Accounting for Unemployment: The Long and Short of It

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June 25, 2012
PRELIMINARY and INCOMPLETE

Abstract

Shimer (2012) accounts for the volatility of unemployment based on a model of homogeneous unemployment. Using data on short-term unemployment he finds that most of unemployment volatility is accounted for by variations in the exit rate from unemployment. The assumption of homogeneous exit rates is inconsistent with the observed negative duration dependence of unemployment exit rates for the U.S. labor market. I describe a simple model of heterogeneous unemployment with short-term and long-term unemployed, and use data on the duration distribution of unemployment to account for entry to and exit from the unemployment pool. This alternative account confirms that most of unemployment volatility is due to variations of exit rates from unemployment, but it also reveals that most of unemployment volatility is due to the volatility of long-term unemployment rather than short-term unemployment.

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From 2008 to 2010 unemployment in the United States more than doubled from about 7.5 million unemployed to close to 16 million unemployed. At the same time the share of long-term unemployed, that is, those unemployed for more than 26 weeks, more than doubled from less than 20 percent to more than 40 percent. This comovement between total unemployment and the share of long-term unemployment is not unusual: in every previous recession the share of long-term unemployed has increased with total unemployment, Figure 1. In the public and among policy makers we often see two reactions to this observation. First, long-term unemployed are seen as different from the rest of the unemployed in that they presumably have a lower chance of exiting unemployment. Furthermore, the high unemployment rate is attributed to the presence of these long-term unemployed. Second, long-term unemployment feeds on itself and creates future unemployment in that the chance that an unemployed worker will exit the unemployment state declines with the duration of being unemployed.

Figure 1. Unemployment Rate and Long-Term Unemployed

We use a simple unemployment accounting framework to provide some perspective on these two interpretations of long-term unemployment. First, we point out that the positive correlation of the unemployment rate and the share of long-term unemployed does not necessarily imply that the long-term unemployed are different from the rest of the unemployed. A simple model of unemployment that assumes homogeneity among the unemployed cannot, however, account quantitatively for the observed increase in long-term unemployment. We then extend the accounting framework slightly and assume that there are two types of unemployed: 'short-term' unemployed with a high exit rate from unemployment, and 'long-term' unemployed with a low exit rate. This simple extension allows us to account for the duration distribution of unemployment, and it sheds new light on the sources of unemployment. In particular, we find that variations in the entry and exit rates of long-term unemployed account for most of unemployment volatility.

Suppose that all unemployed workers are identical in their chances of exiting the unemployment state. Even in this model with homogeneous unemployment the unemployment rate and the share of long-term unemployed will be positively correlated if changes in the unemployment rate are mainly due to changes in the exit rate from unemployment. Simply put, if it gets harder to find a job then relatively more unemployed will be around for a long time. Shimer (2012) argues that most of the variation in the unemployment rate is indeed driven by variations in the exit rate.

Even though variations in a common exit rate can account qualitatively for the correlation between the unemployment rate and the share of long-term unemployment, this model cannot account quantitatively for changes in the overall duration distribution of unemployment.

This failure is associated with the observed negative duration dependence of unemployment, that is, observed exit rates from unemployment appear to decline with the duration of unemployment. Observed duration dependence can be due to 'true duration dependence,' that is, the exit rates for all unemployed simply decline with unemployment duration, but it does not have to be. An alternative interpretation of observed duration dependence is 'unobserved heterogeneity' among the unemployed. In this case, unemployed are assumed to differ in their exit rates from the time they become unemployed. Even if an individual's exit rate is not changing over time, the composition of the pool of unemployed with the same unemployment duration is changing over time. In particular, the share of unemployed with low exit rates is increasing over time, and the average exit rate of the pool is declining with the time members have been unemployed.

