the competition among different game titles. In our model, the competition is captured in a reduced-form fashion through the cumulative number of newly introduced games since the release of the focal game. However, if the substitutability between two games is mainly determined through product characteristics, our approach suffers from a mis-specification bias. There are two reasons why we take the reduced-form approach. First, Nair (2007) finds in the US video game market data that the substitutability between different games is generally low. Second, our main objective is to examine the substitutability between new and used versions of the same game. Thus, we believe that the substitutability across different games is of second-order importance.

Our modeling approach in this paper does not utilize the supply-side competition between video game publishers and used game retailers. This approach is suitable for our research purpose for two reasons. First, it avoids potential biases due to the mis-specification of the supply-side competitive structure. Second, since our main interest is the impact of eliminating used game markets on video game publishers' profits, our policy experiments do not involve the competition between video game publishers and used game retailers. However, if the demand-side model in this paper is combined with a supply-side competition between video game publishers and used game retailers, we are able to explore many more interesting implications. For example, our estimated elasticities show that if the opening of used game markets were delayed by several weeks, it could avoid the cannibalization from used copies yet maintain the future selling opportunity for consumers. This remedy was actually proposed by video game publishers during the used video game lawsuit in Japan, but was not adopted. To examine such a policy change, we need to model the supply-side competition between video game publishers and used game retailers. Furthermore, it is both theoretically

and empirically important to investigate the optimality of flat-pricing strategy in the presence of used goods markets. A unique setting provided by the Japanese video game market allows us to conduct some test. Finally, in our data, we observe an increasing inventory level of used copies over time, and it is theoretically interesting to consider a model that rationalizes the behavior and test it using the data. Again, since previous theoretical and empirical studies consistently assume a market clearing condition on used goods markets, no such theory has been developed. We believe these are important research topics to explore in the future.

References

Berry, Steven, James Levinsohn, Ariel Pakes. 1995. Automobile Prices in Market Equilibrium. *Econometrica* **63**(4) 841–890.

Berry, Steven T. 1994. Estimating Discrete-Choice Models of Product Differentiation. *RAND Journal of Economics* **25**(2) 242–262.

Carranza, Juan Esteban. 2010. Product Innovation and Adoption in Market Equilibrium: The Case of Digital Cameras. *International Journal of Industrial Organization* **28**(6) 604–618.

Chen, Jiawei, Susanna Esteban, Matthew Shum. 2011. How Much Competition is a Secondary Market. Working Paper, Department of Economics, University of California, Irvine.

Chevalier, Judith, Austan Goolsbee. 2009. Are Durable Goods Consumers Forward Looking? Evidence from the College Textbook Market. *Quarterly Journal of Economics* **124**(4) 1853–1884.

Ching, Andrew, Susumu Imai, Masakazu Ishihara, Neelam Jain. 2011. A Practitioner's Guide to Bayesian Estimation of Discrete Choice Dynamic Programming Models. Forthcoming in *Quantitative Marketing and Economics*.

Ching, Andrew, Masakazu Ishihara. 2010. The Effects of Detailing on Prescribing Decisions under Quality Uncertainty. *Quantitative Marketing and Economics* **8**(2) 123–165.

Ching, Andrew T. 2010a. A Dynamic Oligopoly Structural Model for the Prescription Drug Market After Patent Expiration. *International Economic Review* **51**(4) 1175–1207.

Ching, Andrew T. 2010b. Consumer Learning and Heterogeneity: Dynamics of Demand for Prescription Drugs After Patent Expiration. *International Journal of Industrial Organization* **28**(6) 619–638.

Cho, Sungjin, John Rust. 2010. The Flat Rental Puzzle. *Review of Economic Studies* **77**(2) 560–594.

Desai, Preyas, Oded Koenigsberg, Devavrat Purohit. 2004. Strategic Decentralization and Channel Coordination. *Quantitative Marketing and Economics* **2**(1) 5–22.

Desai, Preyas, Devavrat Purohit. 1998. Leasing and Selling: Optimal Marketing Strategies for a Durable Goods Firm. *Management Science* **44**(11) S19–S34.

Desai, Preyas, Devavrat Purohit. 1999. Competition in Durable Goods Markets: The Strategic Consequences of Leasing and Selling. *Marketing Science* **18**(1) 42–58.

Engers, Maxim, Monica Hartmann, Steven Stern. 2009. Annual Miles Drive Used Car Prices. *Journal of Applied Econometrics* **24**(1) 1–33.

Erdem, Tulin, Susumu Imai, Michael P. Keane. 2003. Brand and Quantity Choice Dynamics Under Price Uncertainty. *Quantitative Marketing and Economics* **1**(1) 5–64.

Esteban, Susanna, Matthew Shum. 2007. Durable-Goods Oligopoly with Secondary Markets: The Case of Automobiles. *RAND Journal of Economics* **38**(2) 332–354.

Fevrier, Philippe, Lionel Wilner. 2009. Do Consumers Correctly Expect Price Reductions? Working paper, CREST.

Frederick, Shane, George Loewenstein, Ted O'Donoghue. 2002. Time Discounting and Time Preference: A Critical Review. *Journal of Economic Literature* **40**(2) 351–401.

Ghose, Anindya, Michael D. Smith, Rahul Telang. 2006. Internet Exchange for Used Books: An Empirical Analysis of Product Cannibalization and Welfare Impact. *Information System Research* **17**(1) 3–19.

Goettler, Ronald, Brett R. Gordon. 2011. Does AMD spur Intel to innovate more? Forthcoming in *Journal of Political Economy*.

Gordon, Brett. 2009. A Dynamic Model of Consumer Replacement Cycles in the PC Processor Industry. *Marketing Science* **28**(5) 846–867.

Gowrisankaran, Gautam, Marc Rysman. 2011. Dynamics of Consumer Demand for New Durable Goods. Working Paper, Department of Economics, Boston University.

Hendel, Igal, Aviv Nevo. 2006. Measuring the Implications of Sales and Consumer Inventory Behavior. *Econometrica* **74**(6) 1637–1673.

Hirayama, Taro. 2006. A Sequel to the Used Game Lawsuit. *Research Center for the Legal System of Intellectual Property Column* .

Imai, Susumu, Neelam Jain, Andrew Ching. 2009a. Bayesian Estimation of Dynamic Discrete Choice Models. *Econometrica* **77**(6) 1865–1899.

Imai, Susumu, Neelam Jain, Andrew Ching. 2009b. Supplement to 'Bayesian Estimation of Dynamic Discrete Choice Models'. *Econometrica Supplementary Material* **77**. Http://www.econometricsociety.org/ecta/Supmat/5658 proofs.pdf.

Ishihara, Masakazu, Andrew Ching. 2012. Bayesian Estimation of Non-Stationary Discrete Choice Dynamic Programming Models. Working Paper, Stern School of Business, New York University.

Keane, Michael P., Kenneth I. Wolpin. 1994. The Solution and Estimation of Discrete Choice Dynamic Programming Models by Simulation and Interpolation: Monte Carlo Evidence. *Review of Economics and Statistics* **76**(4) 648–672.

Liang, Meng-Yu. 1999. Does a Second-Hand Market Limit a Durable Goods Monopolist's Market Power? Working Paper, University of Wisconsin-Madison.

Magnac, Thierry, David Thesmar. 2002. Identifying Dynamic Discrete Decision Processes. *Econometrica* **70**(2) 801–816.

Melnikov, Oleg. 2000. Demand for Differentiated Products: The Case of the U.S. Computer Printer Market. Working Paper, Department of Economics, Yale University.

Nair, Harikesh. 2007. Intertemporal Price Discrimination with Forward-Looking Consumers: An Application to the US Market for Console Video-Games. *Quantitative Marketing and Economics* **5**(3) 239–292.

Norets, Andriy. 2009. Inference in Dynamic Discrete Choice Models with Serially Correlated Unobserved State Variables. *Econometrica* **77**(5) 1665–1682.

Park, Sungho, Sachin Gupta. 2009. Simulated Maximum Likelihood Estimator for the Random Coefficient Logit Model Using Aggregate Data. *Journal of Marketing Research* **46**(4) 531–542.

Petrin, Amil, Kenneth Train. 2010. A Control Function Approach to Endogeneity in Consumer Choice Models. *Journal of Marketing Research* **47**(1) 3–13.

Purohit, Devavrat. 1992. Exploring the Relationship between the Markets for New and Used Durable Goods: The Case of Automobiles. *Marketing Science* **11**(2) 154–167.

Rao, Raghunath Singh, Om Narasimhan, George John. 2009. Understanding the Role of Trade-Ins in Durable Goods Markets: Theory and Evidence. *Marketing Science* **28**(5) 950–967.

Rust, John. 1987. Optimal Replacement of GMC Bus Engines: An Empirical Model of Harold Zurcher. *Econometrica* **55**(5) 999–1033.

Rust, John. 1997. Using Randomization to Break the Curse of Dimensionality. *Econometrica* **65**(3) 487–516.

Schiraldi, Pasquale. 2011. Automobile Replacement: a Dynamic Structural Approach. *RAND Journal of Economics* **42**(2) 266–291.

Shiller, Benjamin. 2011. Digital Downloads and the Prohibition of Resale Markets for Information Goods. Working Paper, Department of Economics, Brandeis University.

Shulman, Jeffrey D., Anne T. Coughlan. 2007. Used Goods, Not Used Bads: Profitable Secondary Market Sales for a Durable Goods Channel. *Quantitative Marketing and Economics* **5**(2) 191–210.

Song, Inseong, Pradeep K. Chintagunta. 2003. A Micromodel of New Product Adoption with Heterogeneous and Forward-Looking Consumers: Application to the Digital Camera Category. *Quantitative Marketing and Economics* **1**(4) 371–407.

Tanaka, Migiwa. 2009. Deflation in Durable Goods Markets: an Empirical Model of the Tokyo Condominium Market. Working Paper, Department of Economics, Hong Kong University of Science and Technology.

