

Dynamic Prudential Regulation*

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Abstract

We investigate the design of prudential bank regulation and its effects on the real and financial decisions of banks in a dynamic structural framework. In our model, a representative bank can dynamically alter its asset portfolio by engaging in asset substitution or risk-shifting. The regulator controls the bank's risk-shifting incentives through the threat of intervention in the bank's operations. The optimal regulatory policy, which we characterize analytically, entails an optimal combination of a capital requirement, intervention to control the risk of the bank's portfolio, capital injection, and liquidation of the bank. Under the optimal risk intervention policy, the regulator intervenes when the bank's capital ratio lies inside a "band" consisting of two triggers. We calibrate the model and show that the optimal capital requirement is 23%, which supports the substantially higher capital requirements being proposed in the Basel III accords. Relative to a benchmark unregulated bank, regulation increases bank leverage by 12%, while lowering its credit spread. Capital injection or risk intervention alone has modest impact on the bank's social value. The optimal combination of regulatory policies, however, significantly improves social value by 4% and bank value by 11%. Optimal capital requirements should be counter-cyclical and should be stricter for large banks. Optimal regulation is also significantly affected by monetary and fiscal policies. Overall, our analysis highlights the importance of considering the interactions among different individual regulatory policies as well as the economic environment in designing optimal regulation.

Key Words: capital regulation, asset substitution, bank bailout, prudential regulation

JEL Classification: G20, G21, G28

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I. Introduction

The 2007–2009 financial crisis has engendered a vigorous debate on the prudential regulation of financial institutions. Despite substantial progress in regulation such as the 2010 Dodd-Frank Act and the Basel III agreement a broad consensus has yet to emerge on a number of issues. What forms should regulatory intervention take? At what points should regulators intervene in banks’ operations to control their risk, force liquidation or recapitalization of distressed banks, or bail them out with public funds? What are the relative impacts of different components of regulation—capital regulation, intervention and control of banks’ risk-taking, and capital injection—on banks’ values and social welfare? How sensitive is optimal prudential regulation to the economic environment? Given the inter-temporal nature of banks’ operations and regulatory intervention policies, a dynamic model that is both analytically tractable and calibratable to data can potentially provide qualitative and quantitative guidance on the design and impact of prudential regulation.

We take preliminary steps towards addressing some of the above issues by developing a structural model to analyze the design of prudential bank regulation and its effects on the real and financial decisions of banks. Our objectives are both positive and normative in that we seek to obtain insights into how existing regulatory policies affect the investment and financing decisions of banks as well as the optimal design of prudential regulation. The optimal regulatory policy, which we characterize analytically, is multi-pronged in nature in that it entails imposition of a capital requirement, intervention to control the risk of a bank’s portfolio, injection of capital after the bank becomes insolvent, and liquidation of the bank when it is no longer optimal to keep it active. Our calibration of the model shows that, relative to a benchmark unregulated bank, regulation increases bank leverage substantially, while lowering its credit spread. The optimal capital requirement is consistent with the substantially higher capital requirements proposed in the Basel III accords and advocated by recent studies such as Admati, DeMarzo, Hellwig and Pfleiderer (2011). While capital injection and investment intervention *independently* have modest effects on social value, their optimal combination significantly improves social and bank values suggesting that regulation has substantial economic benefits provided that the entire gamut of intervention tools are appropriately employed. Further, we show that regulation should not be independent of monetary and fiscal policies as well as the economic environment.

In our continuous-time model, a representative bank operates in a regulated environment. The bank’s capital structure consists of equity and debt, and its debt structure requires it to make a fixed payment (per unit time) to debt holders. As in prior literature, there are additional deadweight costs of equity issuance relative to debt issuance (e.g. Giammarino, Lewis and Sappington (1993)). The bank can dynamically alter the distribution of its earnings by changing its investment portfolio. For simplicity, the bank can choose one of two “projects” at any date, where the term “project” is a metaphor for the bank’s portfolio of investments. The bank’s total earnings (before interest payments) evolve as a lognormal process under either of the projects. The first project has a higher

NPV and lower volatility than the second one, thereby creating incentives for asset substitution. The bank’s project choices are only observable by the regulator if it chooses to intervene in the bank’s operations. We allow for a broad gamut of regulatory intervention tools that include the imposition of a capital requirement, intervention in the bank’s project choices, capital injection when the bank is insolvent, and liquidation of the bank. Regulation has potential benefits because, while the bank’s management cares about shareholder value, the regulator cares about the bank’s “social value” that internalizes the claims of shareholders, debtholders and tax-payers. Further, the regulator internalizes the effects of systemic risk — the externality that a bank imposes on the economy through its financial distress.

In the hypothetical environment in which the bank is unregulated, the bank chooses its capital structure at date zero and dynamically chooses its projects to maximize shareholder value. If the bank’s earnings are insufficient to meet debt payments, it can raise additional funds by issuing equity,¹ but incurs costs due to its financial distress. The bank’s capital structure balances the tradeoff between the cost advantage of debt against financial distress costs that include the excess costs of equity issuance in financial distress, the (endogenous) costs associated with asset substitution, and the deadweight costs of liquidation. We solve the bank’s stochastic control problem and obtain a closed-form analytical characterization of the unregulated bank’s optimal behavior. The optimal strategy of the unregulated bank is characterized by three earnings thresholds: the *switching threshold* at which the bank switches between the two projects, the *liquidity threshold* at which debt payments consume all of the bank’s earnings, and the *insolvency threshold* at which the bank’s equity value endogenously falls to zero so that it can no longer meet debt payments. The bank optimally chooses the low-risk project whenever its earnings are higher than the switching threshold, and the high-risk project below the threshold. Because shareholders effectively hold a convex claim, it is optimal for the bank to increase risk when its earnings are sufficiently low to lower the probability of insolvency. We rigorously establish the existence of a *single* threshold at which the bank optimally switches projects that is not a priori obvious given the dynamic nature of the bank’s problem. The bank services debt payments entirely with internal funds when its earnings are above the liquidity threshold, and issues new equity to cover debt obligations when earnings are below the liquidity threshold, but above the insolvency threshold. The bank is unable to continue servicing debt when its earnings fall to the insolvency threshold so that the bank is liquidated.

Next, we analyze the *fully regulated* environment in which the bank is completely controlled by the regulator whose objective is to maximize the bank’s “social” value that internalizes the claims of shareholders, debt holders, and tax payers. To incorporate the systemic effects of the bank on the economy, we also allow the equity issuance, financial distress and liquidation costs in the regulator’s objective function to differ from those of the unregulated bank. We analytically show that

¹It can also liquidate some of its assets, but this is typically costlier than issuing equity because it reduces its earnings capacity (e.g. see Leland (1994)).

the optimal policy of the regulator is described by three thresholds as in the unregulated bank’s problem (although their values differ from those for the unregulated bank) as well as an additional *liquidation threshold*. When the bank’s earnings fall below the insolvency threshold, the regulator takes over the bank and injects capital until either earnings fall further to the liquidation threshold or they increase so that the bank becomes solvent again. In particular, our results demonstrate that, in a dynamic environment, it could also be optimal for the regulator to permit asset substitution by increasing risk when the bank’s earnings are sufficiently low. The intuition is that, because the regulator optimally injects capital when the bank’s earnings fall below the insolvency threshold, the regulator effectively becomes the residual claimant and, therefore, has a “locally” convex objective function akin to that of shareholders of an unregulated bank. Consequently, it could be optimal for the regulator to increase risk to reduce the probability of liquidation and the associated liquidation costs. Nevertheless, because the regulator cares about the bank’s social value, it has lower incentives to engage in asset substitution than the bank’s shareholders so that the regulator’s optimal switching threshold is always lower than that of the unregulated bank.

By comparing the unregulated bank’s problem with that of the regulator, we derive the optimal *regulatory policy* that implements the first best policy described above. The optimal regulatory policy comprises of three components: *capital injection*, *capital regulation*, and *risk intervention*. Capital injection by the regulator prevents costly premature liquidation of banks and the associated social costs. In the presence of a capital injection policy, the regulator imposes an initial capital requirement by setting a cap on the initial leverage ratio of the bank to prevent ex ante risk-taking of the bank in the form of excessive borrowing. The risk intervention policy reduces costs of ex post risk-shifting of the bank and has a “band” structure: the regulator intervenes and forces the bank to choose the low risk project when the bank’s earnings lie in between the unregulated bank’s optimal switching threshold and the regulator’s optimal switching threshold because it is precisely in this interval where the unregulated bank’s and regulator’s optimal project choices differ. Since the latter threshold is lower than the former, the regulator always intervenes nontrivially. In our model, the regulator/social planner provides implicit deposit insurance through injecting capital into an insolvent bank and thus embodies the role of the deposit insurer. We also show that incorporating explicit deposit insurance does not change the optimal regulatory policies in the model.

To quantitatively analyze the impact of prudential regulation, we calibrate the model to data on U.S. banks over the period 1991 – 2008. In the aftermath of the financial crisis, it has become clear that banks were poorly regulated during this period, but many banks were nevertheless subsequently “bailed out” through injection of public funds. Consequently, the model that we calibrate to data is an “intermediate” one where the bank is free to choose its capital structure and projects, but nevertheless implicitly enjoys the protection provided by public capital injection after the bank is no longer able to raise capital to meet debt requirements. In our calibration, we match banks’ average earnings to asset ratio, risk, leverage, and yield spread. In addition, we use the ratio of the economic costs to the fiscal costs of banking crises reported in Hoggarth, Reis, and Saporta (2002)

to calibrate the systemic risk parameters in the model. Our calibration approach, therefore, allows us to differentiate the private and systemic financial distress costs.

Regulation has a substantial impact on bank leverage. Under the calibrated baseline parameters, the fully regulated bank has an optimal leverage of 77%, while the unregulated bank's leverage is 65%. The credit spread (39 bp) of the regulated bank is, however, lower than that of the unregulated bank (67 bp). The reduction in the bank's asset risk due to the regulator's intervention together with the regulator's commitment of capital injection enable the regulated bank to issue much more debt with a lower spread than the unregulated bank. The optimal capital requirement, which is equal to the initial equity ratio in the fully regulated environment, is 23%, substantially higher than that suggested by the Basel II accords, and consistent with the significantly higher capital requirements proposed by the Basel III committee. This finding also supports the proposal of Admati et al (2011) that the capital requirement be set much higher than current regulatory levels (they recommend capital ratios between 20% and 30% in their conclusions). The optimal intervention ratio, which is the level below which the regulator intervenes in the bank's management, is 6.9%. The predicted intervention ratio is broadly consistent with the FDIC definitions of "undercapitalization" and "significant undercapitalization" ratios (8% and 6%) that trigger monitoring and restrictions on banks' activities.²

From an ex ante perspective, optimal regulation increases a bank's value by 11% relative to the unregulated bank, while it increases the bank's "social" value (that also incorporates the costs arising from systemic risk) by 4%. Regulation has an even bigger quantitative impact ex post. We also analyze the relative quantitative impacts of different regulatory tools. Risk intervention on its own improves a bank's social value by only 0.4%, while capital injection on its own reduces it by 1.9%. Together the evidence suggests that there are significant interactive effects of different regulatory tools.

The analytical characterizations of the bank's and regulator's policies greatly facilitate the exploration of various "comparative static" relationships that generate implications for the sensitivity of optimal regulation to changes in the economic environment. The optimal capital requirement decreases with the systemic cost of bank equity that provides a rationale for capital requirements during downswings in the business cycle as the systemic cost is likely to be higher during economic recessions (e.g. Dewatripont, Rochet, and Tirole (2010)). The capital requirement also depends negatively on the bank's private equity issuance cost. As higher taxes increase the private cost of equity,³ our results imply that lower capital requirements should be imposed in a high-tax-rate regime, which implies that bank capital regulation should not be independent of fiscal policy as suggested by Admati et al. (2011). The optimal capital requirement varies negatively and significantly with the risk-free rate implying that bank regulation should also not be independent of

²Details of the relevant rules and regulation can be found on the FDIC website at <http://www.fdic.gov/regulations/laws/rules/2000-1300.html>

³Taxes do not substantially affect the external or systemic cost of equity because they are transfers between different agents.

monetary policy. Banks that operate high NPV and high risk projects should be subject to higher capital requirements, which suggests setting stricter capital requirements on larger banks who are more likely to adopt such projects than smaller banks. We also find that the capital requirement increases with the social cost of financial distress, but is virtually independent of the bank’s private distress cost, suggesting that regulatory policy should be mainly concerned with the systemic risk of the banking sector (e.g. Acharya, Cooley, Richardson, and Walter (2010)).

Finally, we examine the robustness of our results to an extension of the model that allows for the bank to dynamically alter its capital structure with some transaction costs. In the extended model, the optimal *dynamic* capital requirement is 33%, which reflects the fact that the bank can optimally sustain lower leverage levels because it has the option of increasing its debt when its earnings increase sufficiently. The qualitative and quantitative implications of the basic model remain robust to the extended model.

II. Literature Review

We develop a dynamic model to both qualitatively and quantitatively investigate the design and impact of prudential regulation. We build on insights provided by a number of prior studies in the literature. One stream of the literature stresses the role of bank capital in inducing banks’ monitoring efforts (Diamond (1984), Giammarino, Lewis, and Sappington (1993), Holmström and Tirole (1997), Allen, Carletti, and Marquez (2011), Mehran and Thakor (2011), Acharya, Mehran, and Thakor (2011)). Another strand of the literature considers the banks’ liquidity provision role in accepting demand deposits (Diamond and Dybvig (1983), Gorton and Winton (1995), Diamond and Rajan (2000)). In our model, the bank’s special roles in the economy such as liquidity provision are embodied in the excess costs of equity relative to debt and the systemic costs of the bank’s financial distress.

A substantial literature studies the risk-shifting incentives of banks stemming from the early studies of Merton (1977), Kareken and Wallace (1978), and Sharpe (1978).⁴ While Koehn and Santomero (1980) and Kim and Santomero (1988) show that a risk-averse banker may actually select a riskier portfolio when facing capital adequacy requirements, Rochet (1992) points out that the limited liability of the bank implies that capital regulation is still helpful in reducing risk-taking behavior. Dewatripont and Tirole (1994) propose the “representation hypothesis” that regulators monitor and supervise banks on behalf of small depositors who lack the incentives or resources to do so. Marshall and Prescott (2001, 2006) consider state-contingent regulation when the bank has moral hazard in project selection. Also related are studies of shareholder-debtholder conflicts and asset substitution for non-financial firms (Jensen and Meckling (1976), Leland (1998)). In our model, we consider the risk-shifting incentives of banks both in terms of increasing leverage ex ante and increasing portfolio risk ex post.

⁴See Santos (2001, 2006) for reviews of this literature.

We contribute to the above streams of the literature in a number of respects. First, we develop a continuous-time model in which the regulator has a wide gamut of intervention tools consistent with those observed in reality. Second, the bank can dynamically engage in asset substitution, and its capital structure reflects the tradeoff between the excess costs of issuing equity against financial distress costs that include the agency costs arising from asset substitution and liquidation costs. Third, we analytically characterize the optimal regulatory policy, and calibrate our model to data to obtain quantitative insights into the design and value of prudential regulation as well as the relative effects of different components of regulation on the bank's value and its social value.

A few recent studies quantitatively analyze the impact of bank capital regulation, using dynamic models with calibrated parameters. Zhu (2008) and Van den Heuvel (2009) develop dynamic models to evaluate the macroeconomic impact of capital regulation. De Nicolò, Gamba, and Lucchetta (2011) considers the impacts of capital regulation, liquidity requirements, and taxation on banks' behavior and social welfare. They abstract away from the effects of asset substitution. Analogous to the above studies, we too develop a dynamic "calibratable" model that can be used to carry out a quantitative analysis of the design and impact of prudential regulation. Because our model allows analytical solutions, we are able to obtain additional insights on the optimal prudential regulation policies and the behavior of banks in response to these policies.

III. The Model

We develop a continuous-time model of a financial institution that we refer to as a "bank" for concreteness. The time horizon is $[0, \infty)$. Our focus is on shareholder-debtholder agency conflicts so we assume that the bank's management behaves in the interests of its shareholders. Hereafter, we refer to the bank's management as the "bank."

A. The Bank's Earnings and Project Choices

The bank can dynamically alter the distribution of its earnings by choosing its portfolio of projects that we hereafter refer to as its "project." At any date t , the bank can choose one of two possible projects, 1 or 2. In the (infinitesimal) time period $[t, t + dt]$, the bank's *total earnings* (before debt payments) C evolve as follows under the two projects:

$$dC_t = \bar{\mu}_1 C_t dt + \sigma_1 C_t d\bar{B}_t, \text{ if project 1 is chosen,} \quad (1)$$

$$dC_t = \bar{\mu}_2 C_t dt + \sigma_2 C_t d\bar{B}_t, \text{ if project 2 is chosen.} \quad (2)$$

In the above, $\bar{\mu}_1$ and $\bar{\mu}_2$ are the drifts or expected growth rates of earnings under projects 1 and 2, respectively; σ_1 and σ_2 are the volatilities, and \bar{B}_t is a standard Brownian motion that we assume to be the same for the two projects (without loss of generality) to simplify the notation.