We propose a simple model of unobserved heterogeneity and use it to account for the contributions of long-term unemployed to overall unemployment. There are two types of unemployed: 'short-term' unemployed with a high exit rate from unemployment, and 'long-term' unemployed with a low exit rate from unemployment. Newly unemployed can enter the unemployment pool as either of the two types, and while unemployed 'short-term' unemployed can switch type and become 'long-term' unemployed. This model contains two special cases: ex-ante heterogeneity only and ex-post heterogeneity only. With ex-ante heterogeneity short-term unemployed do not switch type while being unemployed, and with ex-post heterogeneity only 'short-term' unemployed enter the unemployment pool. Ex-ante heterogeneity corresponds to the 'unobserved heterogeneity' explanation of duration dependence, and ex-post heterogeneity corresponds to the 'true duration dependence' explanation of duration dependence.

Ex-ante heterogeneity has an interpretation that is consistent with unemployment reflecting the degree of 'mismatch' in the economy. A worker may lose his job for some idiosyncratic reason related to the employer, or a worker's job loss may be due to structural change and represent a permanent loss of human capital. We would expect that the worker's exit rate from unemployment will be relatively higher in the first case. A relatively bigger inflow of long-term unemployed would then indicate relatively more 'mismatch' in the economy. We find that fluctuations in the entry and exit rate of 'long-term' unemployed workers account for more than half of unemployment rate volatility, and that transitions from 'short-term' to 'long-term' unemployment are not a quantitatively important source of unemployment volatiltiy. Indeed, the results for the general framework are quite close to the model with ex-ante heterogeneity only. One should note, however, that exit rates of 'short-term' unemployed and 'long-term' unemployed tend to be positively correlated and volatility of the combined exit rates also accounts for more than half of unemployment volatility.

We can decompose the group of long-term unemployed into those that are unemployed for

more or less than one year. Before the mid-1980s less than half of the long-term unemployed were unemployed for more than one year, but this pattern has reversed since the mid-1980s. In particular, following the 2007-09 recession close to seventy percent of the long-term unemployed have been unemployed for more than one year. To account for this emerging group of the very-long term unemployed the simple two-type model is modified to include a third type. Estimating the modified model reveals that the exit rates from unemployment for the very long-term unemployed have been essentially zero in the past and that their share increased after the 2007-09 recession due to a spike in the transition rates from the long-term unemployed. Given the low exit rates of very long-term unemployed this decomposition suggests persistent high unemployment for some time after 2010.

The fact that the 'long-term' unemployed account for most of total unemployment also suggests that standard estimates of income risk that are based on transition rates derived from short-term unemployment data will understate actual income risk. We provide some suggestive calculations on how the expected present value of being (un)employed is affected by variations of the estimated transition rates. These calculations indicate that accounting for unobserved heterogeneity and using the information in the unemployment duration distribution can increase the income loss in recessions by a factor of ten relative to standard measures.

Accounting separately for 'short-term' and 'long-term' unemployed affects our interpretation of the 'quality' of the pool of unemployed workers. Since in a recession the exit rate of 'long-term' unemployed declines relatively more, the share of 'long-term' unemployed increases in a recession. Suppose that 'long-term' unemployed have lower exit rates because whenever they match with a potential employer they are less likely to be a productive match. If in a matching framework employers cannot distinguish ex-ante between 'short-tern' and 'long-term' unemployed then an increasing share of long-term unemployed reduces the expected quality of a match and thereby the incentive to post vacancies. This negative correlation between the unemployment rate and the average quality of the pool of unemployed has then the potential to amplify the volatility of unemployment exit rates, Shumer (2004).

Existing empirical work on long-term unemployment and duration dependence estimates functional forms for exit rates from unemployment using micro data. The standard approach is to estimate a Multiplicative Proportional Hazard (MPH) model for exit rates, that is, the exit rate is the product of terms that depend on (1) observable individual characteristics other than unemployment duration, (2) a calendar time effect, (3) the duration of unemployment, and (4) an unobserved individual fixed effect. Heckman and Singer (1984) provide an early survey for this approach, and Machin and Manning (1999) provide a more recent survey with an emphasis on long-term unemployment and negative duration dependence in Europe.