Tedeschi, Bob. 2004. Online Battle of Low-Cost Books. *The New York Times* (July 12).

Villas-Boas, J. Miguel, Russell S. Winer. 1999. Endogeneity in Brand Choice Models. *Management Science* **45**(10) 1324–1338.

Yao, Song, Carl F. Mela, Jeongwen Chiang, Yuxin Chen. 2011. Determining Consumers' Discount Rates With Field Studies. Working Paper, Kellogg School of Management, Northwestern University.

Yin, Shuya, Saibal Ray, Haresh Gurnani, Animesh Animesh. 2010. Durable Products with Multiple Used Goods Markets: Product Upgrade and Retail Pricing Implications. *Marketing Science* **29**(3) 540–560.

Table 1: Summary statistics

	Average	S.D.	Min	Max
Price of new copies (in JPY)	7,613.1	629.1	7,140	9,240
Price of used copies (in JPY)	4,515.3	1,087.8	2,219	7,433
Resale value of used copies (in JPY)	2,828.1	1,182.7	1,036	6,547
Weekly sales of new copies	100,650.4	259,022.3	2,772	2,236,881
Weekly sales of used copies	7,184.6	6,478.8	458	62,734
Ratio of cumulative sales of used to new copies	0.107	0.063	0.021	0.286
Weekly quantity sold by consumers	8,121.4	8,436.8	1,012	55,830
Weekly inventory of used copies	31,022.5	28,347.7	0	129,462
Market size (installed base)	14,866,067.6	6,097,167.2	746,971	20,822,775
Weekly # new game introduction	7.01	4.02	0	17
Dummy for story-based games	0.700	0.470	0	1
Dummy for multi-player games	0.450	0.510	0	1
Critic rating (in 10-point scale)	8.99	0.656	7.75	10
User rating	56.4	9.20	41.6	67.4

Note: USD 1 ≈ JPY 100

Table 2: Regressions for the price of used games and resale value ($t = 2$)

	price of used copies		resale value	
variable	estimate	s.e.	estimate	s.e.
price of new copies	0.784*	0.083	0.839	0.129
dummy for story-based games	208.9	114.9	100.5	177.8
dummy for multi-player games	150.1	120.3	140.4	186.2
critic rating	127.0	85.5	132.4	132.3
user rating	-12.8	6.31	-13.4	9.76
constant	-548.2	732.2	-1899.6	1133.2
Adjusted R-squared	0.921		0.840	
# observations	20		20	

Note: * $p < 0.05$

Table 3: Regressions for the price of used games, resale value, and inventory level ($t > 2$)

	price of used copies		resale value		inventory	
variable	estimate	s.e.	estimate	s.e.	estimate	s.e.
lagged value	0.958*	0.005	0.928*	0.005	0.958*	0.006
lagged inventory	-2.22E-03*	2.39E-04	-1.71E-03*	2.45E-04	-	-
dummy for story-based games	5.44	16.5	-26.6	16.8	1581.9*	472.5
dummy for multi-player games	-18.1	17.4	-14.3	17.7	-470.7	497.4
critic rating	29.0*	10.8	28.0*	11.0	1214.4*	306.8
user rating	-2.26*	0.731	-0.876	0.744	-93.8*	20.85
constant	54.6	97.8	-14.6	97.3	-4376.2	2733.4
Adjusted R-squared	0.987		0.988		0.984	
# observations	647		647		667	

Note: * $p < 0.05$

Table 4: Demand estimates

	full model		demand-only model	
	mean	s.d.	mean	s.d.
preference parameters				
discount factor (β)	0.902	0.004	0.905	0.001
price sensitivity (α)	2.87E-04	1.14E-05	3.02E-04	7.66E-06
costs for buying a used copy				
intercept for type-1 (λ_{01})	-0.363	0.144	-0.134	0.112
intercept for type-2 (λ_{02})	1.45	0.054	1.41	0.106
inventory level coefficient (λ_1)	2.30	0.091	2.42	0.096
adjustment parameter (λ_2)	5.30E-04	5.52E-05	6.66E-04	9.95E-05
costs for selling a used copy				
type-1 (μ_1)	9.83	0.049	9.59	0.120
type-2 (μ_2)	3.15	0.035	2.98	0.050
seasonal dummies				
golden week (early May) (γ_1)	0.103	0.012	0.088	0.023
christmas (late Dec.) (γ_2)	0.275	0.003	0.271	0.033
outside options				
intercept (π_0)	0.283	0.037	0.231	0.042
competitive effect (π_1)	0.190	0.008	0.183	0.011
depreciation rates				
potential buyers				
1st-week intercept (φ_1)	-1.27	0.087	-1.17	0.066
2nd-week intercept (φ_2)	-2.19	0.062	-2.04	0.093
time since 3rd week (logged) (φ_3)	-10.2	0.030	-10.2	0.085
game owners				
intercept (δ_0)	1.78	0.013	1.76	0.014
story-based (δ_1)	-0.745	0.005	-0.727	0.020
multi-player (δ_2)	0.291	0.011	0.271	0.022
critic rating (δ_3)	0.015	0.006	0.013	0.015
user rating (δ_4)	-8.61E-06	0.002	-0.002	0.002
ownership duration (logged) (δ_5)	-0.547	0.005	-0.560	0.014
proportion of type-1 consumers (ψ_1)	0.725	0.009	0.756	0.011
parameters for error terms				
within-group correlation (η)	0.247	0.019	0.203	0.041
s.d.(measurement error)				
sales new (ε_1)	101965.8	11437.4	84009.6	11249.7
sales used (ε_2)	3016.2	149.0	2888.1	135.7
volume sold by consumers (ε_s)	1379.8	93.5	1199.4	104.7
s.d.(unobserved aggregate shocks)				
sales new (ξ_1)	0.700	0.192	0.496	0.103
sales used (ξ_2)	0.219	0.024	0.215	0.022
volume sold by consumers (ξ_s)	0.251	0.023	0.184	0.011

Note: 20 game-specific intercepts (γ^g) are not reported.

Table 5: Estimates for pseudo-pricing policy functions

	mean	s.d.
pseudo-pricing policy function parameters		
price of used copies		
intercept (ω_{p0})	8.67	0.115
consumption value to potential buyers (ω_{p1})	0.039	0.023
avg. consumption value to owners (ω_{p2})	-0.336	0.061
unobserved shock to buying new copy (ω_{p3})	0.002	0.008
unobserved shock to buying used copy (ω_{p4})	0.010	0.017
unobserved shock to selling (ω_{p5})	0.503	0.063
avg. size of potential buyers (ω_{p6})	-1.53E-08	3.93E-10
avg. size of owners (ω_{p7})	1.31E-06	1.98E-07
inventory of used goods (ω_{p8})	-2.56E-06	6.63E-08
cumulative # competing games (ω_{p9})	-0.002	5.88E-05
golden week seasonal dummy (ω_{p10})	-0.100	0.021
christmas seasonal dummy (ω_{p11})	0.069	0.018
s.d.(ν_{pt})	0.072	0.021
resale value of used copies		
intercept (ω_{r0})	7.93	0.224
consumption value to potential buyers (ω_{r1})	0.101	0.043
avg. consumption value to owners (ω_{r2})	-0.432	0.103
unobserved shock to buying new copy (ω_{r3})	-0.001	0.011
unobserved shock to buying used copy (ω_{r4})	0.012	0.025
unobserved shock to selling (ω_{r5})	0.894	0.172
avg. size of potential buyers (ω_{r6})	-1.27E-09	1.46E-09
avg. size of owners (ω_{r7})	2.13E-06	3.75E-07
inventory of used goods (ω_{r8})	-4.88E-06	5.42E-20
cumulative # competing games (ω_{r9})	-0.003	5.68E-05
golden week seasonal dummy (ω_{r10})	-0.134	0.033
christmas seasonal dummy (ω_{r11})	0.157	0.032
s.d.(ν_{rt})	0.121	0.033

Table 6: Proportion of Predicted Quantities and Relative Market Share by Consumer Type

weeks in release	sales of new copies type 1	sales of new copies type 2	sales of used copies type 1	sales of used copies type 2	volume sold to retailers type 1	volume sold to retailers type 2	relative market share of used to new copy type 1	relative market share of used to new copy type 2
1	0.611	0.389						
2	0.606	0.394	0.943	0.057	0.004	0.996	0.098	0.009
3	0.591	0.409	0.937	0.063	0.004	0.996	0.634	0.062
4	0.576	0.424	0.932	0.068	0.004	0.996	1.511	0.150
5	0.570	0.430	0.930	0.070	0.005	0.995	2.129	0.211
6	0.570	0.430	0.930	0.070	0.005	0.995	2.458	0.243
7	0.571	0.429	0.930	0.070	0.005	0.995	2.735	0.271
8	0.572	0.428	0.931	0.069	0.006	0.994	2.713	0.270
9	0.575	0.425	0.931	0.069	0.006	0.994	2.679	0.267
10	0.577	0.423	0.932	0.068	0.007	0.993	2.629	0.264
11	0.578	0.422	0.932	0.068	0.007	0.993	2.807	0.283
12	0.579	0.421	0.932	0.068	0.007	0.993	2.990	0.302
13	0.578	0.422	0.932	0.068	0.008	0.992	3.225	0.324
14	0.578	0.422	0.931	0.069	0.008	0.992	3.409	0.344
15	0.577	0.423	0.931	0.069	0.009	0.991	3.646	0.367