The market prices of risk, ς_1, ς_2 , of the two projects and the risk-free rate, r , are constants.

It follows from the principle of *risk-neutral valuation* (see Duffie (2001)) that the market prices of claims to the bank's earnings are given by the expected discount payoffs where the discounting is at the risk-free rate and the expectation is under the risk-neutral measure. Moreover, by Girsanov's theorem, the bank's earnings evolve as follows under the two projects in the risk-neutral measure:

$$dC_t = (\bar{\mu}_1 - \varsigma_1 \sigma_1) C_t dt + \sigma_1 C_t dB_t, \text{ if project 1 is chosen,} \quad (3)$$

$$dC_t = (\bar{\mu}_2 - \varsigma_2 \sigma_2) C_t dt + \sigma_2 C_t dB_t, \text{ if project 2 is chosen.} \quad (4)$$

In the above, B is a Brownian motion under the risk-neutral measure and

$$\begin{aligned} \mu_1 &= \bar{\mu}_1 - \varsigma_1 \sigma_1 \\ \mu_2 &= \bar{\mu}_2 - \varsigma_2 \sigma_2 \end{aligned} \quad (5)$$

are the projects' *risk-neutral* or *risk-adjusted* drifts. To ensure that asset prices are well-defined, we assume that

$$r > \mu_1, \mu_2 \quad (6)$$

Note from (5) that the risk-neutral drifts could well be close to zero or even negative. As our focus is on shareholder-debtholder agency conflicts arising from *asset substitution* or risk-shifting, we assume that project 1 has a higher risk-neutral drift and lower volatility than project 2, that is,

$$\mu_1 > \mu_2 \text{ and } \sigma_1 < \sigma_2. \quad (7)$$

It is easy to see from (3), (4) and (7) that the NPV of the total future earnings from project 1 at any date is greater than that of project 2. Project 2, therefore, has higher risk and lower NPV than project 1. Our analysis can be easily generalized to accommodate the scenario where $\mu_1 > \mu_2$ and $\sigma_1 > \sigma_2$, but the tradeoffs are less interesting and relevant to the real-world regulation of institutions where the mitigation of value-destroying asset substitution plays an important role. Note from (1), (2) and (5) that we could have the *actual drift*, $\bar{\mu}_1$, of project 1 being less than the *actual drift*, $\bar{\mu}_2$, of project 2, consistent with the usual risk-return tradeoff ("high risk- high return") in the physical probability measure, while the *risk-neutral drift* of project 1 is greater.

Henceforth, without loss of generality, we work under the risk-neutral measure. The bank's earnings as well as the project parameters $\mu_1, \mu_2, \sigma_1, \sigma_2$ are observable by all market participants. The bank's actual project choices are, however, only observable by the regulator if it intervenes in the bank's operations. We show later that the regulatory intervention policy can be designed so that the bank's dynamic project choices are optimal from the standpoint of the regulator. Consequently, there is no deadweight loss due to the fact that the regulator can only observe the bank's project choices when it intervenes in the bank's management. The information filtration generated by the earnings process C is \mathcal{F}_t .

B. Capital Structure

The bank's capital structure comprises of equity and debt. For simplicity, we assume that the bank's debt structure requires it to make a constant payment of θ per unit time to debtholders.⁵ The bank's capital structure is determined by the parameter θ .

As in several prior studies (e.g. Giammarino et al. (1993)), equity issuance is costly relative to debt issuance. These costs include direct costs such as the tax disadvantage of equity relative to debt as well as *shadow* costs arising from various sources such as the comparative advantage that banks possess in raising capital through deposits rather than equity, investors' demand for liquid demand deposits, etc. Some of these costs are more naturally interpreted as "flow" costs, while others are easier to interpret as "lump sum" costs that are incurred whenever equity is issued. It would greatly complicate the notation and exposition to separately model "lump sum" and "flow" costs, and it is also hard to independently calibrate the corresponding parameters in our quantitative analysis. Consequently, we choose to model the excess costs of equity as "flow" costs throughout. It should be borne in mind, however, that we can alternately model these as "lump sum" costs or a combination of "flow" and "lump sum" costs. The alternate modeling choices do not affect the main insights of the study.

More precisely, in any period $[t, t + dt]$, as long as the bank is solvent, the cash flow to debt holders is θdt and the cash flow to equity holders is

$$\text{Net cash flow to equity holders} = (C_t - \theta)dt - \lambda_1(C_t - \theta)dt \text{ if } C_t \geq \theta \quad (8)$$

$$= (C_t - \theta)dt - \lambda_2(\theta - C_t)dt \text{ if } C_t < \theta, \quad (9)$$

where

$$\lambda_1, \lambda_2 > 0 \quad (10)$$

In (8), λ_1 is the excess cost of equity modeled as a flow cost. In (9), λ_2 is the proportional cost associated with the bank's financial distress when its earnings are insufficient to meet debt payments. We allow for the bank to raise funds in financial distress by issuing equity and/or borrowing from the central bank. To keep the model parsimonious, we assume that either channel is equally costly for the bank so that we can assume henceforth that the bank issues additional equity in financial distress. We can also allow for the bank to liquidate assets in financial distress, but this is usually costlier than issuing equity because it lowers the bank's cash flow generating capacity (see Leland (1994)).

The bank services debt payments entirely as long as it is solvent. If the bank's earnings are insufficient to meet debt payments, it can issue additional equity. After the equity value falls to zero, the regulator services debt payments by injecting capital until it is no longer optimal to do

⁵More generally, a bank's debt structure includes demand deposits as well as longer-term debt obligations. The explicit incorporation of the different components of a bank's debt structure is well beyond the scope of this study.

so. We assume that the bank cannot easily reduce its debt. In reality, a substantial portion of bank debt comprises of demand deposits that are difficult to close down without the bank incurring substantial costs such as those arising from a loss of reputation because of its important role as a provider of liquidity. Further, if the bank retires debt (either demand deposits or other debt), it usually must do so at par value that is significantly higher than the market value of its debt when it is in financial distress, that is, the value of continuing to service its existing debt as long as it is able to do so.⁶ In the main model, we also assume that the bank does not increase its debt after the initial capital structure decision for simplicity. In Section E and the online Appendix, we extend the model to allow for the possibility that the bank can dynamically adjust its debt structure by issuing more debt when its earnings increase sufficiently. The qualitative and quantitative implications of the main model remain robust to the extended model.

The bank services debt by issuing equity until an \mathcal{F}_t -stopping time τ_B when it becomes insolvent (recall that \mathcal{F}_t is the information filtration generated by the bank's earnings process.) For $t > \tau_B$, the regulator takes control of the bank (in effect, it becomes the owner of the bank), injects capital, and continues to do so until a stopping time $\tau_L \geq \tau_B$. Hence, the bank services debt for $t < \tau_B$, and the regulator services debt by injecting capital for $t \in (\tau_B, \tau_L)$. Note that, in the region (τ_B, τ_L) , the bank's equity value is negative, that is, the regulator effectively holds a negative equity stake in the firm. The bank's assets are liquidated at $t = \tau_L$ when the regulator decides to stop injecting capital, and debtholders receive the resulting liquidation payoff.

There are deadweight liquidation costs that we model by assuming that debtholders receive a proportion $1 - \alpha$; $\alpha \in (0, 1)$ of the *unlevered bank value* at the liquidation threshold. More precisely, it follows immediately from (3) that the value of the (hypothetically) unlevered bank at the liquidation threshold is

$$V_{\tau_L}^{\text{unlevered}} = \frac{(1 - \lambda_1)C_{\tau_L}}{r - \mu_1}. \quad (11)$$

The payoff to debtholders is

$$D_{\tau_L} = (1 - \alpha) \frac{(1 - \lambda_1)C_{\tau_L}}{r - \mu_1} \quad (12)$$

In reality, bank debt comprises of demand deposits that are protected by deposit insurance (up to a limit) as well as longer-term debt obligations that are not insured. Our framework actually incorporates the possibility of (implicit) deposit insurance because we allow the regulator to inject capital after the bank goes bankrupt, and this is also the version of the model that we calibrate. In our setting, the regulator is essentially the “social planner” who also embodies the role of the deposit insurer. We also show in the online appendix that the optimal regulatory policies and thus the normative implications of our study are not affected by the explicit incorporation of deposit insurance.

⁶The banks' shareholders will be worse off if they retire debt at par value or at the market value after debt retirement (Leland (1994), Admati et al (2012)).

C. Regulation

The regulator observes the bank's earnings and can choose to intervene in the bank's management. The regulator's intervention policy could involve controlling the bank's risk as well as injecting capital to keep the bank solvent. As discussed in the previous section, the regulator injects capital in the region (τ_B, τ_L) .

We denote the bank's strategy by an \mathcal{F}_t -adapted process

$$\Pi = (P, \tau_B). \quad (13)$$

In the above, $P_t \in \{1, 2\}$ is the bank's project choice at date t , and τ_B is the \mathcal{F}_t -stopping time at which the bank stops issuing equity to service debt and declares insolvency.

We denote the regulator's intervention policy by an \mathcal{F}_t -adapted process

$$\Pi^{reg} \equiv (P^{reg}, \tau_L). \quad (14)$$

In the above, $P_t^{reg} \in \{1, 2\}$ is the regulator's project choice at date t and τ_L is the \mathcal{F}_t -stopping time at which the regulator stops injecting capital and liquidates the bank. If the regulator forces the bank to choose a particular project at date t , then $P_t = P_t^{reg}$. Note that the above formulation is quite general and encompasses the possibilities that the regulator chooses not to intervene in the bank's operations at a particular date t and control its project/risk choice.

The proportional deadweight social cost of capital injection by the regulator is λ_2^{social} , which could differ from the cost of capital injection by the bank in financial distress. In other words, if the regulator injects capital k_t , the social cost is $\lambda_2^{social} k_t$. The wedge between λ_2^{social} and λ_2 reflects the effects of *systemic risk*. The regulator internalizes the effects of the externality that the bank imposes on the economy because of its financial distress.

D. Equity, Debt, Bank and Social Values

Let Π and Π^{reg} denote strategies for the bank and the regulator, respectively, as defined in the previous section. Suppose that the bank's debt level is θ . At any date t , the value of the bank's debt is

$$D_t(\theta, \Pi, \Pi^{reg}) = E_t \int_t^{\tau_L} e^{-r(u-t)} \theta du + E_t[e^{-r(\tau_L-t)} D_{\tau_L}], \quad (15)$$

where E_t denotes the conditional expectation at date t . The debt payoff D_{τ_L} upon liquidation is given by (12).

By (9), the bank's equity value at date t is

$$S_t(\theta, \Pi, \Pi^{reg}) = E_t \int_t^{\tau_B} e^{-r(u-t)} \left[\begin{array}{l} 1_{C_u(\Pi; \Pi^{reg}) \geq \theta} (1 - \lambda_1)(C_u(\Pi, \Pi^{reg}) - \theta) + \\ 1_{C_u(\Pi; \Pi^{reg}) < \theta} (1 + \lambda_2)(C_u(\Pi, \Pi^{reg}) - \theta) \end{array} \right] du. \quad (16)$$

The upper limit of the integral above is the stopping time $\tau_B \leq \tau_L$ at which the bank becomes insolvent. In the integrand, we explicitly indicate the fact that the earnings process C_u depends on the bank's and the regulator's strategies.

The bank's total value at date t is the sum of its equity and debt values, and is given by

$$\begin{aligned} F_t(\theta, \Pi, \Pi^{reg}) &= D_t(\theta, \Pi, \Pi^{reg}) + S_t(\theta, \Pi, \Pi^{reg}) \\ &= E_t \int_t^{\tau_B} e^{-r(u-t)} \left\{ \left[\begin{array}{l} 1_{C_u(\Pi, \Pi^{reg}) \geq \theta} ((1 - \lambda_1)C_u(\Pi, \Pi^{reg}) + \lambda_1\theta) + \\ 1_{C_u(\Pi, \Pi^{reg}) < \theta} ((1 + \lambda_2)C_u(\Pi, \Pi^{reg}) - \lambda_2\theta) \end{array} \right] \right\} ds \\ &\quad E_t \int_{\tau_B}^{\tau_L} \theta ds + E_t[e^{-r(\tau_L-t)} D_{\tau_L}]. \end{aligned} \quad (17)$$

Finally, we define the bank's *social* value at date t , which also includes the net cash flows to the regulator when it takes control of the bank during the interval $[\tau_B, \tau_L]$ and injects capital.

$$\begin{aligned} F_t^{social}(\theta, \Pi, \Pi^{reg}) &= E_t \int_t^{\tau_L} e^{-r(u-t)} \left\{ \left[\begin{array}{l} 1_{C_u(\Pi, \Pi^{reg}) \geq \theta} ((1 - \lambda_1^{social})C_u(\Pi, \Pi^{reg}) + \lambda_1^{social}\theta) + \\ 1_{C_u(\Pi, \Pi^{reg}) < \theta} ((1 + \lambda_2^{social})C_u(\Pi, \Pi^{reg}) - \lambda_2^{social}\theta) \end{array} \right] \right\} ds \\ &\quad + E_t[e^{-r(\tau_L-t)} D_{\tau_L}^{social}], \end{aligned} \quad (18)$$

where

$$D_{\tau_L}^{social} = (1 - \alpha^{social}) \frac{(1 - \lambda_1^{social})C_{\tau_L}}{r - \mu_1}. \quad (19)$$

The bank's social value (18) differs from its total value (17) in a number of important dimensions. First, the excess cost of equity in "good" states from the regulator's standpoint, λ_1^{social} , could differ in general from λ_1 , reflecting the fact that a portion of the excess cost of equity, such as the tax disadvantage of equity, are simply wealth transfers that do not affect the bank's social value (see Duffie (2001, Chapter 8) and Admati et al. (2011)). On the other hand, other costs of equity issuance such as those affecting a bank's role as an intermediary in issuing deposits are deadweight costs that are not simply transfers. The regulator, therefore, acknowledges the important role that banks play as financial intermediaries and, in particular, providers of liquidity. Second, the financial distress cost λ_2^{social} from the regulator differs, in general, from λ_2 because the regulator incorporates the effects of systemic risk — the externality that the bank imposes on the economy through its financial distress. For similar reasons, the proportional liquidation cost α^{social} as seen

by the regulator also differs from the proportional liquidation cost, α , incurred by the bank.

We first analyze the benchmark scenario where the bank is completely unregulated, that is, the regulator’s intervention policy is the “null” policy of no intervention at all. Hence, the bank’s equity, debt and total values are only functions of the debt level θ and the bank’s project choices Π . Further, because the regulator injects no capital at all, the liquidation time τ_L coincides with the insolvency time τ_B . Given a debt level θ , the bank chooses its optimal strategy—its dynamic project choices and the insolvency time—to maximize its equity value, that is, the bank’s optimal strategy solves

$$\Pi^*(\theta) = \arg \max_{\Pi} S_0(\theta, \Pi). \quad (20)$$

In the above, we explicitly indicate the fact that the bank’s optimal strategy depends on the bank’s debt level. Note that the regulator’s intervention strategy does not appear at all as an argument because it is the “null” policy of no intervention. The bank’s capital structure, which is determined by the debt level θ maximizes its total value at date zero, that is, the bank’s optimal capital structure solves

$$\theta^* = \arg \max_{\theta} F_0(\theta, \Pi^*(\theta)) \quad (21)$$

Next, we consider the “full regulation” or “first best” scenario in which the regulator completely controls the bank and chooses its capital structure as well as its projects to maximize the bank’s social value. Given a debt level θ , the regulator’s optimal strategy—its project choices and liquidation time—solves

$$\Pi^{*reg}(\theta) = \arg \max_{\Pi} F_0^{social}(\theta, \Pi^{reg}). \quad (22)$$

The regulator chooses the bank’s capital structure to maximize the bank’s social value, that is,

$$\theta^{*reg} = \arg \max_{\theta} F_0^{social}(\theta, \Pi^{*reg}(\theta)) \quad (23)$$

To analyze the effects of different regulatory intervention tools—capital regulation, intervention in the bank’s project choices, and capital injection—we also explore a number of “intermediate” scenarios in which one or more of these tools are “turned off.” To avoid complicating the exposition here, we discuss these scenarios when we analyze them later.

IV. The Unregulated Bank

In the hypothetical scenario where the bank is completely unregulated, the bank’s dynamic project choices, the insolvency time, and capital structure solve (20) and (21). We first derive the bank’s optimal dynamic project and insolvency time choices given a capital structure described by the debt level θ and then derive the bank’s optimal capital structure. The following proposition analytically characterizes the bank’s equity and debt values as well as the insolvency threshold for

a given switching policy described by a trigger C_S .