For some recent applications see van den Berg and van der Klaauw (2001), and Abbring, van den Berg and van Ours (2002). We replace functional form assumptions on the hazard function with a non-parametric two-type support for exit rates and the deterministic duration dependence with a Poisson process that determines the transition from the high exit rate type to the low exit rate type.

None of the empirical work that we are aware of has evaluated the contributions of time varying entry and exit rates to overall unemployment in a framework with unobserved heterogeneity in unemployment. An early precursor of our approach is Darby, Haltiwanger and Plant (1985) who speculate on the possibility that changes in the relative inflow rates of short-term and long-term unemployed account for variations of total unemployment and unemployment duration. Shimer (2012) suggests that unobserved heterogeneity is present in unemployment and observes that different measures of unemployment duration (mean or median) yield different values for the level of the unemployment exit rate but all measures yield pro-cyclical exit rates. Baker (2002) and Shimer (2012) argue that correcting for composition effects based on observable characteristics is not important for the measured cyclicality of exit rates and unemployment duration. For the 2007-09 recession Aaronson, Mazumder, and Schechter (2010) and Elsby, Hobijn, and Sahin (2010) also find that changes in the observed demographic composition of unemployment have had a limited impact on the aggregate unemployment duration.

In section 1 we review Shimer's (2012) accounting framework with homogeneous unemployment and show that it cannot account for the duration distribution of unemployment. In section 2 we describe our model with heterogeneous unemployment, how it can be used to estimate entry rates to and exit rates from unemployment, and how variations in these transition rates contribute to overall unemployment volatility. In section 3 we confirm that most of the results we obtain for aggregate unemployment also apply to more narrowly defined demographic groups: male workers of different ages, and workers in different industries and occupations. In section xx we discuss the robustness of our results to measurement error concerning the possible misclassification of unemployed and the misreporting of unemployment duration. In section 5 we provide some estimates on how unemployment heterogeneity might affect income volatility and how it might affect the matching model's ability to generate significant volatility of unemployment exit rates. Section 6 concludes.

1. Long-term unemployment with homogeneous job seekers

Shimer (2012) proposes a simple framework to account for the dynamics of unemployment. This accounting scheme uses observations on total unemployment and short-term unemployment to construct measures of the entry rates into unemployment and the exit rates from

unemployment. These measures indicate that exit rates from unemployment are a more important source of unemployment variation than are entry rates into unemployment.[1] We now review a simplified version of Shimer's (2012) unemployment accounting scheme and obtain measures of the inflow and outflow rates for unemployment. We then show that in this simple framework total unemployment and long-term unemployment are positively correlated, even though all unemployed workers have the same chance of finding a job. This result obtains because changes in the exit rate from unemployment are the main source of changes in total unemployment. Based on the measured transition rates we then calculate the implied duration distribution for an homogeneous pool of unemployed. We find that this model does not capture the duration distribution well: the model significantly under-predicts the number of long-term unemployed.

1.1. A simple framework for unemployment accounting

Consider the following simple model of total unemployment in continuous time. All unemployed are homogeneous, newly unemployed enter the unemployment pool at a rate $f(s)$, and the current unemployed exit the pool according to a Poisson process with arrival rate $\lambda(s)$. The differential equation for total unemployment, $u(s)$, is

$$\dot{u}(s) = f(s) - \lambda(s)\, u(s). \tag{1.1}$$

Suppose that the entry and exit rates are constant, then the steady state measure of unemployment is

$$u = \frac{f}{\lambda}. \tag{1.2}$$

Given a constant exit rate, at any point in time the average duration of an unemployment spell is $\bar{D} = 1/\lambda$, and the share of unemployed that have been unemployed for longer than duration D is $\exp(-\lambda D)$. Thus if changes in unemployment are mainly driven by the exit rate from unemployment, then higher unemployment will be associated with a higher average duration of unemployment and a shift in the duration distribution towards longer unemployment spells.