Table 7: Elasticities

type of elasticities	E.1 elasticities of demand w.r.t. new-copy price	E.2 elasticities of demand w.r.t. new-copy price	E.3 elasticities of demand w.r.t. used-copy price	E.4 elasticities of demand w.r.t. used-copy price	E.5 elasticities of demand w.r.t. used-copy inventory	E.6 elasticities of demand w.r.t. used-copy inventory	E.7 elasticities of supply w.r.t. resale value
weeks in release	sales of new copies	sales of used copies	sales of new copies	sales of used copies	sales of new copies	sales of used copies	volume sold to retailers
1	-1.99						
2	-2.20	0.651	0.045	-2.22			1.43
3	-2.35	0.432	0.159	-2.05	-0.065	0.842	1.33
4	-2.45	0.296	0.240	-1.92	-0.055	0.485	1.26
5	-2.50	0.239	0.272	-1.84	-0.029	0.232	1.21
6	-2.51	0.229	0.276	-1.80	-0.019	0.185	1.14
7	-2.52	0.221	0.274	-1.75	-0.015	0.150	1.08
8	-2.53	0.215	0.273	-1.72	-0.014	0.125	1.03
9	-2.53	0.215	0.268	-1.69	-0.010	0.098	0.991
10	-2.53	0.211	0.266	-1.66	-0.009	0.085	0.932
11	-2.54	0.203	0.260	-1.59	-0.007	0.072	0.875
12	-2.55	0.197	0.258	-1.55	-0.006	0.059	0.842
13	-2.55	0.193	0.252	-1.49	-0.005	0.052	0.796
14	-2.56	0.186	0.249	-1.45	-0.004	0.048	0.763
15	-2.57	0.176	0.248	-1.41	-0.006	0.060	0.728
Average	-2.46	0.262	0.239	-1.72	-0.019	0.192	1.03

Table 8: Proportion of Switchers to New Copies due to a 1% Increase in Used-Copy Price

weeks in release	aggregate	type 1	type 2
1			
2	0.326	0.325	0.404
3	0.197	0.187	0.366
4	0.135	0.121	0.320
5	0.114	0.100	0.297
6	0.102	0.088	0.280
7	0.094	0.080	0.268
8	0.096	0.082	0.275
9	0.096	0.083	0.275
10	0.099	0.085	0.280
11	0.094	0.081	0.276
12	0.090	0.077	0.267
13	0.085	0.072	0.255
14	0.081	0.068	0.250
15	0.078	0.065	0.243
Average	0.12	0.108	0.290

Table 9: Percentage change in profits due to elimination of used game market

	Average	S.D.	Min	Max
Under observed flat-prices				
% change in publisher's profits	-7.09%	3.98%	-14.0%	1.84%
Under optimal flat-prices				
% change in new-copy price (from observed price)	-32.7%	9.88%	-40.0%	8.00%
% change in publisher's profits	10.9%	5.40%	-0.801%	21.0%

Figure 1: Average quantities demanded for new video games

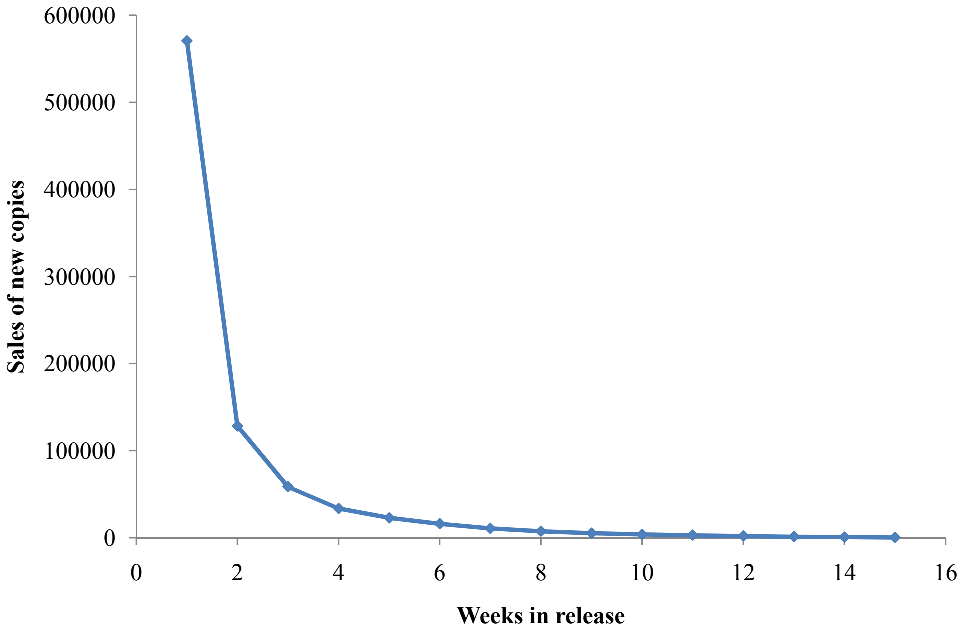


Figure 2: Average quantities demanded and supplied and inventory level for used video games

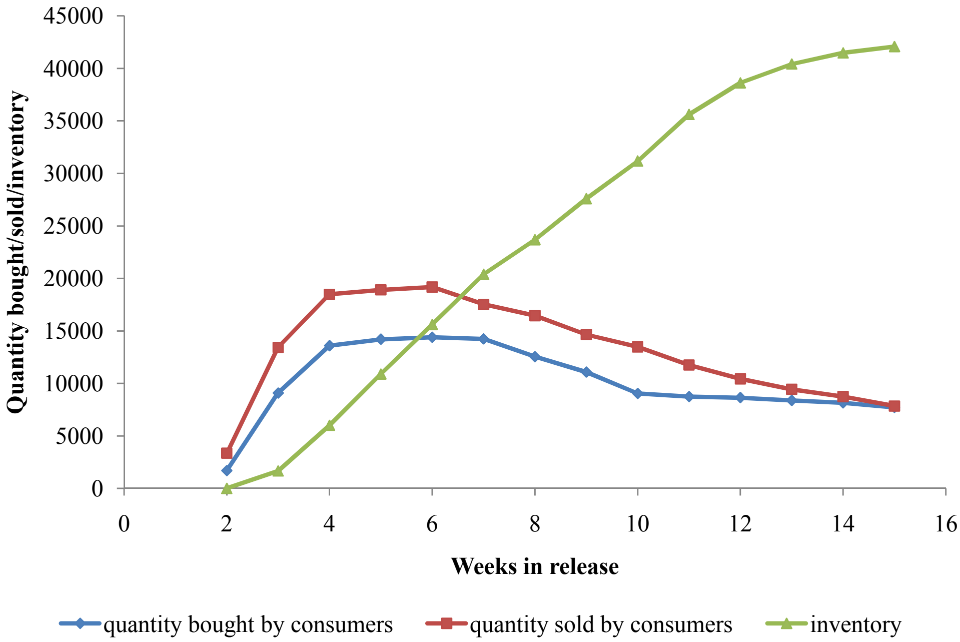


Figure 3: Average price and resale value of used video games

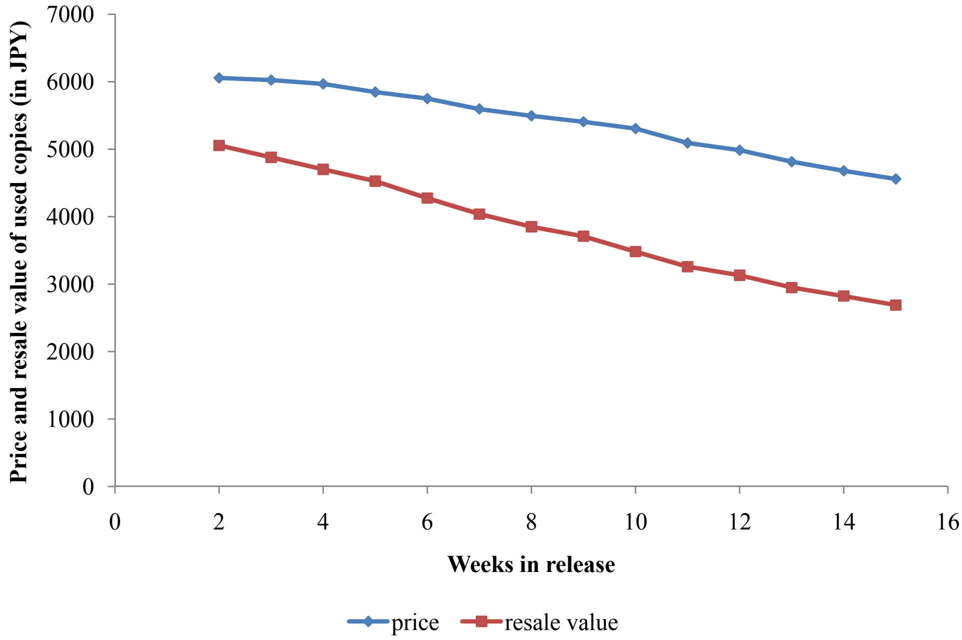


Figure 4: Relative market share of used to new copies and price differential between new and used copies in week 2