Proposition 1 *[Equity and Debt Values for Given Switching Policy] Suppose the debt level is θ and the bank adopts a policy where it chooses project 1 if $C_t \geq C_S$ and project 2 for $C_t < C_S$. Suppose that $C_S \geq \theta$. (For brevity, we show here the representation for the case $C_S \geq \theta$. In the case where $C_S < \theta$, equity values have similar representations that we present in the appendix.)*

(i) *The bank's equity value at any date t is a continuously differentiable function of the current earnings level C_t and is given by*

$$S_t = S(C_t) = \begin{cases} \frac{(1-\lambda_1)C_t}{r-\mu_1} - \frac{(1-\lambda_1)\theta}{r} + A_1(C_S, \theta)C_t^{\gamma_1^-}, & \text{if } C_t \geq C_S, \\ \frac{(1-\lambda_1)C_t}{r-\mu_2} - \frac{(1-\lambda_1)\theta}{r} + A_2(C_S, \theta)C_t^{\gamma_2^-} + A_3(C_S, \theta)C_t^{\gamma_2^+}, & \text{if } C_S > C_t \geq \theta \\ \frac{(1+\lambda_2)C_t}{r-\mu_2} - \frac{(1+\lambda_2)\theta}{r} + A_4(C_S, \theta)C_t^{\gamma_2^-} + A_5(C_S, \theta)C_t^{\gamma_2^+}, & \text{if } \theta > C_t \geq C_B(C_S, \theta), \end{cases} \quad (24)$$

Furthermore, the equity value satisfies the following value matching and smooth pasting conditions at the insolvency threshold $C_B(C_S, \theta)$

$$S(C_B(C_S, \theta)) = S'(C_B(C_S, \theta)) = 0. \quad (25)$$

(ii) *The bank's debt value at any date t is continuously differentiable and given by*

$$D_t = D(C_t) = \begin{cases} \frac{\theta}{r} + B_1(C_S, \theta)C_t^{\gamma_1^-}, & \text{if } C_t \geq C_S, \\ \frac{\theta}{r} + B_2(C_S, \theta)C_t^{\gamma_2^-} + B_3(C_S, \theta)C_t^{\gamma_2^+}, & \text{if } C_S > C_t \geq C_B(C_S, \theta). \\ D(C_B(C_S, \theta)) = \frac{(1-\alpha)(1-\lambda_1)C_B(C_S, \theta)}{r-\mu_1} \end{cases} \quad (26)$$

The coefficients $A_i(C_S, \theta); i = 1, \dots, 5$; $B_j(C_S, \theta); j = 1, \dots, 3$, and the insolvency level $C_B(C_S, \theta)$ are determined by the conditions that the equity and debt values are continuously differentiable for all $C_t > C_B(C_S, \theta)$ and the value-matching and smooth-pasting conditions (25). In the above, $\gamma_i^+, \gamma_i^-; i = 1, 2$ are the positive and negative roots, respectively, of the equation

$$\frac{\sigma_i^2}{2}\gamma^2 + (\mu_i - \frac{\sigma_i^2}{2})\gamma - r = 0; i = 1, 2. \quad (27)$$

Note that the coefficients $A_i(\cdot), B_i(\cdot)$, and the insolvency level $C_B(\cdot)$ depend on the switching trigger C_S and the debt level θ . The asset values and the insolvency level can be analytically characterized in closed form.

The following result shows that the optimal dynamic project choice policy of the bank has the structure assumed in Proposition 1, that is, there is a single time-independent trigger at which the bank switches between the two projects.

Theorem 2 [*Optimal Switching Policy*] *Given an initial choice of debt level θ , there exists a unique cash flow threshold $C_S^*(\theta)$ such that the bank optimally chooses project 1 when $C_t \geq C_S^*(\theta)$ and project 2 when $C_S^*(\theta) > C_t > C_B^*(\theta)$. $C_B^*(\theta)$ is the insolvency trigger that is determined in Proposition 1 when the switching trigger is $C_S^*(\theta)$. The unique switching threshold $C_S^*(\theta)$ is characterized by the second-order “super contact” condition,*

$$\left. \frac{\partial^2 S}{\partial C^2} \right|_{C=C_S^*(\theta)+} = \left. \frac{\partial^2 S}{\partial C^2} \right|_{C=C_S^*(\theta)-}. \quad (28)$$

By the discussion in Section A, in the absence of any frictions, the bank would always choose project 1 because it has a higher NPV than project 2. In the presence of shareholder-debtholder agency conflicts, however, the bank’s shareholders have the incentive to engage in asset substitution, thereby diverting cash flows from debtholders. Asset substitution potentially occurs when the bank’s earnings are below a threshold so that the bank is sufficiently leveraged. Note that, depending on the drifts and volatilities of the two projects, it may be optimal for the bank to always choose project 1, that is, not engage in risk-shifting at all. This scenario corresponds to the case where the switching threshold $C_S^*(\theta)$ coincides with the insolvency threshold $C_B^*(\theta)$. Condition (28) is equivalent to requiring that the equity value is twice differentiable throughout. As we show in the proof of Theorem 2, the uniqueness of the switching threshold is non-trivial and depends on the fact that the excess cost of equity is a general concave function in the level of cash flows.

Given the debt structure θ , the initial leverage ratio of the bank is $Lev_0 = D(C_0; \theta)/F(C_0; \theta)$, where we also indicate the dependence of the debt value and bank value on the debt level θ . The debt value and bank value are determined by Proposition 1 with the switching trigger set to the optimal switching trigger $C_S^*(\theta)$. We define the bank’s leverage ratio at the switching point $C_S^*(\theta)$ as the *switching leverage*. The switching leverage is given by

$$Switching\ Leverage = D(C_S^*(\theta); \theta)/F(C_S^*(\theta); \theta). \quad (29)$$

As described by Theorem 2, the bank has an incentive to engage in asset substitution by selecting the riskier, lower NPV project 2 once its leverage reaches the switching leverage. Therefore, the switching leverage is of importance to regulators who would like to limit the agency costs of ex post asset substitution. The following proposition provides a simple and useful characterization of the switching point and leverage for a given debt level θ .

Proposition 3 1) *The default boundary $C_B^*(\theta)$ and switching trigger $C_S^*(\theta)$ are linear functions of θ , that is,*

$$C_S^*(\theta) = c_S \theta; \quad C_B^*(\theta) = c_B \theta, \quad . \quad (30)$$

2) *The switching leverage is a constant that does not depend on the initial choice of θ .*

Intuitively, the banker chooses the switching point in proportion to the initial debt level. When the debt level is high, agency costs are larger, and the banker would choose to switch to the risky project at a higher threshold. The switching leverage, however, is not sensitive to the initial choice of debt level. Rather, it is sensitive to the relative characteristics (drifts and volatilities) of the two projects. The result of Proposition 3 is useful because it provides a justification for some aspects of the FDIC rules and Basel accords where intervention by regulators can be triggered when a bank's leverage reaches a pre-specified threshold *regardless of its initial leverage level*.

The unregulated bank's optimal capital structure choice is determined by solving (21). The switching and insolvency triggers at the optimal capital structure, θ^{unreg} (the superscript indicates that this is the scenario with no regulation) are, therefore, given by

$$C_S^{unreg} = C_S^*(\theta^{unreg}) = c_S \theta^{unreg}, \quad C_B^{unreg} = C_B^*(\theta^{unreg}) = c_B \theta^{unreg}, \quad . \quad (31)$$

where the constants c_S and c_B are defined in Proposition 3. Since the optimal capital structure cannot be analytically characterized in closed form, we compute it numerically in our subsequent analysis of the calibrated model.

V. The Fully Regulated Bank

In the scenario in which the regulator completely controls the bank's operations, the regulatory intervention policy is the policy of "total intervention" that is the polar opposite of the "null" intervention policy considered in the previous section. Such a policy achieves the first-best outcome for social welfare. From the regulator's standpoint, the bank's capital structure, project choices, the insolvency time, and the liquidation time should maximize the bank's social value, that is, they should solve

$$(\theta^{reg}, \Pi^{reg}) = \arg \max_{(\theta, \Pi)} F_0^{social}(\theta, \Pi). \quad (32)$$

In the above, the superscript "reg" indicates the fact that this is the scenario where the regulator completely controls the bank's operations.

As in the previous section, we first derive the regulator's dynamic project choices for a given capital structure, and then derive the optimal capital structure. It turns out that the regulator's optimal project choice policy has a similar structure to that of the bank in the previous section. There exists a threshold level of the bank's earnings such that it is optimal for the regulator to choose project 1 above the threshold and project 2 below the threshold. The regulator's optimal "switching threshold" differs, in general, from the bank's switching threshold derived in the previous section. The following proposition analytically characterizes the bank's social value for a given switching policy described by a trigger C_S . We omit its proof because it follows using exactly the same arguments used to prove Proposition 1.

Proposition 4 *[Equity, Debt and social Values for Given Switching Policy] Suppose the debt level is θ and the regulator adopts a policy where it chooses project 1 if $C_t \geq C_S$ and project 2 for $C_t < C_S$. Suppose that $C_S \geq \theta$. (For simplicity, we show here the representation for the case $C_S \geq \theta$. In the case where $C_S < \theta$, social values have similar representations that we present in the appendix.)*

(i) The bank's social value at any date t is a continuously differentiable function of the current earnings level C_t and is given by

$$F_t^{\text{social}} = F^{\text{social}}(C_t) = \begin{cases} \frac{(1-\lambda_1^{\text{social}})C_t}{r-\mu_1} + \frac{\lambda_1^{\text{social}}\theta}{r} + X_1(C_S, \theta)C_t^{\gamma_1^-}, & \text{if } C_t \geq C_S, \\ \frac{(1-\lambda_1^{\text{social}})C_t}{r-\mu_2} + \frac{\lambda_1^{\text{social}}\theta}{r} + X_2(C_S, \theta)C_t^{\gamma_2^-} + X_3(C_S, \theta)C_t^{\gamma_2^+}, & \text{if } C_S > C_t \geq \theta, \\ \frac{(1+\lambda_2^{\text{social}})C_t}{r-\mu_2} - \frac{\lambda_2^{\text{social}}\theta}{r} + X_4(C_S, \theta)C_t^{\gamma_2^-} + X_5(C_S, \theta)C_t^{\gamma_2^+}, & \text{if } \theta > C_t \geq C_L(C_S, \theta). \end{cases} \quad (33)$$

Furthermore, the social value satisfies the following value matching and smooth pasting boundary conditions at the liquidation threshold $C_L(C_S, \theta)$,

$$F^{\text{social}}(C_L(C_S, \theta)) = \frac{(1 - \alpha^{\text{social}})(1 - \lambda_1^{\text{social}})C_L(C_S, \theta)}{r - \mu_1}, \quad (34)$$

$$F^{\text{social}'}(C_L(C_S, \theta)) = \frac{(1 - \alpha^{\text{social}})(1 - \lambda_1^{\text{social}})}{r - \mu_1}. \quad (35)$$

The coefficients $X_i(C_S, \theta); i = 1, \dots, 5$ in the above are determined by the conditions that the social value is continuously differentiable throughout and the value matching boundary condition (34). The liquidation level $C_L(C_S, \theta)$ is determined by the smooth pasting condition (35) for the social value at the liquidation level.

(ii) The bank's equity value at any date t is given by (24) where the switching trigger is C_S . The endogenous insolvency threshold at which the equity value falls to zero is $C_B(C_S, \theta)$, which is determined by the smooth pasting condition in (25).

(iii) The bank's debt value at any date t is

$$D_t = D(C_t) = \begin{cases} \frac{\theta}{r} + B_1(C_S, \theta)C_t^{\gamma_1^-}, & \text{if } C_t \geq C_S, \\ \frac{\theta}{r} + B_2(C_S, \theta)C_t^{\gamma_2^-} + B_3(C_S, \theta)C_t^{\gamma_2^+}, & \text{if } C_S > C_t \geq C_L(C_S, \theta). \end{cases} \quad (36)$$

$$D(C_L(C_S, \theta)) = \frac{(1-\alpha^{\text{social}})(1-\lambda_1^{\text{social}})C_L(C_S, \theta)}{r-\mu_1}$$

Comparing Proposition 1 and 4, we note that the insolvency trigger is now replaced by the liquidation trigger $C_L(C_S, \theta)$.

The following result shows that the optimal dynamic project choice policy of the regulator has a structure similar to that of the bank, that is, there is a single time-independent trigger at which the regulator switches between the two projects.

Theorem 5 *[Regulator's Optimal Switching Policy] Given an initial choice of debt level θ , there*

exists a unique cash flow threshold $C_S^{*reg}(\theta) \leq \theta$ such that the regulator optimally chooses project 1 when $C_t \geq C_S^{*reg}(\theta)$ and project 2 when $C_S^{*reg}(\theta) > C_t > C_L^{*reg}(\theta)$. $C_L^{*reg}(\theta)$ is the liquidation trigger that is determined in Proposition 1 when the switching trigger is $C_S^{*reg}(\theta)$. The unique switching threshold $C_S^{*reg}(\theta)$ is characterized by the second-order “super contact” condition,

$$\left. \frac{\partial^2 F^{social}}{\partial C^2} \right|_{C=C_S^{*reg}(\theta)+} = \left. \frac{\partial^2 F^{social}}{\partial C^2} \right|_{C=C_S^{*reg}(\theta)-}. \quad (37)$$

The above theorem shows that, contrary to received wisdom in anecdotal discussions of prudential regulation, it could actually be optimal for the regulator too to engage in asset substitution if the bank’s earnings are below a threshold. The theorem, however, shows that the regulator’s optimal switching trigger is never greater than the debt level, θ . If the regulator finds it optimal to engage in asset substitution, therefore, it always does so when the bank is in financial distress, which is not the case for the unregulated bank as shown by Theorem 2 and its proof. It is worth mentioning that the regulator’s optimal switching trigger could actually lie below the insolvency threshold at which the bank’s equity value falls to zero, that is, it could lie in the “capital injection” region.

The intuition for why the regulator could also find it optimal to engage in asset substitution is as follows. First, when it injects capital, the regulator effectively holds a (negative) *equity stake* in the bank. The regulator’s objective function in this region is, therefore, akin to that of shareholders of the unregulated bank in the previous section. Consequently, when the bank’s earnings fall sufficiently far, it could be optimal for the regulator to increase risk to lower the expected costs of capital injection after insolvency. Second, by increasing risk, the regulator also delays liquidation and, thereby, lowers the probability of liquidation of the bank and the associated liquidation costs.

Because the regulator cares about the bank’s social value rather than just its equity value, one might expect that it has lower incentives to engage in asset substitution than the bank’s shareholders. As the following proposition shows, however, this is only ensured under additional conditions on the characteristics of the bank’s projects as well as its private and social cost parameters.

Proposition 6 [*Comparison of Bank’s and Regulator’s Optimal Switching Policies*]

Suppose $\frac{1-\lambda_1^{social}}{1-\lambda_1} \geq \frac{1+\lambda_2^{social}}{1+\lambda_2}$ and $\gamma_2^+ > \gamma_1^+$, then the optimal switching trigger for the regulator is less than or equal to the optimal switching trigger for the unregulated bank, that is,

$$C_S^{*reg}(\theta) \leq C_S^*(\theta). \quad (38)$$

If the conditions of the proposition are violated, it could actually be optimal for the regulator to switch to the high-risk project earlier than the bank’s shareholders. In particular, if $\frac{1-\lambda_1^{social}}{1-\lambda_1} < \frac{1+\lambda_2^{social}}{1+\lambda_2}$, which is true, for example, when the social distress cost λ_2^{social} is large, the incidence of asset substitution could actually be *greater* in the regulator’s problem than the bank’s problem. Consequently, rather surprisingly, asset substitution might have significantly positive *social value*.

Analogous to Proposition 3, the following proposition shows that the switching, insolvency, and liquidation thresholds are linear in the debt level θ , and the switching leverage is constant. We omit the proof because it follows along the lines of the proof of Proposition 3.

Proposition 7 1) *The insolvency threshold $C_B^{*reg}(\theta)$, the switching threshold $C_S^{*reg}(\theta)$, and the liquidation threshold $C_L^{*reg}(\theta)$ are all proportional to the debt level θ ,*

$$C_B^{*reg}(\theta) = c_B^{reg}\theta, \quad C_S^{*reg}(\theta) = c_S^{reg}\theta, \quad C_L^{*reg}(\theta) = c_L^{reg}\theta. \quad (39)$$

where $c_B^{reg}, c_S^{reg}, c_L^{reg}$ are constants independent of θ .

2) *The switching leverage is a constant that does not depend on the initial choice of θ .*

The regulator chooses the bank's capital structure to maximize the social value, that is, the optimal debt level θ^{reg} solves

$$\theta^{reg} = \arg \max_{\theta} F_0^{social}(\theta), \quad (40)$$

where the bank's social value, $F_0^{social}(\theta)$, is given by Proposition 4 with the switching trigger set to $C_S^{*reg}(\theta)$. The switching, insolvency and liquidation triggers at the optimal capital structure are, therefore, given by

$$C_S^{reg} = C_S^{*reg}(\theta^{reg}) = c_S^{reg}\theta^{reg}, \quad C_B^{reg} = C_B^{*reg}(\theta^{reg}) = c_B^{reg}\theta^{reg}, \quad C_L^{reg} = C_L^{*reg}(\theta^{reg}) = c_L^{reg}\theta^{reg}. \quad (41)$$

VI. Optimal Regulatory Policy

We now use the results of the previous two sections to describe the optimal intervention policy of the regulator.