Assume that the instantaneous entry and exit rates remain constant during a unit of time, that is, $\lambda(s) = \lambda_t$ and $f(s) = f_t$ for $s \in (t-1, t]$ where t denotes the end of a unit time period and is an integer. In the following we will interpret a unit time period as a month. The dynamics of total unemployment u_t^m and short-term unemployment $u_{t,1}^m$, i.e. the number of unemployed that have been unemployed for less than one month is then given

[1] The results of Shimer (2012) on the relative importance of inflow and outflow rates for the determination of unemployment have been discussed by Elsby et al (2009) and Fujita and Ramey (2009).

by

$$u_t^m = \int_0^1 f_t e^{-\lambda_t s} ds + e^{-\lambda_t} u_{t-1}^m = f_t \frac{1 - e^{-\lambda_t}}{\lambda_t} + e^{-\lambda_t} u_{t-1}^m = u_{t,1}^m + \left(1 - \bar{\lambda}_t\right) u_{t-1}^m \tag{1.3}$$

Note that the measured unit inflow rate, $u_{t,1}^m$, combines the effects of the underlying instantaneous inflow rates and exit rates. We use data on total unemployment and short-term unemployment, $\{u_t^m, u_{t,1}^m\}$, together with the unemployment transition equation (1.3), to reconstruct the instantaneous entry and exit rates, $\{f_t, \lambda_t\}$.

Assume that workers are either employed or unemployed, that is, we are not considering movements in and out of the labor force. Then our definition of the inflow rate is not truly exogenous relative to the outflow rate. There can be workers that cycle through repeated unemployment-employment spells within a month. Shimer (2012) takes this possibility into account and estimates job separation rates that remain constant during the month, $\sigma(s) = \sigma_t$ for $s \in (t-1, t]$. Assuming a constant labor force during the month, $l_t = u(s) + n(s)$, this procedure implies the following law of motion for employment

$$\dot{n}(s) = \lambda_t u(s) - \sigma_t n(s) = \lambda_t u(s) - f(s), \tag{1.4}$$

and the entry rate to unemployment is time-varying.[2] For the U.S. labor market employment is large relative to unemployment and the implied exit rate from employment is small relative to the exit rate from unemployment, such that the number of workers who go through repeated unemployment-employment spells within a month is quite small. We do not report the numbers here, but for all practical purposes the employment exit rate from the Shimer (2012) procedure and the normalized unemployment entry rate are indistinguishable, $\sigma_t \simeq f_t/n_t$. In the following we will use our simplified approach to account for unemployment inflows since it allows for a closed form solution of the unemployment accounting for heterogeneous workers.

With some abuse of terminology we will occasionaly refer to our normalized entry rates to unemployment as job separation rates and our unemployment exit rates as job finding rates. Since there are transitions of workers in and out of the laborforce, strictly speaking this is not a correct interpretation. Some of the measured inflows into unemployment come from out of the labor force and some of the measured outflows from unemployment go out of the labor force.[3]

We construct (normalized) entry and exit rates for unemployment using monthly obser-

[2] Substituting for employment in (1.4) defines a differential equation for unemployment.

[3] Shimer (2012) briefly discusses results from an accounting framework for labor market transitions that includes transitions in and out of the labor force. Hornstein (2012) also discusses how results in the extended framework are affected by measurement error, namely misclassification.

vations on (un)employment from the Monthly Household Data part of the BLS Employment and Earnings survey: Employment, Table A-3, and Duration of Unemployment, Table A-12. The data are seasonally adjusted and cover the period from January 1950 to March 2011.[4] The entry and exit rates for unemployment implied by our accounting procedure are displayed in Figure 2, panels A and B.[5] The panels display the monthly transition probabilities based on the quarterly averages of monthly flow transition rates. For example, the probability that a worker will exit unemployment within a month is $1 - \exp(-\lambda)$, where λ is the quarterly average of monthly exit rates.