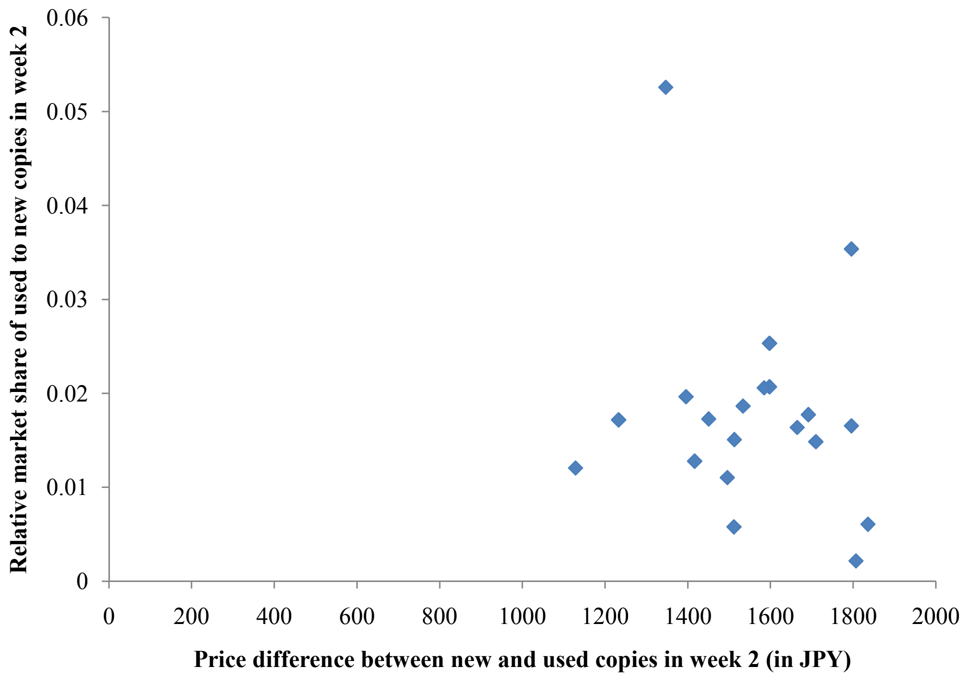


Figure 5: Observed versus Predicted Sales of New Copies

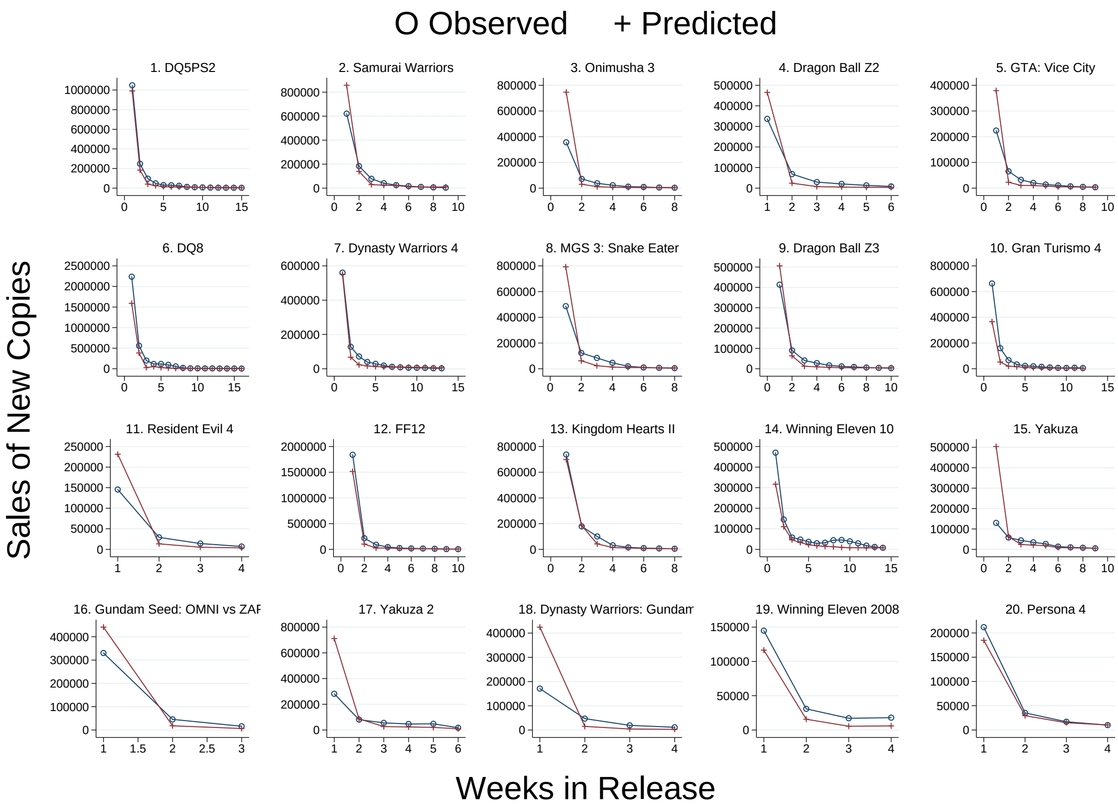


Figure 6: Observed versus Predicted Sales of Used Copies

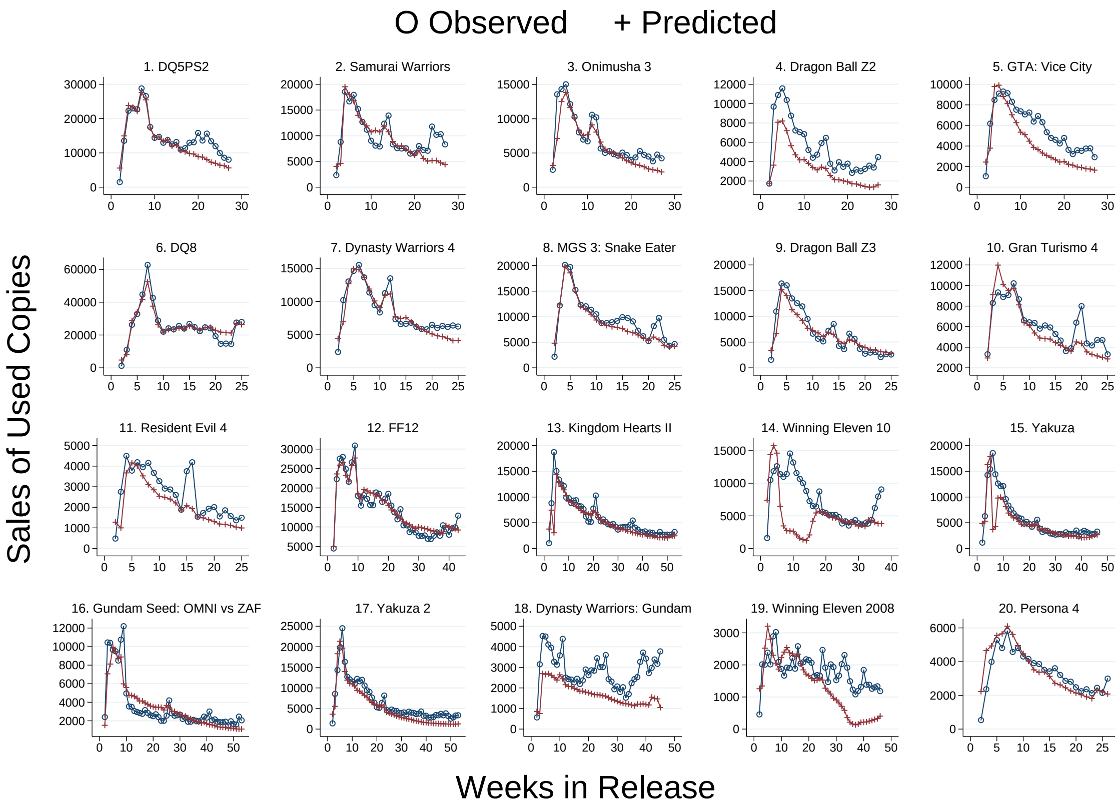


Figure 7: Observed versus Predicted Volume Sold to Retailers by Consumers

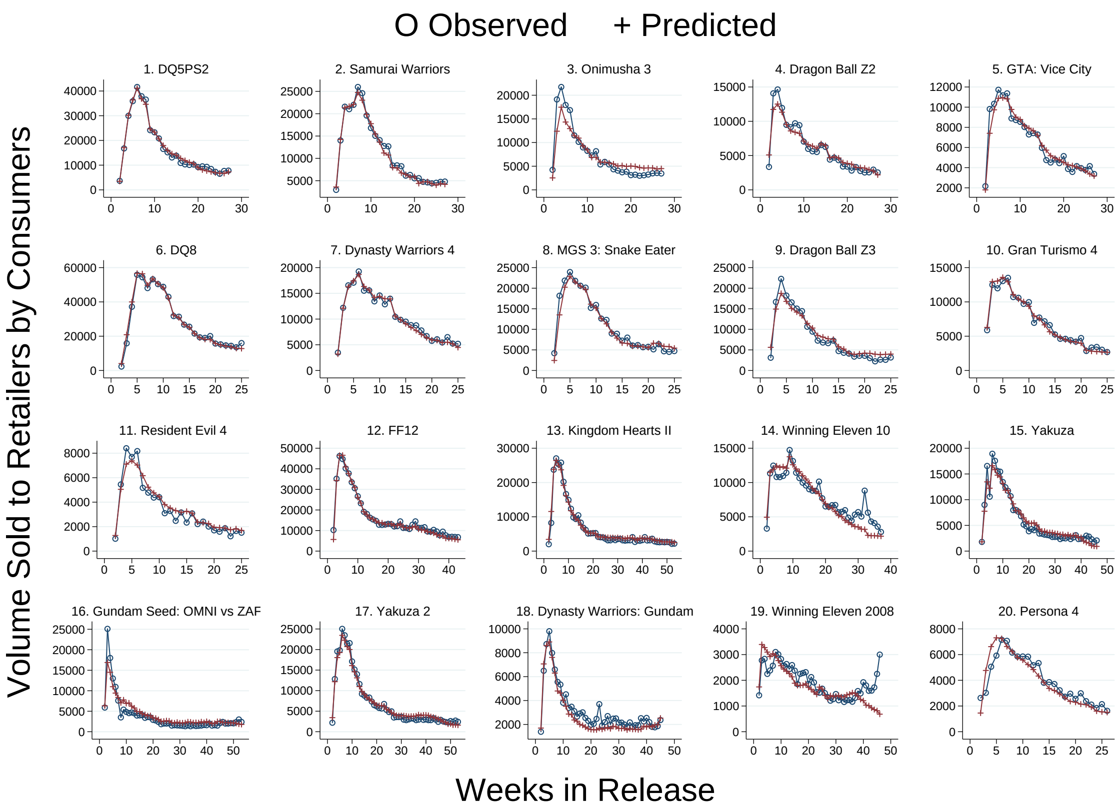


Figure 8: Observed versus Predicted Prices of Used Copies

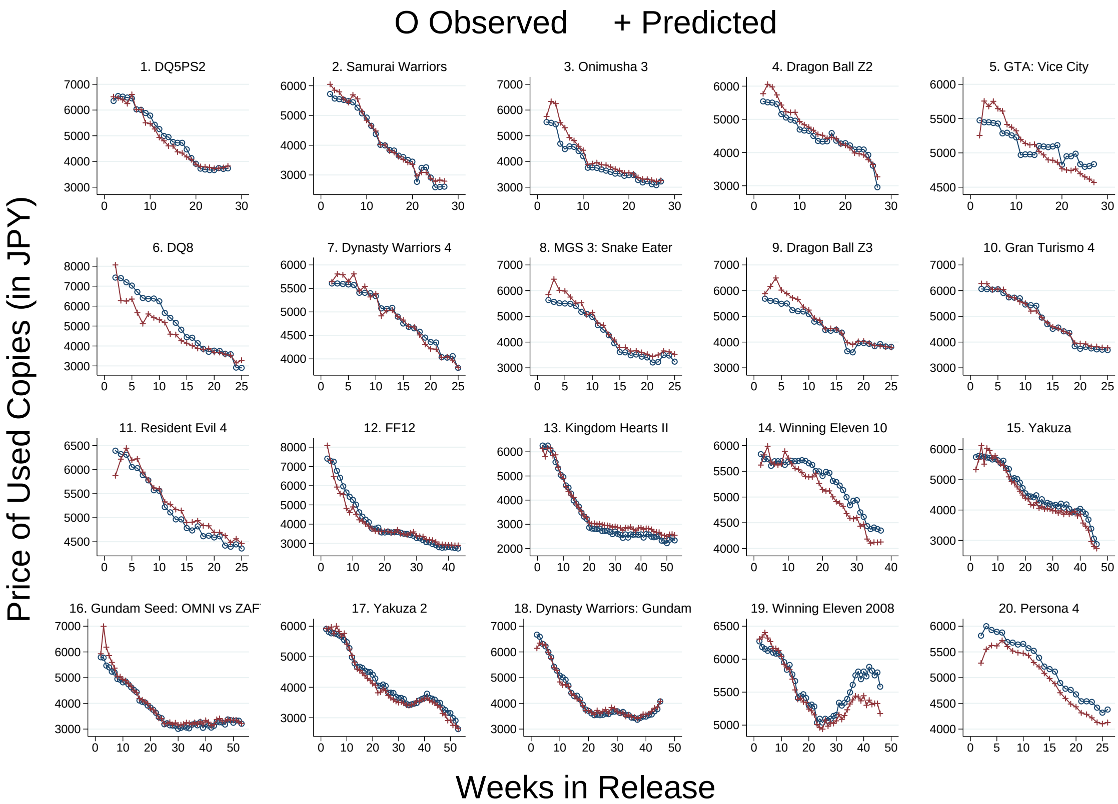