First, by (40), it is optimal for the regulator to force the bank to choose a debt level θ^{reg} . For future reference, we define the *initial capital requirement* to be $1 - D(\theta^{reg})/F(\theta^{reg})$, i.e., one minus the initial leverage ratio or the equity capital ratio.

Second, by Theorem 2, given the debt level θ^{reg} , the bank chooses a project choice policy where the switching trigger is $C_S^*(\theta^{reg})$. However, by Proposition 5, from the standpoint of the regulator, it is optimal to switch projects at C_S^{reg} . By Proposition 6, $C_S^{reg} < C_S^*(\theta^{reg})$. In other words, the regulator has lower incentives to shift risk than the bank's shareholders. To align the bank's project choices with that of the regulator, the regulator intervenes when the bank's earnings cross the threshold $C_S^*(\theta^{reg})$ and forces the bank to choose the lower risk project 1. We define the *intervention ratio* to be $1 - Lev_{C_S^*(\theta^{reg})}$, or the capital ratio of the bank at the switching leverage.

Third, by Proposition 5, it is optimal for the regulator to choose the higher risk project 2 when earnings fall below the threshold C_S^{reg} . Consequently, it is optimal for the regulator to stop intervening in the bank's operations and allow it to shift risk in this region. Hence, the regulator follows a "band policy," that is, it intervenes in the bank's project choice in the region $(C_S^{reg}, C_S^*(\theta^{reg}))$.

Fourth, by (41), the insolvency level at which the equity value falls to zero is C_B^{reg} and the liquidation level is C_L^{reg} . The regulator, therefore, allows the bank to issue equity to service debt payments as long as the earnings exceed C_B^{reg} . When earnings fall below C_B^{reg} , but exceed C_L^{reg} , it is optimal for the regulator to inject capital to keep the bank afloat.

Fifth, if earnings hit the level C_L^{reg} , it is optimal for the regulator to liquidate the bank's assets and pay off debtholders with the liquidation proceeds.

Figure 1 plots sample paths for a fully regulated bank to illustrate the above optimal regulatory policy. The bank begins with an earnings level above the debt level θ^{reg} and chooses the low-risk project 1. When earnings hit the *liquidity threshold* θ^{reg} , the bank begins to issue external equity in order keep servicing debt. When earnings further declines to *the bank's switching threshold* $C_S^*(\theta^{reg})$, despite the bank's incentive to switch to the high-risk project 2, the regulator steps in and forces the bank to continue with the low-risk project 1. For illustrative purposes, the figure plots a particular case where asset substitution occurs when the bank is in financial distress, i.e., $\theta^{reg} > C_S^*(\theta^{reg})$. In general, however, it is possible for the bank to engage in asset substitution even when it is not in financial distress, i.e., $\theta^{reg} \leq C_S^*(\theta^{reg})$. When the bank's earnings reaches the *insolvency threshold* C_B^{reg} , the bank's equity value falls to zero and the regulator bails out the bank by starting to inject capital into the bank. The regulator switches the bank's project to the high-risk project 2 when earnings reaches *the regulator's switching threshold* C_S^{reg} , and liquidates the bank if earnings further deteriorates to the *liquidation threshold* C_L^{reg} . The figure shows that the regulator's intervenes in the bank's project choices when earnings lie between the intervention band $[C_S^{reg}, C_S^*(\theta^{reg})]$. In another sample path plotted in the figure, the bank's earnings improve and the bank stops issuing equity when earnings rise to the liquidity threshold θ^{reg} .

VII. Quantitative Analysis

To obtain a reasonable set of baseline parameter values for our quantitative analysis of the effects of regulation, we calibrate the model to data on U.S. banks over the period 1991–2008. It has become evident, over most of this time period, U.S. banks were only weakly regulated, if at all. Yet, many distressed banks were bailed out with public funds after the crisis. Moreover, a substantial portion of bank liabilities comprises of demand deposits that are insured (up to a limit). Consequently, the model that we calibrate to the data is an “intermediate” one where the bank chooses its capital structure and projects, but enjoys the “safety net” provided by capital injection after the bank's equity value falls to zero.

Without loss of generality, we normalize the bank's initial earnings, C_0 —the state variable in the model—to 1. The set of model parameters that need to be calibrated is

$$\Gamma \equiv \left\{ r, \mu_1, \mu_2, \sigma_1, \sigma_2, \lambda_1, \lambda_1^{social}, \lambda_2, \lambda_2^{social}, \alpha, \alpha^{social} \right\}$$

We set the risk-free rate r to 0.035 to match the average short-term U.S. treasury rate over the period 1991 – 2008.⁷ Tax shields on debt interest payments represent one of the sources of equity issuance costs faced by individual banks. However, from a social standpoint, tax shields are merely wealth transfers, which would suggest that λ_1^{social} should be less than λ_1 . However, the bank’s personal costs of equity issuance may not completely reflect the “general equilibrium” effects of the redistribution of wealth, and the relative composition of securities issued by the bank among heterogeneous investors some of whom prefer information-insensitive debt-like securities, while others prefer risky equity-like securities. These effects could cause λ_1^{social} to even exceed λ_1 . Since it is difficult (if not impossible) to calibrate λ_1^{social} , we set it equal to λ_1 in our baseline analysis, and subsequently explore the effects of varying its value. The liquidation cost $\alpha = 0.20$ is in the ranges estimated by Andrade and Kaplan (1998) and Davydenko, Strebulaev, and Zhao (2011).

We adopt a relatively novel approach to calibrate the social distress and liquidation cost parameters λ_2^{social} and α^{social} by trying to match the ratio of economic costs to fiscal costs of bailouts incurred by a banking crisis. By examining a sample of banking crises around the world, Hoggarth, Reis, and Saporta (2002) estimate the average annual fiscal costs incurred in a banking crisis is 3.8% of the country’s GDP and the average annual economic costs (GDP losses) of a banking crisis is 4.4% of GDP. Hoggarth et al (2002) also estimate that about 85% of the economic costs are directly caused by the banking crisis. Therefore, the ratio of economic costs to fiscal costs of a banking crisis is on average $3.8\% \times 0.85 / 4.4\% = 1.02$. In our model, we proxy for the fiscal and economic costs by using the ex ante expected costs of capital injection and the expected social distress costs, respectively, that are given by

$$InjCost_0 = E_0 \left[\int_{\tau_B}^{\tau_L} e^{-rt} (C_t - \theta) dt \right], \quad (42)$$

$$DistressCost_0^{social} = E_0 \left[\int_0^{\tau_L} e^{-rt} \lambda_2^{social} (C_t - \theta) 1_{C_t \leq \theta} dt \right]. \quad (43)$$

There are closed-form representations of the above two quantities in our model (the formulae are provided in the online appendix). To reduce the number of free parameters, we also assume that

$$\frac{\alpha^{social}}{\alpha} = \frac{\lambda_2^{social}}{\lambda_2}, \quad (44)$$

so that the ratio of the social to the private liquidation costs are equal to the ratio of the social to the private financial distress costs.

To calibrate the project return and risk characteristics, we use Compustat data for U.S. banks over the sample time period . In the model, the bank chooses the low risk project except possibly in financial distress. Accordingly, we set the risk, σ_1 , of the low-risk project to 0.126 to match the

⁷We consider this time period because the Basel I Accord was published in 1988 and enforced by G-10 countries in 1992. The risk-free rates are downloaded from Kenneth French’s website.

asset risk of the average bank in the sample.⁸ We set the risk of the high-risk project σ_2 to 0.148, which corresponds to the 75th percentile of the asset risk of banks in the sample. In the absence of default risk, the bank's value is

$$V_0 = \frac{(1 - \lambda_1)C_0}{r - \mu_1} + \frac{\lambda_1\theta}{r}, \quad (45)$$

where θ is the interest payment. The ratio of the average bank's earnings to assets and interest payments to assets are 3.8% and 2.8%, respectively. Setting C_0/V_0 to 3.8% and θ/V_0 to 2.8% in (45), respectively, we obtain

$$\mu_1 = r \left(1 - \frac{(1 - \lambda_1)0.038}{r - 0.028\lambda_1} \right) \quad (46)$$

To avoid introducing additional arbitrary parameters in our analysis, we assume that there is a single parameter π such that the projects' risk-neutral drifts are given by

$$\mu_1 = r - \pi\sigma_1 \quad (47)$$

$$\mu_2 = r - \pi\sigma_2, \quad (48)$$

From (48), we have

$$\frac{r - \mu_1}{\sigma_1} = \frac{r - \mu_2}{\sigma_2} \quad (49)$$

We then calibrate the projects' risk-neutral drifts μ_1, μ_2 , and the cost parameters $\lambda_1, \lambda_2, \lambda_2^{social}$, and α^{social} as follows. We calibrate these parameters by matching the average leverage and credit spread of banks and the economic-fiscal cost ratio. The average leverage ratio of banks in the sample is 87%. Krishnan, Ritchken, and Thomson (2006) find that the credit spread of 10-year bonds issued by average highly rated (A- or better) banks is 106 bp. We choose $\mu_1, \mu_2, \lambda_1, \lambda_2, \lambda_2^{social}$, and α^{social} so that the equations (44), (46) and (49) are satisfied, and the optimal initial leverage ratio, the credit spread, and the ratio of expected social distress costs to expected capital infusion costs match the empirical values (87%, 106 bp, and 1.02) as closely as possible. We thereby obtain $\mu_1 = -0.02\%$, $\mu_2 = -0.62\%$, $\lambda_1 = 0.288$, $\lambda_2 = 0.442$, $\lambda_2^{social} = 0.6$, and $\alpha^{social} = 0.272$. It is important to note that, because μ_1 and μ_2 are the projects' *risk-neutral* or *risk-adjusted* drifts, they can be substantially lower than the drifts of the projects under the *actual* or physical measure. In particular, they can be zero or even negative. The following table shows the calibrated parameters.

⁸We calculate the asset risk by the formula $\sigma_A = \sqrt{(\theta_D\sigma_D)^2 + (\theta_E\sigma_E)^2}$, where θ_D and σ_D are the fraction and risk of bank debt, and θ_E and σ_E are the fraction and risk of equity. We assume that the risk of deposit debt is zero. The debt risk σ_D is estimated to be 0.08, the same as the long-term treasury bond, and the equity risk is estimated by monthly stock returns of U.S. banks over the period 1991–2010.

	Parameter	Calibrated Value
Risk-free rate	r	0.035
Risk-neutral/Risk-adjusted drift of low-risk project	μ_1	-0.0002
Volatility of low-risk project	σ_1	0.126
Risk-neutral/Risk-adjusted drift of high-risk project	μ_2	-0.0062
Volatility of high-risk project	σ_2	0.148
Excess Costs of Equity	$\lambda_1, \lambda_1^{social}$	0.288
Private Distress Cost of Equity	λ_2	0.442
Social Distress Cost of Equity	λ_2^{social}	0.600
Private Liquidation Cost	α	0.200
Social Liquidation Cost	α^{social}	0.272

A. Impact of Regulation

In our first set of numerical experiments, we compare the fully regulated bank with the fully unregulated bank. The parameters of the model take their baseline values described in the previous sub-section. We compute the optimal leverage ratios and risk-shifting policies of the unregulated and fully regulated bank, respectively, as described in Sections IV and V, respectively. Table I displays the results.

First, we note that the optimal leverage of the regulated bank (77%) is much higher than that of the unregulated bank (65%), but its credit spread is lower (39 bp versus 67 bp), which is due to two factors. First, the regulator’s intervention in the bank’s project choices substantially reduces the risk of the bank. Second, the promise of capital injection by the regulator reduces the effective cost of debt. These factors enable the regulated bank to borrow much more than an unregulated bank, but with a lower credit risk. By the discussion in Section VI, the optimal leverage of the regulated bank implies an initial capital requirement of 23% imposed by the regulator, which is consistent with the Basel III committee’s proposal to require banks to hold much more equity relative to the Basel II requirements, and with the much higher capital ratios proposed by recent studies such as Admati et al (2011).

Second, the regulator prevents asset substitution from happening by intervening in the bank’s management at the switching threshold of the bank, or at the intervention ratio of 6.9%. Interestingly, consistent with Theorem 5, the regulator also finds it optimal to engage in risk-shifting when the bank’s earnings are sufficiently low (0.327). In the baseline model, the risk-switching threshold of the regulator is less than the insolvency/bailout threshold (0.542). In other words, the regulator shifts risk when its equity stake in the bank is negative. The risk-shifting behavior of the regulator is consistent with anecdotal evidence of an increase in the risk of Irish banks *after* regulators took over their operations.

Third, the ex ante benefit of optimal regulation is economically significant and equals 4% of the

bank value. Because there are additional deadweight social costs of capital injection, the bank's value (equity value plus debt value) is greater than the social (or social) value of the bank. Optimal regulation boosts the bank's value by 11%, implying huge benefits for the equity and debt holders of the bank. The benefits to the bank's claimholders are, however, partially offset by a significant external social cost of capital injection, equaling 7% of bank value.

Fourth, regulation has an even greater impact on the bank's value *ex post* at the insolvency boundary. The ex post debt value at insolvency increases from the case with no capital injection to the case where the regulator injects capital by 83%.⁹ Even after excluding the external social cost of injection, the net benefit of the bank bailout is 29% of the value of the unbailed bank.

B. Impact of Individual Regulatory Policies

We further explore the effects of different regulatory tools by analyzing environments in which the regulator has less than the full set of regulatory tools considered in the previous section. By considering individual regulatory policies, we isolate the effects of different components of bank regulation. Table II compares the solutions in several environments where the regulator only employ certain subsets of regulatory tools.

Capital Injection

We first study the scenario in which the regulator can promise to make capital injections but has no control over the bank's project choices or its leverage. Column 1 of Table II reports the solution to the regulator's problem in this environment. In this setting, the regulator injects capital after equity value drops to zero, thereby providing implicit insurance to debt holders. This allows the bank to choose a much higher initial leverage (87%) than in the unregulated environment (65%). Capital injection alone actually reduces the bank's social value by -1.9%. The ex ante costs of capital injection arises from the fact that the bank is able to engage in asset substitution without intervention by the regulator. Unconstrained asset substitution by the bank not only decreases social value, but also leads the regulator to inject less capital because the bank is more risky and less worthy of salvation.

Risk Intervention

We now examine the environment in which the regulator can restrict the bank's risk choices, but does not inject capital into the bank. This scenario seems particularly relevant given the vigorous academic and political debate on the rationales for capital injection.

Column 3 of Table II reports the optimal policies in such an environment, and compares them with those in the unregulated environment. Because the regulator does not incur external capital

⁹Note that at the insolvency boundary or later, the bank value is equal to the debt value of the bank.

injection costs in this setup, the bank value is equal to the social value. Therefore, the optimal initial leverage for the bank is the same as that for the regulator. This optimal leverage is 66%, slightly higher than that in the unregulated case (65%), because of the reduced asset substitution problem. Ex ante, capital regulation in this setup improves firm/social value by only 0.4%.

Comparing Column 2 and 3 in Table II, we see that capital injection should not be used alone as a regulatory tool, and prudential regulation that aims at reducing asset substitution is more effective. The combination of these policies together with the capital requirement (Column 6 of Table II), however, yields a much greater ex ante benefit (4%) than the benefit of capital regulation (0.4%) without injection. Therefore, our results suggest that there is a large impact of the interaction between capital regulation, risk intervention, and capital injection policies, when they are all optimally chosen.

Regulation without Initial Capital Requirement

We next consider the environment where the regulator optimally intervenes in the bank's project choices, injects capital, and liquidates the bank, but does not impose an initial capital requirement. Studying such an environment allows us to understand the role of the initial capital requirement when other mechanisms such as investment intervention and capital injection are in place.

Column 4 of Table II compares the policies in such an environment with those in the unregulated environment. When the initial leverage is unrestricted, the optimal leverage chosen by the bank is 100% and the bank value increases by 31% compared to the unregulated case. Meanwhile, the social value of the bank decreases by 20% relative to the unregulated bank. The intuition behind these results is that because investment intervention and capital injection by the regulator protect debtholders, the bank exploits these protection mechanisms by increasing its debt as much as possible, thereby substantially increasing the expected social costs of capital injection. These results illustrate the necessity of imposing a minimum initial capital requirement, especially when other regulatory tools that aim at reducing bank risk are in place.

Regulation without Risk Intervention

Finally, we consider the environment in which the regulator imposes initial capital requirement, injects capital and liquidates the bank optimally, but does not intervene in the bank's project choices until it becomes insolvent.¹⁰ Comparing this scenario with the full regulation case will allow us to evaluate the incremental value of risk intervention policy.

Column 5 of Table II presents the optimal policies in this environment. The optimal capital

¹⁰The two remaining possible combinations of individual regulatory policies are identical to two scenarios we consider above. The case where the regulator imposes only capital requirement is the same as the case of the unregulated bank because without bailout, the bank optimizes capital structure in the same way as the regulator. Similarly, the other case where the regulator imposes capital requirement and intervene in the bank's risk choices is the same as the case where the regulator implements only the risk intervention policy.

requirement is 28%, higher than that under full regulation. Without risk intervention, the bank's unconstrained asset substitution practices increase default risk and thus a higher capital requirement is necessary. The social benefit of regulation in this environment is 2.3%, or 1.6% less than that in the full regulation case. Therefore, risk intervention adds significant social benefit, even when the initial capital requirement is optimally imposed. We also note that the incremental value of risk intervention (1.6%) over other policies is substantially higher than the social benefit of risk intervention over the unregulated bank (0.4%). This indicates that when multiple regulatory policies are employed together, there can be significant interactive (in this case positive) effects on the social value of the bank.

C. Impact of Changes in Initial Capital Requirement

We now analyze the scenario where the regulator imposes an initial capital requirement that may be suboptimal, but administers other regulatory policies optimally. Figure 2 presents the impact of the initial capital requirement in such an environment. Recall that the optimal leverage is equal to one minus the initial capital requirement. When initial capital requirement varies, the intervention ratio stays constant (Proposition 7 shows that the switching leverage and thus the intervention ratio is independent of initial capital structure). The ex ante social benefit of regulation decreases substantially when the initial capital requirement is relaxed. For example, when the initial capital requirement is equal to 8%, i.e., the Basel II requirement, the ex ante social benefit decreases by 3.5% of bank value and almost vanishes. Our result thus suggests that adopting the optimal capital requirement, which is much higher than those required by the Basel II accords, can improve social welfare substantially.

D. Comparative Statics

We now examine how variations in the values of key underlying parameters affect the optimal regulatory policies and the associated benefits. The closed-form solution of the model greatly simplifies the computation of these “comparative static” relationships. Figure 3 shows the impact of changes in the parameters on optimal regulation.

The Systemic and Internal Costs

Panels A and B of Figure 3 display the effects of the social cost of equity issuance λ_1^{social} and the bank's internal cost λ_1 . The initial capital requirement decreases with and is very sensitive to the social cost λ_1^{social} . When λ_1^{social} is high, the regulator has a strong preference for the bank to preform its welfare-improving functions, such as providing demand deposits, and therefore, imposes a less strict capital requirement on the bank. As a consequence, at times when the functions of banks are crucial, for example, during an economic depression or credit crunch, the regulator

should optimally reduce the capital requirement of banks. Our model thus provides a rationale for lower capital requirements during recessions that is proposed by many critiques of the Basel II requirement (see e.g. Drumond (2009) and Dewatripont, Rochet, and Tirole (2010)).

The initial capital requirement also depends negatively on the bank's personal cost of equity λ_1 (to a lesser extent than λ_1^{social}), for a very different reason. The (unreported) optimal regulatory initial debt level is virtually independent of λ_1 , which implies that the optimal regulatory initial leverage is higher (and capital requirement is lower) for higher λ_1 since the bank's equity value depends negatively on λ_1 . The intervention ratio is relatively insensitive to changes in λ_1^{social} or λ_1 . The intuition is that the incentives for risk-shifting, which determine the optimal switching trigger and, therefore, the capital adequacy ratio, mainly depend on the project characteristics, and not on the other parameters of the model. The ex ante social benefits of regulation increase sharply as λ_1^{social} increases due to the higher initial leverage which allows the bank to perform its functions better. In contrast, the social benefits are not sensitive to changes in λ_1 as the regulator does not incorporate the costs that are only internal to the bank.

Here we can relate our findings to tax policy and capital regulation. When the government imposes a higher corporate tax rate on banks, it makes the bank's internal cost of equity λ_1 higher, but should not affect the social cost λ_1^{social} as the tax policy is simply a wealth transfer.¹¹ Therefore, our comparative statics suggest that in a high-tax-rate regime, capital requirements should be less stringent, because higher tax rates decrease banks' equity values, but do not affect debt values substantially. Importantly, our results suggest that bank capital regulation should not be independent of fiscal policy as suggested by Admati et al. (2011).

Panels C and D show the comparative statics for the systemic cost λ_2^{social} and the bank's personal distress cost λ_2 . In contrast to the result with λ_1^{social} , when λ_2^{social} increases, the optimal initial capital requirement increases due to the higher systemic cost of distress leading to a lower optimal initial leverage. Since the regulator is concerned only with the systemic cost of distress, the optimal regulatory policies and social benefits are virtually independent of the bank's internal distress cost λ_2 .

Panels E and F show the effects of the social and personal costs of liquidation α^{social} and α . Surprisingly, the optimal initial capital requirement changes little with α^{social} or α , contrary to the usual intuition that leverage should decrease with default costs. The reason is that the regulator is able to inject capital, delay liquidation, and provide significant insurance to the debt holders, offsetting the negative impact on debt value and optimal leverage of high liquidation cost. Consistent with the above intuition, for higher liquidation cost α^{social} , the ex ante benefits of regulation is larger.

¹¹This is a simplified analysis because tax policy may have social benefits or costs associated with redistribution of wealth, which is beyond the scope of this paper.

The Risk-Free Rate

Panel G of Figure 3 shows the impact of varying the risk-free interest rate r . The optimal initial capital requirement decreases with the risk-free rate. This implies that capital regulation should not be independent of monetary policy, and keeping a constant capital requirement as recommended by the Basel II accords could be sub-optimal. From the regulator's point of view, a looser monetary policy should be combined with a tighter capital requirement. Our findings also suggest that it could be sub-optimal to impose the same capital requirements in different countries with different monetary policies.¹²

In contrast with the effects of the risk-free rate on the initial capital requirement, the intervention ratio is relatively insensitive to the risk-free rate, for the same reason discussed above. The social benefit of regulation increases slightly with the risk-free rate, possibly due to the fact that the benefits of reducing asset substitution and capital injection are higher when the initial leverage of the bank is higher, i.e., when initial capital requirement is lower.

Project Risk-Neutral Drifts and Volatilities

Panel H of Figure 3 shows the effects of varying the risk-neutral drift, μ_1 , of the low-risk project. The intervention ratio declines sharply when the risk-neutral drift μ_1 increases. The intuition is that, when the risk-neutral drift of the low-risk project increases, the bank's incentives for risk-shifting decline. The optimal initial capital requirement, however, increases with μ_1 , because a higher risk-neutral drift implies a lower debt capacity of the bank. The social benefit of regulation decreases with μ_1 due to the leverage effect.

Panel I shows the effects of varying the volatility of the low-risk project σ_1 . When σ_1 increases, the bank's average risk increases. Consequently, the optimal leverage declines, and the initial capital requirement increases. Perhaps surprisingly, the intervention ratio declines with σ_1 . As σ_1 increases, the incentives for choosing the high-risk project decline because the wedge between the volatilities of the two projects declines. Consequently, the optimal intervention ratio is lower. In other words, even though the average risk of the bank is higher, the capital ratio at which the regulator intervenes is actually *lower*.

The above results suggest that prudential regulation should depend on the nature of particular operations of financial institutions. For example, larger commercial banks may regularly invest in projects that have higher NPVs and higher volatility than smaller community banks, which implies that they should be subject to higher capital requirements.

Finally, Panels J and K present the comparative statics for the risk-neutral drift, μ_2 , and volatility, σ_2 , of the high-risk project. The intervention ratio is very sensitive to these parameters. The intuition is straightforward. An increase in the risk-neutral drift and/or the volatility of

¹²Acharya (2003) makes a similar point using a very different framework and for different reasons.

the high-risk project *aggravate* the asset substitution problem that leads to higher optimal capital requirement. The initial capital requirement, however, is largely stable with μ_2 and σ_2 , which reflects the fact that the optimal leverage is primarily driven by the characteristics of the low-risk project because this is the project that the bank chooses in “normal” circumstances.

E. Dynamic Debt Restructuring

In this section we present the implications of an extension to the basic model that incorporates the possibility that the bank can dynamically adjust its leverage by issuing more debt when its leverage ratio falls sufficiently (see Goldstein, Ju, and Leland (2001)). We describe the details of the extended model in the online appendix. Table III compares the optimal policies for the unregulated bank and the fully regulated bank, under the baseline parameters. We note that because of the scale-invariant property of the model (see the online Appendix for details), the optimal regulatory initial leverage (67%) is the same as the leverage after each restructuring, which implies a dynamic capital requirement of 33%. The intuition for the lower leverage and higher capital requirement compared to the basic model is that the bank does not have to borrow as much initially when it anticipates future upward restructuring opportunities. Optimal regulation increases social value by 3.1% and bank value by 7.8%. The impact of regulation is, therefore, quantitatively similar to its impact in the basic model (Table I).

VIII. Conclusions

We investigate the optimal design of prudential bank regulation using a continuous-time structural model. The optimal regulatory policy, which we characterize analytically, entails a combination of capital regulation, risk intervention, capital injection, and liquidation. The regulator intervenes to control a bank’s risk-taking when its capital ratio lies in an interval. We calibrate the model to data and, in particular, indirectly infer the private and social costs of banks’ financial distress. Our calibration suggests that the optimal capital requirement supports the substantially higher capital requirements being proposed by the Basel III committee. Prudential regulation can achieve substantial social benefits when it is appropriately designed, and there are significant interactive effects of different regulatory tools. Capital requirements should be counter-cyclical, large banks should be subject to stricter capital requirements, and optimal regulation should not be independent of fiscal and monetary policies.

Appendix. Proofs

Proof of Proposition 1

By (16),

$$S_t = E_t \int_t^{\tau_B} e^{-r(u-t)} [1_{C_u \geq \theta} (1 - \lambda_1)(C_u - \theta) + 1_{C_u < \theta} (1 + \lambda_2)(C_u - \theta)] du. \quad (\text{A1})$$

From the above, the equity value must satisfy the following flow equation (expressed in infinitesimal form for convenience)

$$S_t = (1_{C_t \geq \theta} (1 - \lambda_1)(C_t - \theta) + 1_{C_t < \theta} (1 + \lambda_2)(C_t - \theta)) dt + E_t [e^{-r dt} S_{t+dt}]. \quad (\text{A2})$$

Subtracting the L.H.S. from the R.H.S., dividing throughout by dt and eliminating terms of $o(dt)$, we get

$$E_t \left[\frac{S_{t+dt} - S_t}{dt} - r S_t + (1_{C_t \geq \theta} (1 - \lambda_1)(C_t - \theta) + 1_{C_t < \theta} (1 + \lambda_2)(C_t - \theta)) \right] = 0.$$

Applying Ito's lemma to the above, it follows from (3) and (4) that the equity value $S(C)$ must satisfy the following system of ODEs

$$\begin{aligned} \frac{1}{2} \sigma_1^2 C^2 \frac{d^2 S}{dC^2} + \mu_1 C \frac{dS}{dC} - r S + (1_{C \geq \theta} (1 - \lambda_1)(C - \theta) + 1_{C < \theta} (1 + \lambda_2)(C - \theta)) &= 0 \text{ for } C > C_S \\ \frac{1}{2} \sigma_2^2 C^2 \frac{d^2 S}{dC^2} + \mu_2 C \frac{dS}{dC} - r S + (1_{C \geq \theta} (1 - \lambda_1)(C - \theta) + 1_{C < \theta} (1 + \lambda_2)(C - \theta)) &= 0 \text{ for } C < C_S. \end{aligned} \quad (\text{A3})$$

(i) Suppose that $C_S > \theta$. The solution of the above system of ODEs is as follows.

$$\begin{aligned} S(C) &= A_1 C^{\gamma_1^-} + A_1' C^{\gamma_1^+} + \frac{(1 - \lambda_1)C}{r - \mu_1} - \frac{(1 - \lambda_1)\theta}{r} \text{ for } C > C_S \\ &= A_2 C^{\gamma_2^-} + A_3 C^{\gamma_2^+} + \frac{(1 - \lambda_1)C}{r - \mu_2} - \frac{(1 - \lambda_1)\theta}{r} \text{ for } C_S > C > \theta \\ &= A_4 C^{\gamma_2^-} + A_5 C^{\gamma_2^+} + \frac{(1 + \lambda_2)C}{r - \mu_2} - \frac{(1 + \lambda_2)\theta}{r} \text{ for } C < C_B, \end{aligned} \quad (\text{A4})$$

where γ_i^+, γ_i^- are the positive and negative root, respectively of (27). (It is easy to show that (27) has one positive and one negative root.) In the above, we have suppressed the dependence of the coefficients $A_1, A_1', A_2, A_3, A_4, A_5$ on C_S and θ .

Because $S(C) \sim \frac{(1 - \lambda_1)C}{r - \mu_1}$ as $C \rightarrow \infty$, we must have $A_1' = 0$. Further, because the equity value is zero at insolvency, and insolvency is optimally chosen by the bank to maximize its equity value, it follows from well-known arguments (e.g. see Leland (1994)) that the equity value must satisfy the value matching and smooth pasting conditions (25), which ensure that the equity value is differentiable at the insolvency threshold C_B . The coefficients A_1, \dots, A_5 and the insolvency threshold C_B are determined by the conditions that the equity value must be differentiable at the switching threshold C_S , the debt level θ and the insolvency threshold C_B , that is, we have 6 unknowns and 6 equations.

In the case of $C_S < \theta$, we can use similar arguments to those used above to show that the equity value is given by

$$S_t = S(C_t) = \begin{cases} \frac{(1-\lambda_1)C_t}{r-\mu_1} - \frac{(1-\lambda_1)\theta}{r} + A_1(C_S, \theta)C_t^{\gamma_1^-}, & \text{if } C_t \geq \theta, \\ \frac{(1+\lambda_2)C_t}{r-\mu_1} - \frac{(1+\lambda_2)\theta}{r} + A_2(C_S, \theta)C_t^{\gamma_1^-} + A_3(C_S, \theta)C_t^{\gamma_1^+}, & \text{if } \theta > C_t \geq C_S, \\ \frac{(1+\lambda_2)C_t}{r-\mu_2} - \frac{(1+\lambda_2)\theta}{r} + A_4(C_S, \theta)C_t^{\gamma_2^-} + A_5(C_S, \theta)C_t^{\gamma_2^+}, & \text{if } C_S > C_t \geq C_B(C_S, \theta). \end{cases} \quad (\text{A5})$$

(ii) Using arguments similar to those used for the equity value, the debt value satisfies the following system of ODEs:

$$\begin{aligned} \frac{1}{2}\sigma_1^2 C^2 \frac{d^2 D}{dC^2} + \mu_1 C \frac{dD}{dC} - rD + \theta &= 0 \text{ for } C > C_S \\ \frac{1}{2}\sigma_2^2 C^2 \frac{d^2 D}{dC^2} + \mu_2 C \frac{dD}{dC} - rD + \theta &= 0 \text{ for } C < C_S. \end{aligned} \quad (\text{A6})$$

The general solution to the above system is

$$\begin{aligned} D(C) &= B_1 C^{\gamma_1^-} + B_1' C^{\gamma_1^+} + \frac{\theta}{r} \text{ for } C > C_S \\ &= B_2 C^{\gamma_2^-} + B_3 C^{\gamma_2^+} + \frac{\theta}{r} \text{ for } C < C_S. \end{aligned}$$

Since $D(C) \sim \frac{\theta}{r}$ as $C \rightarrow \infty$, $B_1' = 0$. By (12), we must have

$$D(C_B) = \frac{(1-\alpha)(1-\lambda_1)C_B}{r-\mu_1}. \quad (\text{A7})$$

The coefficients B_1, B_2, B_3 are determined by (A7) and the conditions that the debt value is differentiable at C_S , that is, we have 3 unknowns and 3 equations. Q.E.D.

Proof of Theorem 2

We begin by stating the relevant dynamic programming verification theorem for our analysis.

Proposition A1 *[Dynamic Programming Verification Theorem] Let $S_q(C)$ denote the equity value when the current earnings level is C if the bank follows a switching policy where it chooses project 1 when its earnings exceed q and project 2 when the earnings are below q . Suppose that $S_q(C)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:*

$$\begin{aligned} \max_{i \in \{1,2\}} L_i S_q + (1_{C \geq \theta}(1-\lambda_1)(C-\theta) + 1_{C < \theta}(1+\lambda_2)(C-\theta)) &= 0; \\ S_q(C_B) = S_q'(C_B) &= 0, \end{aligned} \quad (\text{A8})$$

where

$$L_i S_q = \frac{1}{2}\sigma_i^2 C^2 \frac{d^2 S_q}{dC^2} + \mu_i C \frac{dS_q}{dC} - rS_q; i \in \{1,2\} \quad (\text{A9})$$

Then $S_q(C)$ is the optimal equity value function among all possible dynamic project choice policies (including non-stationary policies) and q is the optimal switching trigger.

Since the above follows from the general verification theorem for dynamic programming, we omit its proof for brevity and refer the reader to Fleming and Soner (1992).

We use the following lemma frequently in the proof.