Figure 9: Observed versus Predicted Resale Value of Used Copies

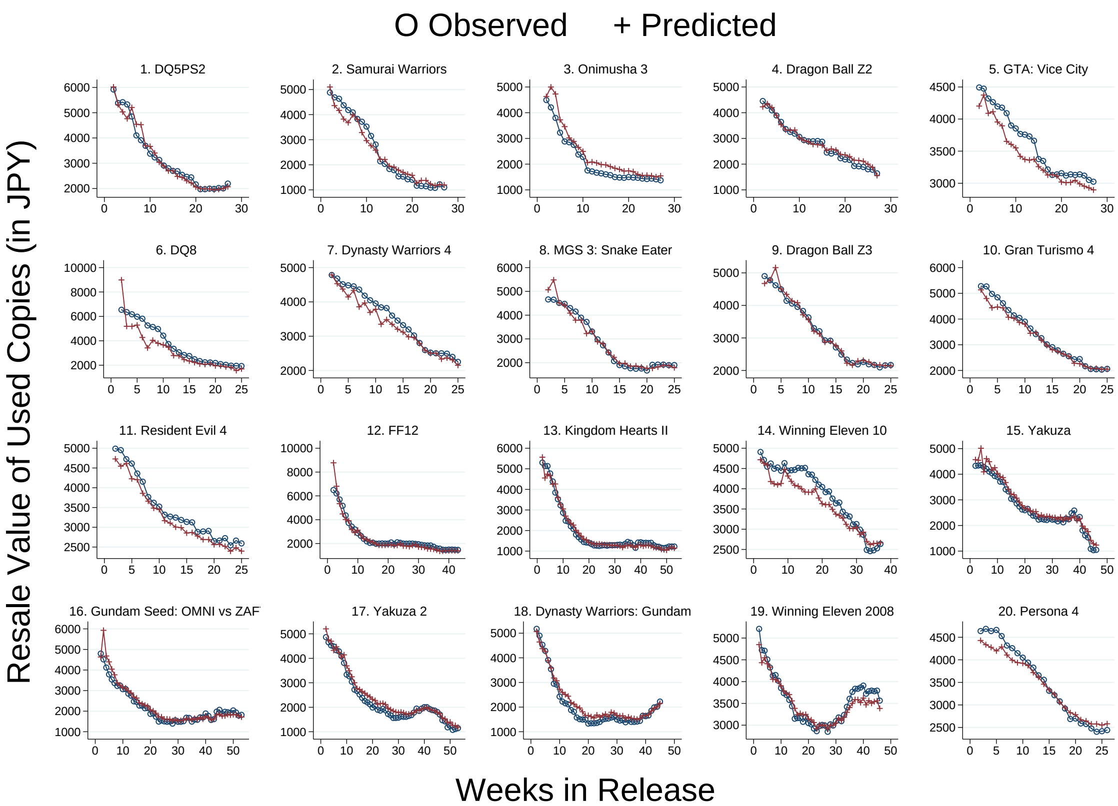


Figure 10: Average percentage change in sales of new copies due to elimination of used game market

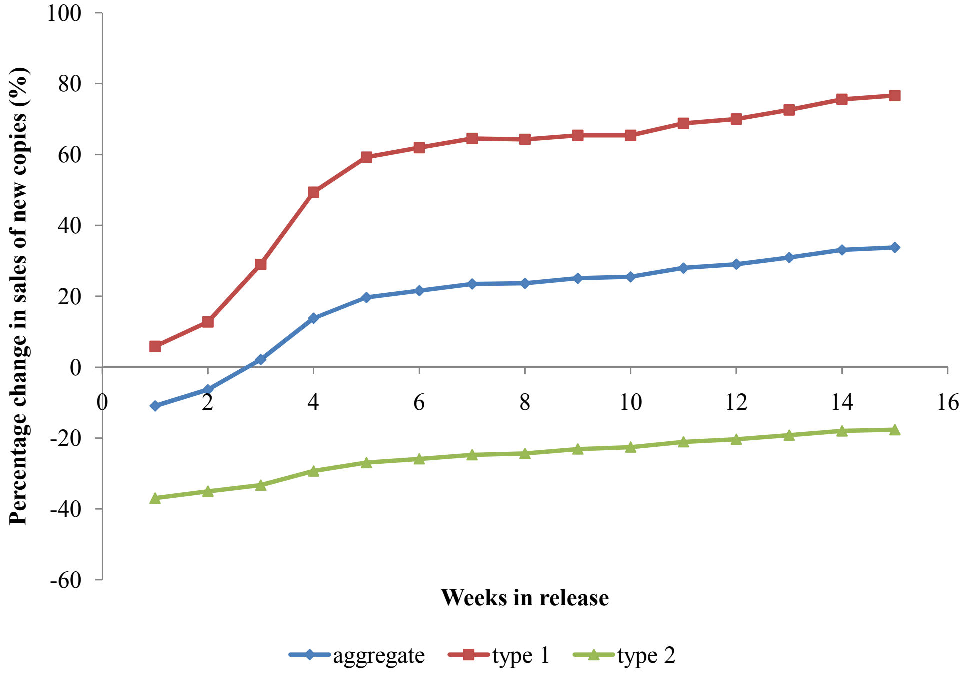
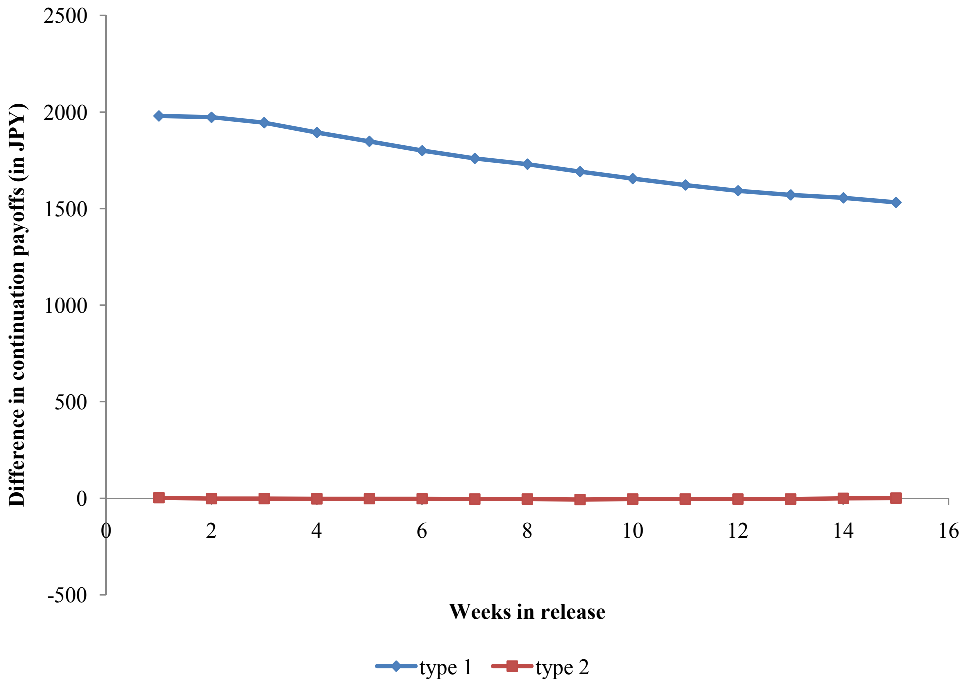


Figure 11: Average value of having the future selling opportunity



A Appendix

A.1 The procedure for the estimation algorithm

This appendix describes the details for the estimation algorithm described in section 5.1.