Lemma A2 *We have*

$$\gamma_i^+ > 1 \text{ for } i \in \{1, 2\} \quad (\text{A10})$$

$$\gamma_1^- < \gamma_2^- \quad (\text{A11})$$

$$r - \mu_i \gamma_i^- > 0 \text{ for } i \in \{1, 2\} \quad (\text{A12})$$

$$r - \mu_i \gamma_i^+ > 0 \text{ for } i \in \{1, 2\} \quad (\text{A13})$$

Proof. By (6), and since γ_i^+, γ_i^- are the roots of (27), we have $0 > \mu_i - r = \frac{1}{2}\sigma_i^2(1)^2 + (\mu_i - \frac{1}{2}\sigma_i^2)(1) - r = \frac{1}{2}\sigma_i^2(1 - \gamma_i^+)(1 - \gamma_i^-)$. Since $\gamma_i^- < 0$, we must have $1 < \gamma_i^+$.

Next, we note that, because $\mu_2 < \mu_1$ and $\sigma_1 < \sigma_2$,

$$\mu_2 - \frac{1}{2}\sigma_2^2 < \mu_1 - \frac{1}{2}\sigma_1^2$$

Because $\gamma_1^- < 0$, it follows that

$$\frac{1}{2}\sigma_2^2(\gamma_1^-)^2 + (\mu_2 - \frac{1}{2}\sigma_2^2)(\gamma_1^-) - r > \frac{1}{2}\sigma_1^2(\gamma_1^-)^2 + (\mu_1 - \frac{1}{2}\sigma_1^2)(\gamma_1^-) - r = 0,$$

Because γ_2^+, γ_2^- are the roots of $\frac{1}{2}\sigma_2^2(x)^2 + (\mu_2 - \frac{1}{2}\sigma_2^2)(x) - r = 0$, we have

$$\frac{1}{2}\sigma_2^2(\gamma_1^-)^2 + (\mu_2 - \frac{1}{2}\sigma_2^2)(\gamma_1^-) - r = \frac{1}{2}\sigma_2^2(\gamma_1^- - \gamma_2^-)(\gamma_1^- - \gamma_2^+) > 0.$$

Since $\gamma_1^- < 0 < \gamma_2^+$, it follows from the last inequality above that we must have $\gamma_1^- < \gamma_2^-$.

To prove the third inequality, we proceed as follows.

$$0 = \frac{1}{2}\sigma_i^2(\gamma_i^-)^2 + (\mu_i - \frac{1}{2}\sigma_i^2)(\gamma_1^-) - r = \frac{1}{2}\sigma_i^2((\gamma_i^-)^2 - \gamma_i^-) + \mu_i(\gamma_1^-) - r$$

Because $\gamma_1^- < 0$, $\frac{1}{2}\sigma_i^2((\gamma_i^-)^2 - \gamma_i^-) > 0$. Consequently, it follows from the above that (A12) must hold.

To prove the fourth inequality, observe that

$$0 = \frac{1}{2}\sigma_i^2(\gamma_i^+)^2 + (\mu_i - \frac{1}{2}\sigma_i^2)(\gamma_1^+) - r = \frac{1}{2}\sigma_i^2((\gamma_i^+)^2 - \gamma_i^+) + \mu_i(\gamma_1^+) - r$$

By (A10), $\frac{1}{2}\sigma_i^2((\gamma_i^+)^2 - \gamma_i^+) > 0$ so that inequality (A13) follows from the above. Q.E.D.

We now proceed with the proof of the main theorem. Because there are different cases to consider, we prove the theorem by stating and proving propositions that deal with each case. The following proposition establishes a *necessary and sufficient* condition for the bank to optimally choose project 1 always, that is, engage in no risk-shifting.

Proposition A3 *[No Asset Substitution] Suppose that*

$$L_2 S_0 + (1 + \lambda_2)(C - \theta)|_{C=C_B(0)+} \leq 0, \quad (\text{A14})$$

where S_0 is the equity value function when the bank always chooses project 1 and $C_B(0)$ is the corresponding endogenous insolvency level. Then it is optimal for the bank to always choose project 1, that is, no asset substitution is optimal.

Proof. By Proposition 7, it suffices to show that

$$L_2 S_0 + (1_{C \geq \theta}(1 - \lambda_1)(C - \theta) + 1_{C < \theta}(1 + \lambda_2)(C - \theta)) \leq 0 \text{ for all } C \geq C_B(0). \quad (\text{A15})$$

By Proposition 1,

$$\begin{aligned} S_0(C) &= A_1 C^{\gamma_1^-} + \frac{(1 - \lambda_1)C}{r - \mu_1} - \frac{(1 - \lambda_1)\theta}{r} \text{ for } C \geq \theta \\ &= A_2 C^{\gamma_1^-} + A_3 C^{\gamma_1^+} + \frac{(1 + \lambda_2)C}{r - \mu_1} - \frac{(1 + \lambda_2)\theta}{r} \text{ for } C_B(0) < C < \theta. \end{aligned} \quad (\text{A16})$$

First, we note that

$$A_1 > 0 \quad (\text{A17})$$

because the equity value function must be greater than $\frac{(1 - \lambda_1)C}{r - \mu_1} - \frac{(1 - \lambda_1)\theta}{r}$, which is the equity value function in the hypothetical scenario where shareholders are not protected by limited liability.

Matching the value and derivative of the function S_0 at $C = \theta$, we have

$$\begin{aligned} A_1 \theta^{\gamma_1^-} + \frac{(1 - \lambda_1)\theta}{r - \mu_1} - \frac{(1 - \lambda_1)\theta}{r} &= A_2 \theta^{\gamma_1^-} + A_3 \theta^{\gamma_1^+} + \frac{(1 + \lambda_2)\theta}{r - \mu_1} - \frac{(1 + \lambda_2)\theta}{r}, \\ \gamma_1^- A_1 \theta^{\gamma_1^-} + \frac{(1 - \lambda_1)\theta}{r - \mu_1} &= \gamma_1^- A_2 \theta^{\gamma_1^-} + \gamma_1^+ A_3 \theta^{\gamma_1^+} + \frac{(1 + \lambda_2)\theta}{r - \mu_1}. \end{aligned} \quad (\text{A18})$$

From the above, we obtain

$$(\gamma_1^+ - \gamma_1^-) A_3 \theta^{\gamma_1^+} = (\lambda_1 + \lambda_2) \theta \left[\frac{\gamma_1^- - 1}{r - \mu_1} - \frac{\gamma_1^-}{r} \right] = (\lambda_1 + \lambda_2) \theta \frac{\mu_1 \gamma_1^- - r}{r(r - \mu_1)}.$$

By (A12), and the fact that $\gamma_1^+ > \gamma_1^-$, the above implies that

$$A_3 < 0. \quad (\text{A19})$$

Again, from (A18), we obtain

$$(\gamma_1^+ - \gamma_1^-) A_1 \theta^{\gamma_1^-} + (\lambda_1 + \lambda_2) \theta \left[\frac{r - \mu_1 \gamma_1^+}{r(r - \mu_1)} \right] = (\gamma_1^+ - \gamma_1^-) A_2 \theta^{\gamma_1^-}.$$

By (A13) and (A17), the above implies that

$$A_2 > 0. \quad (\text{A20})$$

Next, we note that, for $C < \theta$,

$$\begin{aligned}
L_2 S_0 + 1_{C < \theta} (1 + \lambda_2) (C - \theta) &= A_2 \left[\frac{1}{2} \sigma_2^2 (\gamma_1^-)^2 + (\mu_2 - \frac{1}{2} \sigma_2^2) \gamma_1^- - r \right] C^{\gamma_1^-} \\
&\quad + A_3 \left[\frac{1}{2} \sigma_2^2 (\gamma_1^+)^2 + (\mu_2 - \frac{1}{2} \sigma_2^2) \gamma_1^+ - r \right] C^{\gamma_1^+} + \frac{(\mu_2 - \mu_1) C}{r - \mu_1} \\
&= A_2 \frac{1}{2} \sigma_2^2 (\gamma_1^- - \gamma_2^-) (\gamma_1^- - \gamma_2^+) C^{\gamma_1^-} + A_3 \frac{1}{2} \sigma_2^2 (\gamma_1^+ - \gamma_2^-) (\gamma_1^+ - \gamma_2^+) C^{\gamma_1^+} \\
&\quad + \frac{(1 + \lambda_2) (\mu_2 - \mu_1) C}{r - \mu_1}, \tag{A21}
\end{aligned}$$

where the last equality follows from the fact that γ_2^+ and γ_2^- are the roots of (27) for $i = 2$. Since $\gamma_1^- < \gamma_2^- < \gamma_2^+$ by (A11), it follows from (A20) that

$$A_2 \frac{1}{2} \sigma_2^2 (\gamma_1^- - \gamma_2^-) (\gamma_1^- - \gamma_2^+) C^{\gamma_1^-} > 0. \tag{A22}$$

We need to consider two cases.

Case 1: $\gamma_1^+ > \gamma_2^+$

By (A19),

$$A_3 \frac{1}{2} \sigma_2^2 (\gamma_1^+ - \gamma_2^-) (\gamma_1^+ - \gamma_2^+) C^{\gamma_1^+} \leq 0. \tag{A23}$$

Since $\mu_2 < \mu_1$, $\frac{(1 + \lambda_2) (\mu_2 - \mu_1) C}{r - \mu_1} < 0$ and decreases with C . It then follows from (A21), (A22), and (A23) that $L_2 S_0 + 1_{C < \theta} (1 + \lambda_2) (C - \theta)$ decreases with C . Since $L_2 S_0 + (1 + \lambda_2) (C - \theta)|_{C=C_B(0)+} \leq 0$ by (A14), we see that

$$L_2 S_0 + (1 + \lambda_2) (C - \theta) \leq 0 \text{ for all } \theta > C \geq C_B(0).$$

Case 2: $\gamma_1^+ \leq \gamma_2^+$

In this case,

$$A_3 \frac{1}{2} \sigma_2^2 (\gamma_1^+ - \gamma_2^-) (\gamma_1^+ - \gamma_2^+) C^{\gamma_1^+} \geq 0. \tag{A24}$$

By (A21), (A22), (A24), and (A10), the function $L_2 S_0 + 1_{C < \theta} (1 + \lambda_2) (C - \theta)$ tends to ∞ as $C \rightarrow 0$ and as $C \rightarrow \infty$ and has one unique global minimum that must be negative by (A14). We will show that the function is decreasing at $C = \theta$ that, by condition (A14) would imply that it is negative in the region $\theta > C \geq C_B(0)$.

Observe that, for $C > \theta$

$$\frac{d}{dC} [L_1 S_0 + (1 - \lambda_1) (C - \theta)] = \frac{1}{2} \sigma_1^2 C^2 S_0''' + \sigma_1^2 C S_0'' + \mu_1 S_0' + \mu_1 C S_0'' - r S_0' + (1 - \lambda_1) = 0.$$

For $C < \theta$

$$\frac{d}{dC} [L_1 S_0 + (1 + \lambda_2) (C - \theta)] = \frac{1}{2} \sigma_1^2 C^2 S_0''' + \sigma_1^2 C S_0'' + \mu_1 S_0' + \mu_1 C S_0'' - r S_0' + (1 + \lambda_2) = 0.$$

Since S_0 is twice differentiable at $C = \theta$, we have

$$\frac{1}{2}\sigma_1^2 C^2 S_0'''(\theta+) - \frac{1}{2}\sigma_1^2 C^2 S_0'''(\theta-) = \lambda_1 + \lambda_2 > 0.$$

As $\sigma_1 < \sigma_2$,

$$\frac{1}{2}\sigma_2^2 C^2 S_0'''(\theta+) - \frac{1}{2}\sigma_2^2 C^2 S_0'''(\theta-) > \lambda_1 + \lambda_2. \quad (\text{A25})$$

Now note that, by (A25),

$$\begin{aligned} & \frac{d}{dC}[L_2 S_0 + (1 - \lambda_1)(C - \theta)]|_{C=\theta+} - \frac{d}{dC}[L_2 S_0 + (1 + \lambda_2)(C - \theta)]|_{C=\theta-} \\ &= \frac{1}{2}\sigma_2^2 C^2 S_0'''(\theta+) - \frac{1}{2}\sigma_2^2 C^2 S_0'''(\theta-) - (\lambda_1 + \lambda_2) > 0. \end{aligned} \quad (\text{A26})$$

By (A16), for $C > \theta$,

$$L_2 S_0 + (1 - \lambda_1)(C - \theta) = A_1 \frac{1}{2}\sigma_2^2 (\gamma_1^- - \gamma_2^-)(\gamma_1^- - \gamma_2^+) C^{\gamma_1^-} + \frac{(1 - \lambda_1)(\mu_2 - \mu_1)C}{r - \mu_1}.$$

By (A11), (A17), and because $\mu_2 < \mu_1$, the R.H.S. above decreases with C and, in particular, is decreasing at $C = \theta +$. By (A26), therefore, $\frac{d}{dC}[L_2 S_0 + (1 + \lambda_2)(C - \theta)]|_{C=\theta-} < 0$. Consequently,

$$L_2 S_0 + (1 + \lambda_2)(C - \theta) \leq 0 \text{ for all } \theta > C \geq C_B(\theta).$$

It remains to show that

$$L_2 S_0 + (1 - \lambda_1)(C - \theta) < 0 \text{ for all } C > \theta.$$

Because S_0 is twice differentiable at $C = \theta$, it follows from the above that

$$L_2 S_0 + (1 - \lambda_1)(C - \theta)|_{C=\theta+} < 0$$

We have already shown that $L_2 S_0 + (1 - \lambda_1)(C - \theta)$ is decreasing for $C > \theta$. It follows from the above that it must be negative for $C > \theta$.

By Proposition A1, therefore, choosing project 1 throughout is optimal for the bank. Q.E.D.

The following proposition establishes a *necessary and sufficient* condition for the optimal switching trigger to exceed the debt level. The proofs of this and the next proposition involve similar techniques as the proof of Proposition A3 and are included in the online appendix.

Proposition A4 [*Optimal Switching Trigger Greater than Debt Level*] Suppose that

$$L_1 S_\theta|_{C=\theta-} < 0, \quad (\text{A27})$$

where S_θ is the equity value function corresponding to the policy where the manager chooses project 2 for $C < \theta$ and project 1 for $C \geq \theta$. There exists q^* with $\theta < q^* < \infty$ such that the policy of switching projects at q^* is optimal for the bank and S_{q^*} is the corresponding equity value function.

The following proposition completes the remaining step in the proof of the theorem by establishing conditions under which the optimal switching trigger is less than the debt level.

Proposition A5 [*Optimal Switching Trigger Less than Debt Level*] Suppose that

$$L_1 S_\theta|_{C=\theta-} \geq 0, L_2 S_0 + (1 + \lambda_2)(C - \theta)|_{C=C_B(0)+} > 0. \quad (\text{A28})$$

There exists $q^* \leq \theta$ such that the policy of switching projects at q^* is optimal for the bank and S_{q^*} is the corresponding equity value function.

The main theorem follows from Propositions A3, A4 and A5. Q.E.D.

Proof of Proposition 3

By Proposition 1, the equity, debt and bank values for a given switching policy satisfy the following ODE:

$$\frac{1}{2}\sigma_i^2 C^2 \frac{d^2 Y}{dC^2} + \mu_i C \frac{dY}{dC} - rY + y(C, \theta) = 0 \text{ when project } i \in \{1, 2\} \text{ is chosen,}$$

where $y(C, \theta)$ is the corresponding payout flow to equity, debt, or the bank. Note that $y(C, \theta)$ is homogeneous of degree 1 in C and θ , that is,

$$y(\xi C, \xi \theta) = \xi y(C, \theta).$$

Further, the boundary conditions that determine the solution to the corresponding equity, debt and firm value functions are also homogeneous of degree 1 in C and θ . Consequently, the value function must itself be homogeneous of degree 1 in C and θ . It immediately follows from the proof of Theorem 2 that the optimal switching threshold and insolvency threshold must both scale linearly with θ as stated in the proposition. The constancy of the switching leverage follows from the fact that the optimal switching trigger is proportional to θ , and the debt value and bank value are homogeneous of degree 1 in current cash flow level and the debt level. Q.E.D.

Proof of Proposition 4

We state below the representation of the social value function in the case $C_s < \theta$ (that is omitted in the statement of the Proposition).

$$F_t^{social} = F^{social}(C_t) = \begin{cases} \frac{(1-\lambda_1^{social})C_t}{r-\mu_1} + \frac{\lambda_1^{social}\theta}{r} + X_1(C_S, \theta)C_t^{\gamma_1^-}, & \text{if } C_t \geq \theta, \\ \frac{(1+\lambda_2^{social})C_t}{r-\mu_2} - \frac{\lambda_2^{social}\theta}{r} + X_2(C_S, \theta)C_t^{\gamma_1^-} + X_3(C_S, \theta)C_t^{\gamma_1^+}, & \text{if } \theta > C_t \geq C_S, \\ \frac{(1+\lambda_2^{social})C_t}{r-\mu_2} - \frac{\lambda_2^{social}\theta}{r} + X_4(C_S, \theta)C_t^{\gamma_2^-} + X_5(C_S, \theta)C_t^{\gamma_2^+}, & \text{if } C_S > C_t \geq C_L(C_S, \theta). \end{cases} \quad (\text{A29})$$

We omit the proof of the Proposition because it follows using exactly the same arguments used to prove Proposition 1. Q.E.D.

Proof of Theorem 5

As in the proof of Theorem 2, we establish the theorem by stating and proving intermediate propositions. The following proposition states the relevant dynamic programming verification theorem.

Proposition A6 [Verification Theorem for Regulator's Problem] Let $F_q^{social}(C)$ denote the social value when the current earnings level is C if the regulator follows a switching policy where it switches projects at the earnings level q . Suppose that $F_q^{social}(C)$ satisfies the following HJB equation:

$$\max_{i \in \{1,2\}} L_i F_q^{social}(C) + \left[1_{C \geq \theta} \left\{ (1 - \lambda_1^{social})C + \lambda_1^{social}\theta \right\} + 1_{C < \theta} \left\{ (1 + \lambda_2^{social})C - \lambda_2^{social}\theta \right\} \right] = 0;$$

$$F_q^{social}(C_L) = F_q^{social'}(C_L) = 0, \quad (\text{A30})$$

Then $F_q^{social}(C)$ is the optimal social value function among all possible dynamic project choice policies (including non-stationary policies) and q is the optimal switching trigger.

The following proposition establishes the necessary and sufficient condition for the regulator to optimally choose project 1 throughout, that is, engage in no risk-shifting.

Proposition A7 [No Asset Substitution for Regulator] Suppose that

$$L_2 F_0^{social} + (1 + \lambda_2^{social})C - \lambda_2^{social}\theta|_{C=C_L(0)+} \leq 0, \quad (\text{A31})$$

where F_0^{social} is the social value function when the regulator always chooses project 1 and $C_L(0)$ is the corresponding endogenous liquidation level. Then it is optimal for the regulator to always choose project 1, that is, no asset substitution is optimal.

Proof. By (A29), the social value when the regulator always chooses project 1 is given by

$$\begin{aligned} F_0^{social}(C) &= X_1 C^{\gamma_1^-} + \frac{(1 - \lambda_1^{social})C}{r - \mu_1} + \frac{\lambda_1^{social}\theta}{r} \text{ for } C \geq \theta \\ &= X_2 C^{\gamma_1^-} + X_3 C^{\gamma_1^+} + \frac{(1 + \lambda_2^{social})C}{r - \mu_1} - \frac{\lambda_2^{social}\theta}{r} \text{ for } C < \theta. \end{aligned} \quad (\text{A32})$$

The social value function in the hypothetical scenario where the bank's assets are never liquidated and/or debt is completely risk-free is $\frac{(1 - \lambda_1^{social})C}{r - \mu_1} + \frac{\lambda_1^{social}\theta}{r}$. Since $F_0^{social}(C)$ must clearly be less, we must have

$$X_1 < 0. \quad (\text{A33})$$

Next, we note that, for $C > \theta$,

$$L_2 F_0^{social} + (1 - \lambda_1^{social})C + \lambda_1^{social}\theta = \frac{1}{2} \sigma_2^2 X_1 (\gamma_1^- - \gamma_2^-) (\gamma_1^- - \gamma_2^+) C^{\gamma_1^-} + \frac{(1 - \lambda_1^{social})C(\mu_2 - \mu_1)}{r - \mu_1}.$$

Because $\gamma_1^- < \gamma_2^-$ by (A11), it follows from (A33) that the first term on the R.H.S. above is strictly negative. The second term is also strictly negative because $\mu_2 < \mu_1$. Consequently, $L_2 F_0^{social} + (1 - \lambda_1^{social})C + \lambda_1^{social}\theta < 0$, which establishes the verification condition for $C > \theta$.

Since the social value function is differentiable at θ ,

$$\begin{aligned} X_1 \theta^{\gamma_1^-} + \frac{(1 - \lambda_1^{social})\theta}{r - \mu_1} + \frac{\lambda_1^{social}\theta}{r} &= X_2 \theta^{\gamma_1^-} + X_3 \theta^{\gamma_1^+} + \frac{(1 + \lambda_2^{social})\theta}{r - \mu_1} - \frac{\lambda_2^{social}\theta}{r} \\ \gamma_1^- X_1 \theta^{\gamma_1^-} + \frac{(1 - \lambda_1^{social})\theta}{r - \mu_1} &= \gamma_1^- X_2 \theta^{\gamma_1^-} + \gamma_1^+ X_3 \theta^{\gamma_1^+} + \frac{(1 + \lambda_2^{social})\theta}{r - \mu_1} \end{aligned}$$

From the above, we obtain

$$(\gamma_1^+ - \gamma_1^-)X_3\theta\gamma_1^+ = (\lambda_1^{social} + \lambda_2^{social})\theta \left[\frac{\gamma_1^- - 1}{r - \mu_1} - \frac{\gamma_1^-}{r} \right] = (\lambda_1^{social} + \lambda_2^{social})\theta \frac{\mu_1\gamma_1^- - r}{r(r - \mu_1)}.$$

By (A12), and the fact that $\gamma_1^+ > \gamma_1^-$, the above implies that

$$X_3 < 0. \quad (\text{A34})$$

Next, note that, for $C < \theta$,

$$\begin{aligned} L_2F_0^{social} + (1 + \lambda_2^{social})C - \lambda_2^{social}\theta &= \frac{1}{2}\sigma_2^2 X_2(\gamma_1^- - \gamma_2^-)(\gamma_1^- - \gamma_2^+)C^{\gamma_1^-} + \frac{1}{2}\sigma_2^2 X_3(\gamma_1^+ - \gamma_2^-)(\gamma_1^+ - \gamma_2^+)C^{\gamma_1^+} \\ &\quad + \frac{(1 - \lambda_1^{social})C(\mu_2 - \mu_1)}{r - \mu_1}. \end{aligned} \quad (\text{A35})$$

Because F_0^{social} is twice differentiable at $C = \theta$, it follows from our earlier result that $L_2F_0^{social} + (1 - \lambda_1^{social})C + \lambda_1^{social}\theta < 0$ for $C > 0$ that

$$L_2F_0^{social} + (1 + \lambda_2^{social})C - \lambda_2^{social}\theta|_{C=\theta-} = L_2F_0^{social} + (1 - \lambda_1^{social})C + \lambda_1^{social}\theta|_{C=\theta+} < 0 \quad (\text{A36})$$

We need to consider two cases.

Case 1: $\gamma_1^+ \geq \gamma_2^+$.

In this case, the second term on the right hand side of (A35) is non-positive by (A34). The third term is negative because $\mu_2 < \mu_1$. If $X_2 \leq 0$, then the first term is non-positive by (A11) so that the entire expression on the R.H.S. of (A35) is negative for $C < \theta$. If $X_2 > 0$, then the expression on the R.H.S. of (A35) is decreasing with C . In this case, condition (A31) implies that it is again negative for $C_L(0) < C < \theta$.

Case 2: $\gamma_1^+ < \gamma_2^+$

In this case, the second term on the right hand side of (A35) is positive by (A34). If $X_2 < 0$, then the first term is positive by (A11). The function on the R.H.S. of (A35) tends to ∞ as $C \rightarrow 0$ and as $C \rightarrow \infty$ and has a unique local (and global) minimum that is strictly negative by (A31). Condition (A31) and (A36) together imply that $L_2F_0^{social} + (1 + \lambda_2^{social})C - \lambda_2^{social}\theta$ must, in fact, be negative for $C_L(0) < C < \theta$.

In summary, we have shown that F_0^{social} satisfies the HJB equation. Moreover, $L_2F_q^{social}(C) + [1_{C \geq \theta} \{(1 - \lambda_1^{social})C + \lambda_1^{social}\theta\} + 1_{C < \theta} \{(1 + \lambda_2^{social})C - \lambda_2^{social}\theta\}] < 0$ for $C_L(0) < C < \infty$, which implies that choosing project 1 always is the unique optimal policy for the regulator. Q.E.D.

The following proposition shows that, if condition (A31) does not hold, the regulator optimally switches projects at a trigger that is below the debt level.

Proposition A8 [*Optimal Switching Trigger*] Suppose that

$$L_2F_0^{social} + (1 + \lambda_2^{social})C - \lambda_2^{social}\theta|_{C=C_L(0)+} > 0. \quad (\text{A37})$$

There exists $q^* < \theta$ such that it is optimal for the regulator to choose project 2 for $C < q^*$, and project 1 for $C \geq q^*$.

Proof. We split the proof into several steps.

Step 1. Consider the policy where the switching trigger is equal to θ . By (33), the social value function for $C > \theta$ has the form

$$F_{\theta}^{social}(C) = X_1 C^{\gamma_1^-} + \frac{(1 - \lambda_1^{social})C}{r - \mu_1} + \frac{\lambda_1^{social}\theta}{r}.$$

By the same argument used in the proof of Proposition A7,

$$X_1 < 0. \quad (A38)$$

Consequently, for $C > \theta$,

$$L_2 F_{\theta}^{social} + (1 - \lambda_1^{social})C + \lambda_1^{social}\theta = \frac{1}{2}\sigma_2^2 X_1 (\gamma_1^- - \gamma_2^-)(\gamma_1^- - \gamma_2^+)C^{\gamma_1^-} + \frac{(1 - \lambda_1^{social})C(\mu_2 - \mu_1)}{r - \mu_1} < 0, \quad (A39)$$

so that

$$L_2 F_{\theta}^{social} + (1 - \lambda_1^{social})C + \lambda_1^{social}\theta|_{C=\theta+} < 0. \quad (A40)$$

We know that, because project 2 is chosen for $C < \theta$, and project 1 for $C > \theta$,

$$\begin{aligned} L_2 F_{\theta}^{social} + (1 + \lambda_2^{social})C - \lambda_2^{social}\theta|_{C=\theta-} &= 0, \\ L_1 F_{\theta}^{social} + (1 - \lambda_1)C + \lambda_1\theta|_{C=\theta+} &= 0. \end{aligned}$$

From the above equations and (A40), we obtain

$$L_1 F_{\theta}^{social} + (1 + \lambda_2^{social})C - \lambda_2^{social}\theta|_{C=\theta-} > 0. \quad (A41)$$

By (A37) and (A41), the policy of always choosing project 1, and the policy of switching projects at θ are both sub-optimal within the restricted class of “single switching trigger” policies where the switching trigger is less than or equal to θ . By the continuity of the regulator’s objective function, there exists $q^* < \theta$ such that the policy of switching projects at q^* is optimal within the restricted class of “single switching trigger” policies. We will show that the policy is, in fact, *globally optimal* as in the proof of Proposition A5.

Step 2. By arguments similar to those used in Step 2 of the proof of Proposition A5, we must have

$$L_2 F_{q^*}^{social} + (1 + \lambda_2^{social})C - \lambda_2^{social}\theta|_{C=q^*+} = 0, \quad (A42)$$

$$L_1 F_{q^*}^{social} + (1 + \lambda_2^{social})C - \lambda_2^{social}\theta|_{C=q^*-} = 0. \quad (A43)$$

Further, the above imply that $F_{q^*}^{social}$ is twice differentiable at q^* .

Step 3. Using arguments similar to those used in the proof of Proposition A7, we can show that, condition (A42) implies that $L_2 F_{q^*}^{social} + (1 + \lambda_2^{social})C - \lambda_2^{social}\theta < 0$ for $q^* < C < \theta$.

Step 4. It remains to show that $L_1 F_{q^*}^{social} + (1 + \lambda_2^{social})C - \lambda_2^{social}\theta < 0$ for $C_L(q^*) < C < q^*$. By arguments similar to those used in Step 4 of the proof of Proposition A5, we can show that $L_1 F_{q^*}^{social} + (1 + \lambda_2^{social})C - \lambda_2^{social}\theta$ is increasing for $C < q^*$. Condition (A43), therefore, implies that it is negative for $C < q^*$.

Hence, the function $F_{q^*}^{social}$ satisfies all the conditions of the verification theorem. Moreover,

the results that

$$\begin{aligned} L_2 F_{q^*}^{social}(C) + \left[1_{C \geq \theta} \left\{ (1 - \lambda_1^{social})C + \lambda_1^{social}\theta \right\} + 1_{C < \theta} \left\{ (1 + \lambda_2^{social})C - \lambda_2^{social}\theta \right\} \right] &< 0 \text{ for } C > q^*, \\ L_1 F_{q^*}^{social}(C) + \left[1_{C \geq \theta} \left\{ (1 - \lambda_1^{social})C + \lambda_1^{social}\theta \right\} + 1_{C < \theta} \left\{ (1 + \lambda_2^{social})C - \lambda_2^{social}\theta \right\} \right] &< 0 \text{ for } C < q^*, \end{aligned}$$

together imply that switching projects at q^* is the unique optimal policy for the regulator. Q.E.D.

Proof of Proposition 6

To simplify the notation, let q^* and \tilde{q}^* denote the optimal switching triggers in the bank's and regulator's problems, respectively. We need to show that $\tilde{q}^* \leq q^*$. Since the regulator's optimal switching trigger is less than θ by Theorem 5, it suffices to consider the case where $q^* < \theta$. We prove the proposition in two main parts.

Part 1. Let S^{social} be the social equity value function with cost parameters $(\lambda_1^{social}, \lambda_2^{social})$. We will show that the optimal switching point q_1^* defined for the “social shareholders” is lower than q^* .

Because the problem is scaling invariant in $(1 - \lambda_1, 1 + \lambda_2)$, we can rescale the pair $(1 - \lambda_1^{social}, 1 + \lambda_2^{social})$ to $((1 - \lambda_1^{social}) \frac{1 + \lambda_2}{1 + \lambda_2^{social}}, 1 + \lambda_2)$ and the switching trigger for the social shareholders will not change. Since $\frac{1 + \lambda_2^{social}}{1 + \lambda_2} \geq \frac{1 - \lambda_1^{social}}{1 - \lambda_1}$, without loss of generality, we may thus assume $\lambda_1 \geq \lambda_1^{social}$ and $\lambda_2 = \lambda_2^{social}$. Consider a switching policy described by a trigger q , where $q \geq C_B(q)$, where $C_B(q)$ is the endogenous insolvency threshold corresponding to the switching policy. Define the function

$$H_q = S_q^{social} - S_q \quad (\text{A44})$$

By (16), H_q satisfies the following system of ODEs:

$$\begin{aligned} L_1 H_q + (\lambda_1 - \lambda_1^{social})(C - \theta) &= 0 \text{ for } C \geq \theta \\ L_1 H_q &= 0 \text{ for } q \leq C < \theta \\ L_2 H_q &= 0 \text{ for } C_B(q) \leq C < q \end{aligned} \quad (\text{A45})$$

We also have $H_q \geq 0$ since the social cost is less than the private cost of equity ($\lambda_1^{social} \leq \lambda_1$). By arguments similar to those used in the proof of Proposition 1, H_q has the following form:

$$H_q(C) = \begin{cases} W_1 C^{\gamma_1^-} + (\lambda_1 - \lambda_1^{social}) \left(\frac{C}{r - \mu_1} - \frac{\theta}{r} \right), & \text{for } \theta \leq C, \\ W_2 C^{\gamma_1^-} + W_3 C^{\gamma_1^+}, & \text{for } q \leq C < \theta, \\ W_4 C^{\gamma_2^-} + W_5 C^{\gamma_2^+}, & \text{for } C_B(q) \leq C < q. \end{cases} \quad (\text{A46})$$

From the value matching condition at $C = C_B(q)$ and that $H_q \geq 0$, we solve

$$W_4 = -W C_B^{-\gamma_2^-}, W_5 = W C_B^{-\gamma_2^+}, \quad \text{for some } W > 0 \quad (\text{A47})$$

An examination of the proof of Theorem 5 reveals that it suffices to show that

$$L_2 H_q|_{C=q+} \leq 0 \quad (\text{A48})$$

to ensure the “social shareholders” always choose project 1 when $C > q$. It is also straightforward to verify that (A48) is equivalent to

$$L_1 H_q|_{C=q-} \geq 0. \quad (\text{A49})$$

Plugging (A47) into (A49) yields the equivalent condition

$$-(\gamma_2^- - \gamma_1^+)(\gamma_2^- - \gamma_1^-) \left(\frac{q}{C_B} \right)^{\gamma_2^-} + (\gamma_2^+ - \gamma_1^+)(\gamma_2^+ - \gamma_1^-) \left(\frac{q}{C_B} \right)^{\gamma_2^+} \geq 0. \quad (\text{A50})$$

Since $\gamma_2^+ > \gamma_1^+ > \gamma_2^- > \gamma_1^-$, the left hand side of (A50) is always positive and thus the condition holds.

Part 2. We now show that the optimal switching trigger in the regulator’s problem is lower than in the “social” shareholders’ problem, i.e., $\tilde{q}^* \leq q_1^*$. Equivalently, we assume below $\lambda_1 = \lambda_1^{social}$ and $\lambda_2 = \lambda_2^{social}$ and prove that $\tilde{q}^* \leq q^*$.