Let θ_d and θ_s be the vectors of demand-side parameters and pseudo-policy function parameters, respectively. In the context of the present model, the output of the new algorithm in iteration m is

$$H^m = \{\theta_d^l, \theta_s^l, \theta_d^{*l}, \{\tilde{V}_i^l(p_1, \tilde{p}_{2t}^l, \tilde{r}_t^l, \tilde{Y}_t^l, \tilde{C}_t^l, \tilde{\xi}_d^l, t; \theta_d^{*l}) \ \forall i\}_{t=2}^T, \{\{\tilde{W}_i^l(\tilde{r}_t^l, \tilde{Y}_t^l, \tilde{\xi}_s^l, t, \tau; \theta_d^{*l}) \ \forall i\}_{\tau=1}^{t-1}\}_{t=2}^T\}_{l=m-N}^{m-1},$$

where $\tilde{V}_i^l$ and $\tilde{W}_i^l$ are type-i consumer's pseudo-value functions for buying and selling decisions in iteration l, respectively; N is the number of past pseudo-value functions used for approximating the expected value functions; θ_d^l and θ_s^l are the accepted parameter vectors of the demand-side model and the pseudo-policy functions in iteration l, respectively; θ_d^{*l} is the candidate parameter vector for the demand-side model in iteration l; $(\tilde{p}_{2t}^l, \tilde{r}_t^l, \tilde{Y}_t^l, \tilde{C}_t^l)$ are a draw of (serially correlated) state vector at time t in iteration l (e.g., drawn from uniform distribution); $(\tilde{\xi}_d^l, \tilde{\xi}_s^l)$ are drawn from the corresponding normal distributions. We assume $(\tilde{\xi}_d^l, \tilde{\xi}_s^l)$ are *i.i.d.* across time, and thus we can (1) use the same draws for all periods, and (2) the integration of unobserved shocks can be done by the simple average of the past pseudo-value functions.

Type-i consumer's pseudo-value functions for selling decision at time t in iteration m are defined as follows:

$$\tilde{W}_i^m(\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, \tau; \theta_d^{*m}) = E_e \max_{k \in \{0,1\}} \{\tilde{W}_{ik}^m(\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, \tau; \theta_d^{*m}) + e_{ikt}\},$$

where $\tilde{W}_{ik}^m$'s are type-i consumer's pseudo alternative-specific value functions in iteration m, which are given by

$$\begin{aligned} &\tilde{W}_{ik}^m(\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, \tau; \theta_d^{*m}) \\ &\quad = \begin{cases} \alpha \tilde{r}_t^m - \mu_i + \tilde{\xi}_s^m & \text{if selling,} \\ v(t,\tau) + \beta \hat{E}^m[W_i(r', Y', \xi_s', t+1, \tau+1; \theta_d^{*m}) | (\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, \tau)] & \text{if keeping.} \end{cases} \end{aligned}$$

The *pseudo*-expected value function for selling decision, $\hat{E}^m[W_i(.; \theta_d^{*m})|.]$, is defined as the weighted average

of the past pseudo-value functions for selling decision in period $t+1$. It is constructed as follows:

$$\hat{E}^m[W_i(r', Y', \xi'_s, t+1, \tau+1; \theta_d^{*m}) | (\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, \tau)]$$
$$= \sum_{l=m-N}^{m-1} \tilde{W}_i^s(\tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{\xi}_s^l, t+1, \tau+1; \theta_d^{*l}) \frac{K_h(\theta_d^{*m} - \theta_d^{*l}) f(\tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l | \tilde{r}_t^m, \tilde{Y}_t^m)}{\sum_{q=m-N}^{m-1} K_h(\theta_d^{*m} - \theta_d^{*q}) f(\tilde{r}_{t+1}^q, \tilde{Y}_{t+1}^q | \tilde{r}_t^m, \tilde{Y}_t^m)},$$

where $K_h(.)$ is a Gaussian kernel with bandwidth h, and $f(.|.)$ is the transition density estimated in the first step. Note that the kernel captures the idea that one assigns higher weights to the past pseudo-value functions which are evaluated at parameter vectors that are closer to θ_d^{*m}. Also, this weighted average integrates out r', Y', and ξ'_s. In particular, ξ'_s is integrated out by the simple average since ξ_s^l's are drawn from its distribution. In contrast, r' and Y' are integrated out by the weighted average, where weights are given by the transition probabilities.

The type-i consumers' pseudo-value functions for buying decision at time t in iteration m are defined as follows:

$$\tilde{V}_i^m(p_1, \tilde{p}_{2t}^m, \tilde{r}_t^m, \tilde{Y}_t^m, \tilde{C}_t^m, \tilde{\xi}_d^m, t; \theta_d^{*m}) = E_\epsilon \max_{j \in \{0,1,2\}} \{\tilde{V}_{ij}^m(p_1, \tilde{p}_{2t}^m, \tilde{r}_t^m, \tilde{Y}_t^m, \tilde{C}_t^m, \tilde{\xi}_d^m, t; \theta_d^{*m}) + \epsilon_{ijt}\},$$

where $\tilde{V}_{ij}^m$'s are type-i consumer's pseudo alternative-specific value functions in iteration m, which are given by

$$\tilde{V}_{ij}^m(p_1, \tilde{p}_{2t}^m, \tilde{r}_t^m, \tilde{Y}_t^m, \tilde{C}_t^m, \tilde{\xi}_d^m, t; \theta_d^{*m})$$
$$= \begin{cases} v(t,0) - \alpha p_1 + \tilde{\xi}_1^m + \beta \hat{E}^m[W_i(r', Y', \xi'_s, t+1, 1; \theta_d^{*m}) | (\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, 0)] & \text{new copy,} \\ v(t,0) - \alpha \tilde{p}_{2t}^m + \tilde{\xi}_2^m - l_Y(\tilde{Y}_t^m; \lambda_i) + \beta \hat{E}^m[W_i(r', Y', \xi'_s, t+1, 1; \theta_d^{*m}) | (\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, 0)] & \text{used copy,} \\ l_C(\tilde{C}_t^m) + \beta \hat{E}^m[V_i(p_1, p'_2, r', Y', C', \xi'_d, t+1; \theta_d^{*m}) | (p_1, \tilde{p}_{2t}^m, \tilde{r}_t^m, \tilde{Y}_t^m, \tilde{C}_t^m, \tilde{\xi}_d^m, t)] & \text{no purchase.} \end{cases}$$

The *pseudo*-expected future value function for buying decision, $\hat{E}^m[V_i(,;\theta_d^{*m})|.]$, is defined as the weighted average of the past pseudo-value functions for buying decision in period $t+1$, and is constructed as follows:

$$\hat{E}^m[V_i(p_1, p'_2, r', Y', C', \xi'_d, t+1; \theta_d^{*m}) | (p_1, \tilde{p}_{2t}^m, \tilde{r}_t^m, \tilde{Y}_t^m, \tilde{C}_t^m, \tilde{\xi}_d^m, t)]$$
$$= \sum_{l=m-N}^{m-1} \tilde{V}_i(p_1, \tilde{p}_{2t+1}^l, \tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{C}_{t+1}^l, \tilde{\xi}_d^l, t+1; \theta_d^{*l})$$
$$\times \frac{K_h(\theta_d^{*m} - \theta_d^{*l}) f(\tilde{p}_{2t+1}^l, \tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{C}_{t+1}^l | \tilde{p}_{2t}^m, \tilde{r}_t^m, \tilde{Y}_t^m, \tilde{C}_t^m)}{\sum_{q=m-N}^{m-1} K_h(\theta_d^{*m} - \theta_d^{*q}) f(\tilde{p}_{2t+1}^q, \tilde{r}_{t+1}^q, \tilde{Y}_{t+1}^q, \tilde{C}_{t+1}^q | \tilde{p}_{2t}^m, \tilde{r}_t^m, \tilde{Y}_t^m, \tilde{C}_t^m)}.$$

Note again that this weighted average integrates out p'_2, r', Y', C', and ξ'_d.

Each MCMC iteration in the proposed algorithm consists of five blocks:

1. Draw $\sigma_\xi^m = (\sigma_{\xi_1}, \sigma_{\xi_2}, \sigma_{\xi_s})$ directly from their posterior distributions conditional on $\xi_t^{m-1} = (\xi_{1t}^{m-1}, \xi_{2t}^{m-1}, \xi_{st}^{m-1})$ for all observed t and g.

2. Draw ξ_t^m for all observed t and g conditional on the data, σ_ξ^m, θ_d^{m-1} and θ_s^{m-1}. In the Metropolis-Hastings algorithm, the joint-likelihood of the demand-side model and the pseudo-policy functions will be used to compute the acceptance probability.

3. Draw θ_d^m conditional on the data, $\{\xi_t^m\}$ and θ_s^{m-1} using the random-walk Metropolis-Hastings algorithm. In the Metropolis-Hastings algorithm, the joint-likelihood will be used.

4. Draw θ_s^m conditional on the data, $\{\xi_t^m\}$ and θ_d^m using the random-walk Metropolis-Hastings algorithm. In the Metropolis-Hastings algorithm, only the likelihood of the pseudo-policy functions will be used since θ_s^m does not enter the demand-side model.

5. Compute the pseudo-value functions for buying and selling decision problems. Starting from the terminal period, We sequentially compute the pseudo-value functions backwards at only one randomly drawn state point in each period. We store them and update H^m to H^{m+1}.

In deriving the posterior distribution of parameters, we use an inverted gamma prior on σ_ξ, and a flat prior on θ_d and θ_s. Also, note that the likelihood used in the IJC algorithm is called *pseudo*-likelihood as it is a function of *pseudo* alternative-specific value functions. Below, we provide a step-by-step procedure for the five blocks described above.

1. Suppose that we are at iteration m. We start with

$$H^m = \{\theta_d^l, \theta_s^l, \theta_d^{*l}, \{\tilde{V}_i^l(p_1, \tilde{p}_{2t}^l, \tilde{r}_t^l, \tilde{Y}_t^l, \tilde{C}_t^l, \tilde{\xi}_d^l, t; \theta_d^{*l})\ \forall i\}_{t=2}^T, \{\{\tilde{W}_i^l(\tilde{r}_t^l, \tilde{Y}_t^l, \tilde{\xi}_s^l, t, \tau; \theta_d^{*l})\ \forall i\}_{\tau=1}^{t-1}\}_{t=2}^T\}_{l=m-N}^{m-1},$$

where N is the number of past iterations used for expected value approximations.

2. Draw $\sigma_\xi^m = (\sigma_{\xi_1}, \sigma_{\xi_2}, \sigma_{\xi_s})$ directly from their posterior distributions (inverted gamma) conditional on $\xi_t^{g,m-1} = (\xi_{1t}^{g,m-1}, \xi_{2t}^{g,m-1}, \xi_{st}^{g,m-1})$ for all observed t and g.

3. For each observed t and g, draw $\xi_t^{g,m}$ from its posterior distribution conditional on $\sigma_{\xi_1}^m$, θ_d^{m-1}, θ_s^{m-1}, $\{\xi_k^{g,m}\}_{k=1}^{t-1}$, and $\{\xi_k^{g,m-1}\}_{k=t+1}^{T^g}$. We will draw $\xi_{1t}^{g,m}$, $\xi_{2t}^{g,m}$, and $\xi_{st}^{g,m}$ separately. Below, we will describe how to draw $\xi_{1t}^{g,m}$, but the procedure can be applied for drawing $\xi_{2t}^{g,m}$ and $\xi_{st}^{g,m}$.

   (a) Draw $\xi_{1t}^{g,*m}$ (candidate parameter value).

   (b) We compute the pseudo-joint likelihood at $\xi_{1t}^{g,*m}$ conditional on $\{\xi_k^{g,m}\}_{k=1}^{t-1}$, $\xi_{2t}^{g,m-1}$, $\xi_{st}^{g,m-1}$, $\{\xi_k^{g,m-1}\}_{k=t+1}^{T^g}$, θ_d^{m-1} and θ_s^{m-1}. Note that conditional on $\sigma_{\xi_1}^m$, the pseudo-joint likelihood prior to time t does not depend on $\xi_{1t}^{g,*m}$. Thus, we only need to compute the pseudo-joint likelihood at time t and later. To compute the pseudo-joint likelihood, we need to obtain the pseudo-alternative specific value functions for both buying and selling decisions at time t and later: $\tilde{V}_{ij}^m(\cdot, t; \theta_d^{m-1})$ and $\{\tilde{W}_{ik}^m(\cdot, t, \tau; \theta_d^{m-1})\}_{\tau=1}^{t-1}$. To obtain $\tilde{V}_{ij}^m(\cdot, t; \theta_d^{m-1})$, we need to calculate both $\hat{E}^m V_i(\cdot, t+1; \theta_d^{m-1})$ (pseudo-expected value function when consumers choose no option) and $\hat{E}^m W_i(\cdot, t+1, 1; \theta_d^{m-1})$ (pseudo-expected value function when consumers choose to buy a new or used copy), which are computed as the weighted average of past-pseudo value functions evaluated at time $t+1$:

      i. For $\hat{E}^m V_i(\cdot, t+1; \theta_d^{m-1})$, we take the weighted average of $\{\tilde{V}_i^l(p_1, \tilde{p}_{2t+1}^l, \tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{C}_{t+1}^l, \tilde{\xi}_d^l, t+1; \theta_d^{*l})\ \forall i\}_{l=m-N}^{m-1}$ as in Equation (5).

      ii. For $\hat{E}^m W_i(\cdot, t+1, 1; \theta_d^{m-1})$, we take the weighted average of $\{\tilde{W}_i^l(\tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{\xi}_s^l, t+1, 1; \theta_d^{*l})\ \forall i\}_{l=m-N}^{m-1}$ as in Equation (5). Note that since potential buyers at time t will have owned the game for one period when they reach $t+1$, the set of past pseudo-value functions used here only include those evaluated at $\tau = 1$.

   To obtain $\{\tilde{W}_{ik}^m(\cdot, t, \tau; \theta_d^{m-1})\}_{\tau=1}^{t-1}$, we need to calculate $\{\hat{E}^m W_i(\cdot, t+1, \tau+1; \theta_d^{m-1})\}_{\tau=1}^{t-1}$ by the weighted average of the past pseudo-value functions $\{\tilde{W}_i^l(\tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{\xi}_s^l, t+1, \tau+1; \theta_d^{*l})\ \forall i\}_{l=m-N}^{m-1}$ as in Equation (5).[24]

   (c) Similarly, we compute the pseudo-joint likelihood at $\xi_{1t}^{g,m-1}$ conditional on $\{\xi_k^{g,m}\}_{k=1}^{t-1}$, $\xi_{2t}^{g,m-1}$,

[24] Conditional on σ_ξ^m, pseudo alternative-specific value functions do not depend on $\xi_{1t}^{g,*m}$. This is also true for $\xi_{2t}^{g,*m}$ and $\xi_{st}^{g,*m}$. Thus, pseudo alternative-specific value functions can be pre-computed right after step 2.

$\xi_{st}^{g,m-1}$, $\{\xi_k^{g,m-1}\}_{k=t+1}^{T^g}$, θ_d^{m-1} and θ_s^{m-1}.[25]

(d) Based on the pseudo-joint likelihoods at $\xi_{1t}^{g,*m}$ and $\xi_{2t}^{g,m-1}$, we compute the acceptance probability for $\xi_{1t}^{g,*m}$ and decide whether to accept (i.e., set $\xi_{1t}^{g,m} = \xi_{1t}^{g,*m}$) or reject (i.e., set $\xi_{1t}^{g,m} = \xi_{1t}^{g,m-1}$).

(e) Using a similar procedure, draw $\xi_{2t}^{g,m}$ and $\xi_{st}^{g,m}$. One difference in drawing $\xi_{st}^{g,m}$ is that conditional on $\sigma_{\xi_s}^m$, $\xi_{st}^{g,*m}$ does not influence the likelihood function for buying decisions.

4. Use the Metropolis-Hastings algorithm to draw θ_d^m conditional on $\{\xi_t^m\}$ and θ_s^{m-1}.

(a) Draw θ_d^{*m} (candidate parameter vector).

(b) We compute the pseudo-joint likelihood at θ_d^{*m} conditional on $\{\xi_t^m\}$ and θ_s^{m-1} based on the pseudo-alternative specific value functions for both buying and selling decisions at θ_d^{*m}: $\tilde{V}_{ij}^m(\cdot, t; \theta_d^{*m})$ and $\{\tilde{W}_{ik}^m(\cdot, t, \tau; \theta_d^{*m})\}_{\tau=1}^{t-1}$ for all observed t and g. To obtain $\tilde{V}_{ij}^m(\cdot, t; \theta_d^{*m})$, we need to calculate both $\hat{E}^m V_i(\cdot, t+1; \theta_d^{*m})$ and $\hat{E}^m W_i(\cdot, t, 1; \theta_d^{*m})$, which are computed as the weighted average of past-pseudo value functions evaluated at time $t+1$:

i. For $\hat{E}^m V_i(\cdot, t; \theta_d^{*m})$, we take the weighted average of $\{\tilde{V}_i^l(p_1, \tilde{p}_{2t+1}^l, \tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{C}_{t+1}^l, \tilde{\xi}_d^l, t+1; \theta_d^{*l})\ \forall i\}_{l=m-N}^{m-1}$ as in Equation (5).

ii. For $\hat{E}^m W_i(\cdot, t, 1; \theta_d^{*m})$, we take the weighted average of $\{\tilde{W}_i^l(\tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{\xi}_s^l, t+1, 1; \theta_d^{*l})\ \forall i\}_{l=m-N}^{m-1}$ as in Equation (5). Again, note that since potential buyers at time t will have owned the game for one period when they reach $t+1$, the set of past pseudo-value functions used here are all evaluated at $\tau = 1$.

To obtain $\{\tilde{W}_{ik}^m(\cdot, t, \tau; \theta_d^{*m})\}_{\tau=1}^{t-1}$, we only need to calculate $\{\hat{E}^m W_i(\cdot, t+1, \tau+1; \theta_d^{*m})\}_{\tau=1}^{t-1}$ by the weighted average of the past pseudo-value functions $\{\tilde{W}_i^l(\tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{\xi}_s^l, t+1, \tau+1; \theta_d^{*l})\ \forall i\}_{l=m-N}^{m-1}$ as in Equation (5).

(c) Similarly, we compute the pseudo-joint likelihood at θ_d^{m-1} conditional on $\{\xi_t^m\}$ and θ_s^{m-1}.

[25] In a standard Metropolis-Hastings algorithm, this step is not necessary as this value has been computed in the previous iteration. However, the IJC algorithm updates the set of past pseudo-value functions in each iteration. Thus, the pseudo-likelihood at $\xi_{1t}^{g,m-1}$ in iteration $m-1$ will be different from that at $\xi_{1t}^{g,m-1}$ in iteration m.

(d) Based on the pseudo-joint likelihoods at θ_d^{*m} and θ_d^{m-1}, we compute the acceptance probability for θ_d^{*m} and decide whether to accept (i.e., set $\theta_d^m = \theta_d^{*m}$) or reject (i.e., set $\theta_d^m = \theta_d^{m-1}$).

5. Use the Metropolis-Hastings algorithm to draw θ_s^m conditional on $\{\xi_t^m\}$ and θ_d^m.

(a) Draw θ_s^{*m} (candidate parameter vector).