Step 1. Consider a switching policy described by a trigger q , where $q \geq C_B(q)$, where $C_B(q)$ is the endogenous insolvency threshold corresponding to the switching policy. Define the function

$$G_q = F_q^{social} - S_q \quad (\text{A51})$$

By (16) and (18), G satisfies the following system of ODEs:

$$\begin{aligned} L_1 G_q + \theta &= 0 \text{ for } C > q \\ L_2 G_q + \theta &= 0 \text{ for } C_B(q) < C < q \\ L_2 G_q + (1 - \lambda_1)C + \lambda_1 \theta &= 0 \text{ for } C_L(q) < C < C_B(q), \end{aligned} \quad (\text{A52})$$

where $C_L(q)$ is the endogenous liquidation threshold corresponding to the switching policy characterized by the trigger q . By arguments similar to those used in the proof of Proposition 1, G has the following form for $C > q$:

$$G_q(C) = Y_1 C^{\gamma_1^-} + \frac{\theta}{r}. \quad (\text{A53})$$

Since $\frac{\theta}{r}$ is the value of $G_q(C)$ in the hypothetical scenario where debt is completely risk-free and liquidation never occurs, we must have

$$Y_1 < 0. \quad (\text{A54})$$

Next, we note that

$$L_2(G_q) + \theta = \frac{1}{2} Y_1 \sigma_2^2 (\gamma_1^- - \gamma_2^-)(\gamma_1^- - \gamma_2^+) C^{\gamma_1^-} < 0 \quad (\text{A55})$$

because $Y_1 < 0$ and $\gamma_1^- < \gamma_2^-$ by (A11).

Step 2. Suppose that $q^* = C_B(q^*)$, which implies that project 1 is always chosen. By (A14),

$$L_2 S_{q^*} + (1 + \lambda_2)(C - \theta)|_{C=C_B(q^*)+} \leq 0. \quad (\text{A56})$$

An examination of the proof of Theorem 5 reveals that it suffices to show that

$$L_2 F_{q^*}^{social} + (1 + \lambda_2)C - \lambda_2 \theta|_{C=C_B(q^*)+} \leq 0$$

to ensure that the regulator also optimally chooses project 1 for all $C \geq C_B(q^*)$, that is, the regulator's optimal switching trigger (if it exists) is less than $C_B(q^*)$. By (A51) and (A55),

$$\begin{aligned} L_2 F_{q^*}^{social} + (1 + \lambda_2)C - \lambda_2 \theta &= L_2 S_{q^*} + (1 + \lambda_2)(C - \theta) + \\ &\quad L_2(G_{q^*}) + \theta < 0, \end{aligned}$$

where the last inequality above follows from (A56) and (A55).

Step 3. Suppose that $q^* > C_B(q^*)$.

By the proof of Theorem 2,

$$L_2 S_{q^*} + (1 + \lambda_2)(C - \theta)|_{C=q^*+} = 0 \quad (\text{A57})$$

By the proof of Theorem 5, it suffices to show that

$$L_2 F_{q^*}^{social} + (1 + \lambda_2)C - \lambda_2 \theta|_{C=q^*+} \leq 0.$$

The above, however, immediately follows from (A55) and (A57). Q.E.D.

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Table I: The Impact of Optimal Prudential Regulation

This table reports the solutions to the case of the unregulated bank and the case where the regulator optimally implements the full set of regulatory tools.

	Unregulated Bank	Full Regulation
<i>Debt Level and Thresholds</i>		
Initial Coupon Level	0.632	0.776
Switching Boundary (Bank)	0.576	0.708
Switching Boundary (Regulator)		0.327
Insolvency Boundary	0.426	0.542
Liquidation Boundary		0.283
<i>Leverage and Regulatory Ratios</i>		
Initial Leverage	64.5%	76.8%
Initial Capital Requirement		23.2%
Switching/Intervention Leverage	90.2%	93.1%
Intervention Ratio		6.9%
Switching Leverage (Regulator)		161.5%
Spread (bp)	67.2	39.1
<i>Ex Ante Values</i>		
Bank Value	23.490	25.990
Social Value	23.308	24.230
Ex Ante Benefit (Bank Value, %)		10.6%
Ex Ante Benefit (Social Value, %)		3.9%
<i>Ex Post Values at Insolvency</i>		
Ex Post Benefit (Bank Value, %)		82.8%
Ex Post Benefit (Social Value, %)		28.9%

Table II: The Impact of Individual Components of Prudential Regulation

This table reports the key regulatory ratios and ex ante benefits in several cases where different sets of regulatory tools are available. Column 1 reports the case of the unregulated bank. Column 2 reports the case where the regulator injects capital when the bank is insolvent and optimally liquidates the bank, but does not intervenes in project choices or impose initial capital requirement. Column 3 reports the case where the regulator's only action is intervention in the bank's project choices. Column 4 reports the case where the regulator employs all regulatory tools (investment intervention, capital injection, and liquidation) except the initial capital requirement. Column 5 reports the case where the regulator employs all regulatory tools (initial capital requirement, capital injection, and liquidation) except risk intervention. Column 6 reports the case where the regulator optimally employ all regulatory tools.

	Unregulated Bank	Capital Injection Only	Risk Intervention Only	Capital Injection and Risk Intervention	Capital Injection and Capital Requirement	Full Regulation
<i>Leverage and Regulatory Ratios</i>						
Initial Leverage	64.5%	87%	66.0%	100%	71.9%	76.8%
Initial Capital Requirement					28.1%	23.2%
Intervention Ratio		8.1%	9.2%	6.9%	0%	6.9%
<i>Ex Ante Values</i>						
Bank Value	23.490	25.298	23.588	30.777	25.270	25.990
Social Value	23.308	22.871	23.393	18.659	23.886	24.230
Ex Ante Benefit (Bank Value, %)		7.7%	0.4%	31.0%	7.0%	10.6%
Ex Ante Benefit (Social Value, %)		-1.9%	0.4%	-19.8%	2.3%	3.9%

Table III: Model Extension with Dynamic Debt Restructuring

This table reports the solutions to the extension of the main model under which the bank can dynamically adjust capital structure by issuing more debt. The parameters are calibrated to the intermediate case where the regulator injects capital and the bank chooses projects and capital structure. Details of the model and calibration are described in the online appendix.

	Unregulated Bank	Full Regulation
<i>Leverage and Regulatory Ratios</i>		
Initial Leverage/Leverage after Restructuring	55.4%	66.5%
Capital Requirement		33.5%
Leverage before Restructuring	44.6%	43.5%
Switching/Intervention Leverage	98.2%	97.6%
Intervention Ratio		2.4%
<i>Ex Ante Values</i>		
Bank Value	24.051	25.930
Social Value	23.825	24.575
Ex Ante Benefit (Bank Value, %)		7.8%
Ex Ante Benefit (Social Value, %)		3.1%

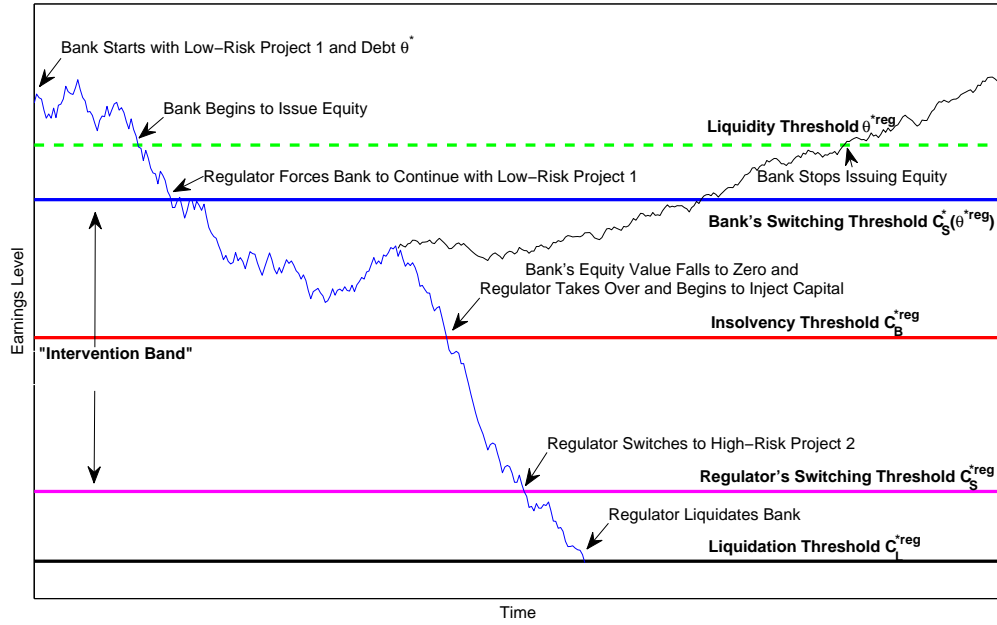


Figure 1: **Optimal Prudential Regulation: An Illustration** This figure plots sample earnings paths and project choices of a fully regulated bank.

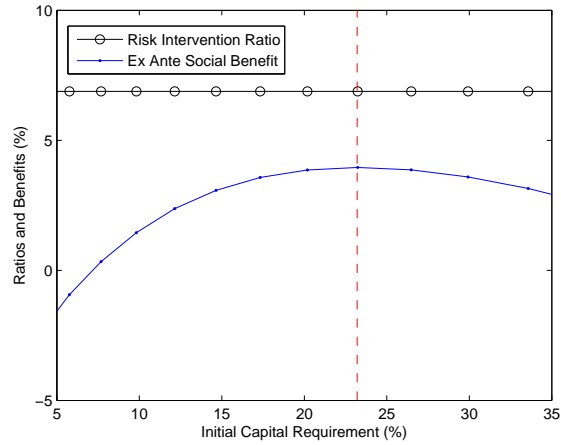


Figure 2: **Impact of Changes in Initial Capital Requirement** This figure plots the changes in optimal regulatory policy and the associated benefits when the initial capital requirement deviates from the optimal level. The model parameters are the baseline parameters. The dashed lines correspond to the case of optimal regulation.

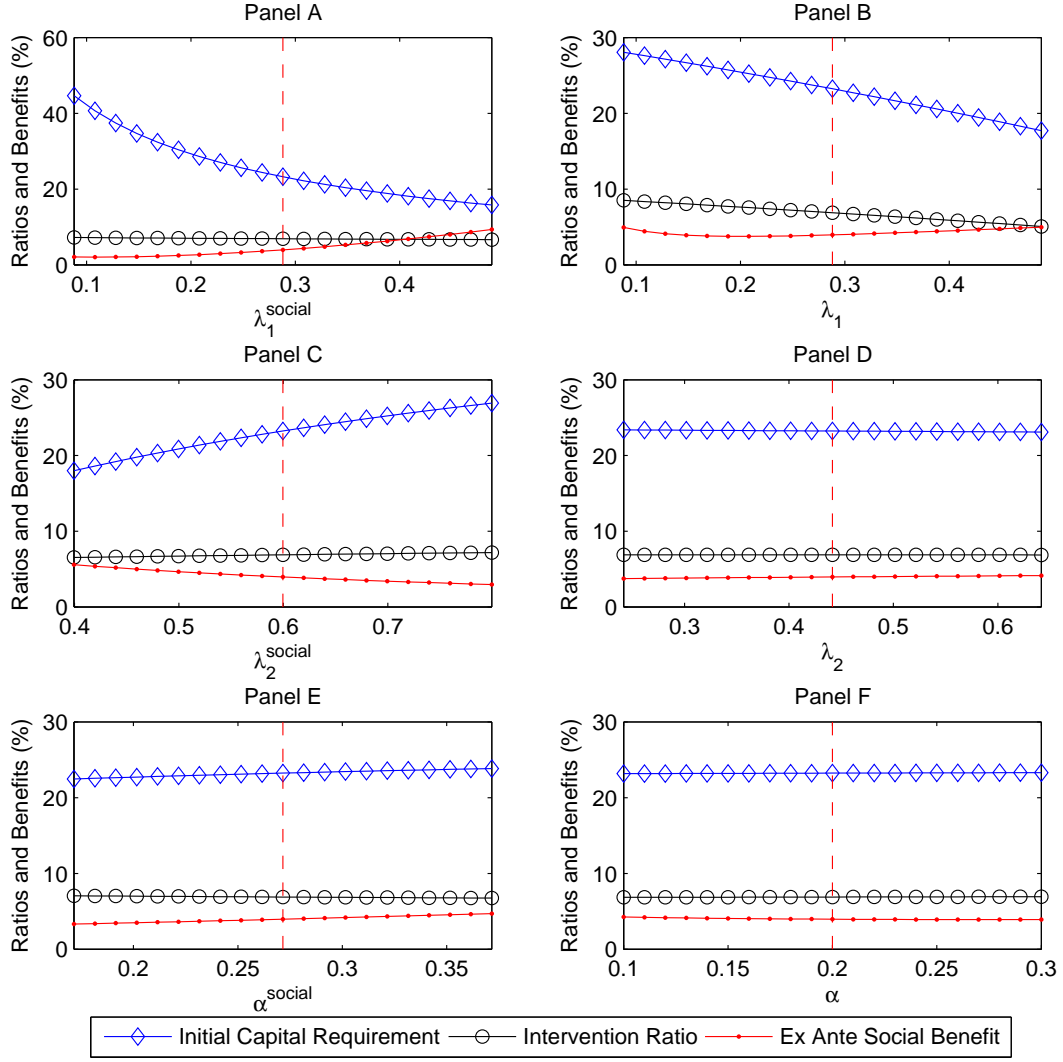


Figure 3: **Optimal Prudential Regulation: Comparative Statics** This figure reports the optimal regulatory ratios and the benefits to the bank and social welfare of optimal regulation under different values of the model's parameters. The dashed lines correspond to the baseline case.

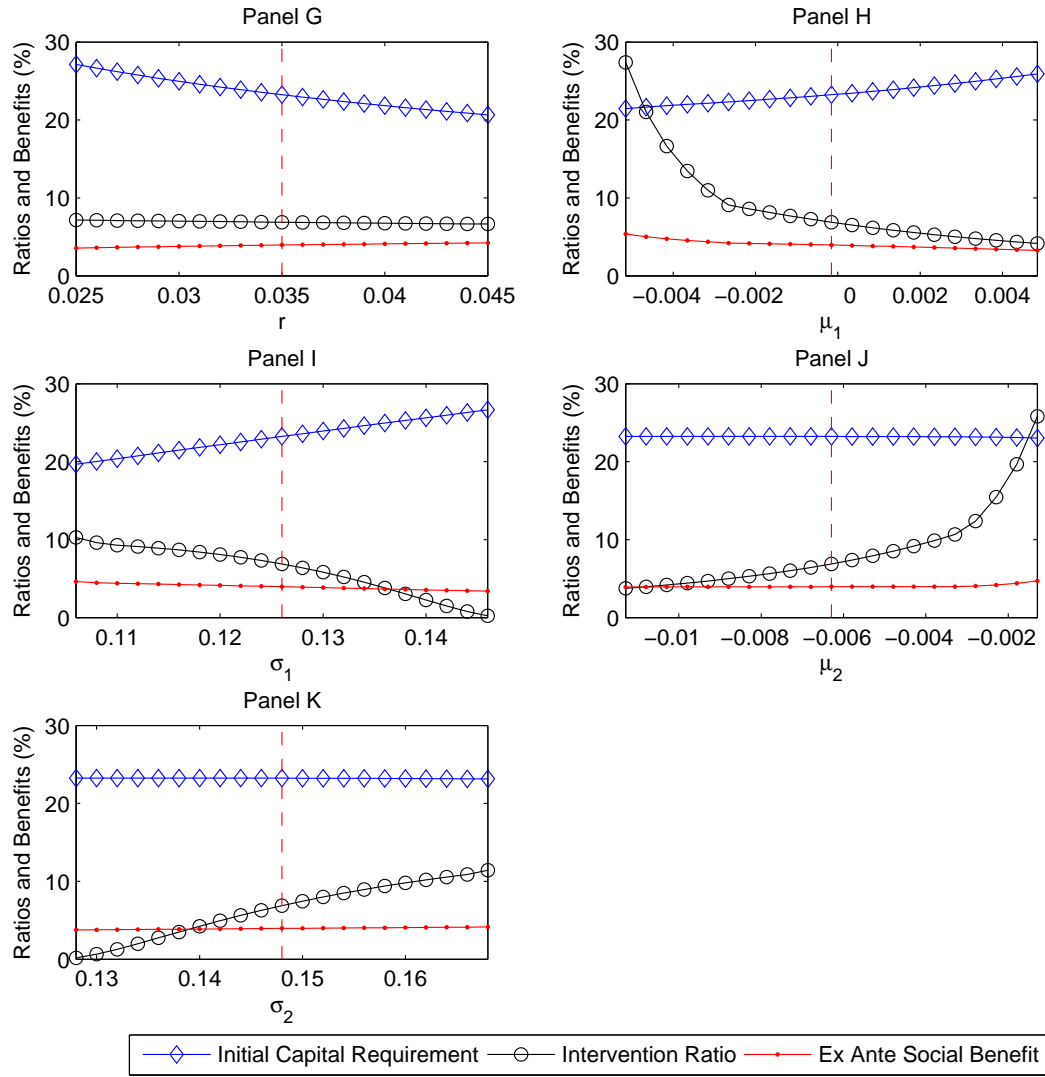


Figure 3: Optimal Prudential Regulation: Comparative Statics (Continued)