(b) We compute the pseudo-likelihood for pseudo-policy functions at θ_s^{*m} conditional on $\{\xi_t^m\}$ and θ_d^m. Note that the pseudo-alternative specific value functions do not depend on θ_s^{*m}, but are required to compute the pseudo-likelihood at θ_s^{*m} since they influence the evolution of equilibrium state variables. However, they have already been computed in step 4(b) (if θ_d^{*m} has been accepted) or 4(c) (if θ_d^{*m} has been rejected), there is no need to re-compute them here to form the pseudo-likelihood for pseudo-policy functions.

(c) To form the acceptance probability of θ_s^{*m}, we need the pseudo-likelihood for pseudo-policy functions at θ_s^{m-1} conditional on $\{\xi_t^m\}$ and θ_d^m. Note that this value has been computed in step 4 and needs not be re-computed here.

(d) Based on the pseudo-likelihood for pseudo-policy functions at θ_s^{*m} and θ_s^{m-1}, we compute the acceptance probability for θ_s^{*m} and decide whether to accept (i.e., set $\theta_s^m = \theta_s^{*m}$) or reject (i.e., set $\theta_s^m = \theta_s^{m-1}$).

6. Compute the pseudo-value functions for buying and selling decision problems.

(a) For each $t = 2, \ldots, T$, make a draw of used-game price ($\tilde{p}_{2t}^m$), resale value ($\tilde{r}_t^m$), inventory level ($\tilde{Y}_t^m$), and cumulative number of newly introduced games ($\tilde{C}_t^m$) from uniform distributions with appropriate upper- and lower-bound (e.g., upper- and lower-bound of observed values).

(b) Make a draw of $\tilde{\xi}_1^m$, $\tilde{\xi}_2^m$, and $\tilde{\xi}_s^m$ from the corresponding distribution based on $\sigma_{\xi_1}^m$, $\sigma_{\xi_2}^m$, and $\sigma_{\xi_s}^m$.

(c) Start from the terminal period T.

i. Compute the value functions $\tilde{V}_i^m(p_1, \tilde{p}_{2T}^m, \tilde{r}_T^m, \tilde{Y}_T^m, \tilde{C}_T^m, \tilde{\xi}_d^m, T; \theta_d^{*m})$ and $\{\tilde{W}_i^m(\tilde{r}_T^m, \tilde{Y}_T^m, \tilde{\xi}_s^m, T, \tau; \theta_d^{*m})\}_{\tau=1}^{T-1}$ for all i. Note that at time T, there is no need to compute

the pseudo-expected value function. Thus, the value functions computed at time T are not pseudo-value functions.

ii. Store $\tilde{V}_i^m(\cdot, T; \theta_d^{*m})$ and $\{\tilde{W}_i^m(\cdot, T, \tau; \theta_d^{*m})\}_{\tau=1}^{T-1}$.

(d) For $t = T-1, \ldots, 2$, compute the pseudo-value function $\tilde{V}_i^m(p_1, \tilde{p}_{2T}^m, \tilde{r}_t^m, \tilde{Y}_t^m, \tilde{C}_t^m, \tilde{\xi}_d^m, t; \theta_d^{*m})$ and $\{\tilde{W}_i^m(\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, \tau; \theta_d^{*m})\}_{\tau=1}^{t-1}$ for all i backwards.

i. To compute $\tilde{V}_i^m(\cdot, t; \theta_d^{*m})$, we need to calculate $\hat{E}^m V_i(\cdot, t+1; \theta_d^{*m})$ and $\hat{E}^m W_i(\cdot, t+1, 1; \theta_d^{*m})$ based on Equations (5) and (5), respectively.

ii. To compute $\{\tilde{W}_i^m(\cdot, t, \tau; \theta_d^{*m})\}_{\tau=1}^{t-1}$, we need to calculate $\{\hat{E}^m W_i(\cdot, t+1, \tau+1; \theta_d^{*m})\}_{\tau=1}^{t-1}$ based on Equation (5).

iii. Store $\tilde{V}_i^m(\cdot, t; \theta_d^{*m})$ and $\{\tilde{W}_i^m(\cdot, t, \tau; \theta_d^{*m})\}_{\tau=1}^{t-1}$.

7. Go to iteration $m+1$.

In our application in Section 6, we set $N = 100$ (# past pseudo-value functions used for the approximation of expected value functions) and $h = 0.01$ (kernel bandwidth).

A.2 The likelihood function

Assuming that the prediction errors, ν_{pt} and ν_{rt}, in Equations (3) and (4) are normally distributed, we obtain the conditional likelihood of observing (p_{2t}^g, r_t^g),

$$f_s(p_{2t}^g, r_t^g | \{M_{it}^{d,g}, \upsilon^g(t,0), \{M_{it}^{s,g}(\tau), \upsilon^g(t,\tau)\}_{\tau=1}^{t-1}\}_{i=1}^{I}, \xi_{1t}^g, \xi_{2t}^g, \xi_{st}^g, Y_t^g, C_t^g; \theta_s)$$

where θ_s is the parameter vector of pseudo-policy functions. Note that (i) $\upsilon^g(t,\tau)$ depends on product characteristics, X_g; (ii) $M_{it}^{d,g}$ (size of potential buyers) and $M_{it}^{s,g}(\tau)$ (size of owners) are a function of X_g, p_1^g, $\{p_{2m}^g, r_m^g, Y_m^g\}_{m=2}^{t-1}$, $\{C_m^g\}_{m=1}^{t-1}$, $\{\xi_{1m}^g\}_{m=1}^{t-1}$, $\{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^{t-1}$, $M_1^{d,g}$ (initial size of potential buyers), and $\{N_m^g\}_{m=2}^{t}$ (potential buyers who entered at time m). Thus, we can rewrite f_s as

$$f_s(p_{2t}^g, r_t^g | \{\xi_{1m}^g\}_{m=1}^{t}, \{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^{t}, Y_t^g, C_t^g, Z_t^g; \theta_s).$$

where $Z_t^g = \{X_g, p_1^g, \{p_{2m}^g, r_m^g, Y_m^g\}_{m=2}^{t-1}, \{C_m^g\}_{m=1}^{t-1}, M_1^{d,g}, \{N_m^g\}_{m=2}^{t}\}$ is a vector of observed variables.

Assume further that the measurement errors, ε_{1t}, ε_{2t}, ε_{st}, in Equations (1) and (2) are normally distributed. Then, the conditional likelihood of observing $(Q_{1t}^g, Q_{2t}^g, Q_{st}^g)$ is written as

$$f_d(Q_{1t}^g, Q_{2t}^g, Q_{st}^g | \{M_{it}^{d,g}, v_i^g(t,0), \{M_{it}^{s,g}(\tau), v_i^g(t,\tau)\}_{\tau=1}^{t-1}\}_{i=1}^{I}, \xi_{1t}^g, \xi_{2t}^g, \xi_{st}^g, p_1^g, p_{2t}^g, r_t^g, Y_t^g, C_t^g; \theta_d),$$

where θ_d is the vector of demand-side parameters. Similar to f_s, f_d can be rewritten as

$$f_d(Q_{1t}^g, Q_{2t}^g, Q_{st}^g | \{\xi_{1m}^g\}_{m=1}^t, \{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^t, p_{2t}^g, r_t^g, Y_t^g, C_t^g, Z_t^g; \theta_d).$$

The joint likelihood of observing $(Q_{1t}^g, Q_{2t}^g, Q_{st}^g, p_{2t}^g, r_t^g)$ is the product of f_s and f_d:

$$\begin{aligned} & l(Q_{1t}^g, Q_{2t}^g, Q_{st}^g, p_{2t}^g, r_t^g | \{\xi_{1m}^g\}_{m=1}^t, \{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^t, Y_t^g, C_t^g, Z_t^g; \theta_d, \theta_s) = \\ & \quad f_d(Q_{1t}^g, Q_{2t}^g, Q_{st}^g | \{\xi_{1m}^g\}_{m=1}^t, \{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^t, p_{2t}^g, r_t^g, Y_t^g, C_t^g, Z_t^g; \theta_d) \times \\ & \quad f_s(p_{2t}^g, r_t^g | \{\xi_{1m}^g\}_{m=1}^t, \{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^t, Y_t^g, C_t^g, Z_t^g; \theta_s). \end{aligned}$$

The likelihood of observing $\mathbf{D} = \{\{Q_{1t}^g\}_{t=1}^{T^g}, \{Q_{2t}^g, Q_{st}^g, p_{2t}^g, r_t^g\}_{t=2}^{T^g}\}_{g=1}^G$ is

$$\begin{aligned} & L(\mathbf{D}|\xi, C, Y, Z; \theta_d, \theta_s) = \\ & \quad \prod_{g=1}^{G} \left[f_d(Q_{11}^g | \xi_{11}^g, C_1^g, Z_1^g; \theta_d) \prod_{t=2}^{T^g} l(Q_{1t}^g, Q_{2t}^g, Q_{st}^g, p_{2t}^g, r_t^g | \{\xi_{1m}^g\}_{m=1}^t, \{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^t, Y_t^g, C_t^g, Z_t^g; \theta_d, \theta_s) \right] \end{aligned}$$

where G is the total number of games, T^g is the length of observations for game g, $Y = \{\{Y_t^g\}_{t=1}^{T^g}\}_{g=1}^G$, $C = \{\{C_t^g\}_{t=1}^{T^g}\}_{g=1}^G$, and $Z = \{\{Z_t^g\}_{t=1}^{T^g}\}_{g=1}^G$.

Note that $\{\xi_{1t}^g\}_{t=1}^{T^g}, \{\xi_{2t}^g, \xi_{st}^g\}_{t=2}^{T^g}$ are unobserved to the econometricians. In the proposed Bayesian framework, these variables are augmented from the corresponding distributions to form the likelihood $L(\mathbf{D}|\cdot)$.