The exit probabilities from unemployment vary between thirty and sixty percent with an average of about forty five percent, and the normalized entry probabilities vary between two and five percent with an average of about three and a half percent. Thus unemployment is of short duration with an average of somewhat more than two months.[6] There is a downward trend in the exit probability from unemployment, and an upward trend in the entry probability to unemployment that is reversed in the 1990s. The declining unemployment exit probability is reflected in the increasing trend for the average duration of unemployment, whereas the trend reversal for the unemployment entry rate in the 1990s accounts for the decline in the average unemployment rate, Table 1.A. The 2007-09 recession is associated with a sharp decline of the unemployment exit rate and a transitory up tick of the unemployment inflow. Comparing Figures 2.A, B and C we can see that periods of high unemployment are associated with low exit rates from unemployment and high entry rates to unemployment.

Figure 2. Homogeneous Unemployment

1.2. Accounting for unemployment volatility

Shimer (2012) argues that the volatility of unemployment exit rates accounts for most of the cyclical fluctuations of the unemployment rate. Shimer (2012) comes to this conclusion using a procedure that linearizes the unemployment rate process around its trend path. We now revisit this question using an alternative procedure to decompose unemployment rate fluctuations. Our alternative procedure confirms that for homogeneous unemployment variations in unemployment exit rates are the most important source of unemployment volatility.

[4] Shimer (2012) argues that there is a break in the data collection process in January 1994 and that short-term unemployment, that is, those that have been unemployed for less than 5 weeks, is understated after January 1994. Shimer (2012) suggests a correction factor of 10 percent and accordingly we move 10 percent of those who have been unemployed for 5 to 14 weeks to the group of people who have been unemployed for less than 5 weeks.

[5] For reasons explained below, Section 2.1, we only discuss transition rates for the period ending December 2009.

[6] If we interpret the entry rate as the job separation rate then employment is of long duration with an average duration of about two and a half years.

This alternative procedure will be quite useful for the analysis of the unemployment rate fluctuations when we allow for heterogeneous unemployment.

We calculate the trend of a variable using a band-pass filter that eliminates fluctuations with periodicity less than twelve years, for example, Baxter and King (1999).[7] The dashed lines in Figure 2, panels A, B and C, display the trends of the unemployment exit and entry rates and the unemployment rate. We also define an alternative trend for the unemployment rate as follows. Given a time path for monthly instantaneous inflow rates and exit rates from unemployment, $x = (f, \lambda)$, equation (1.3) defines a mapping for the path of the unemployment rate, $u = G(x)$. We calculate the trend for each component of x using a band-pass filter, $x^T = \left(f^T, \lambda^T\right)$, and define the alternative trend unemployment rate as $u^T = G\left(x^T\right)$ and the deviation of unemployment from trend as $du^T = u - u^T = G(x) - G\left(x^T\right)$. Next, we calculate the contribution of a variable to the trend deviation of the unemployment rate as

$$du_i^T = G\left(x_i, x_{-i}^T\right) - G\left(x_i^T, x_{-i}^T\right).$$

If G is a linear mapping and the trend filter is linear, as the band-pass filter is, then this alternative procedure yields the same result as applying the trend filter directly to the unemployment rate. Furthermore, the sum of the individual variables' trend deviation contributions sum to the unemployment rate trend deviation. Indeed, for the model with homogeneous unemployment, there is almost no difference between the two definitions of the trend unemployment rate. On the other hand, for the models with heterogeneous unemployment that we consider below the mapping G can be sufficiently nonlinear, such that the residual term

$$r^T = du^T - \sum_i du_i^T$$

would become quantitatively important if we were to follow Shimer's (2012) original procedure.

In Figure 2.D we plot the trend deviations of the unemployment rate and the contributions of the exit rate from unemployment and the (normalized) inflow rate to unemployment. It is quite clear that even though spikes in the unemployment entry rate precede an increase of the unemployment rate, most of the unemployment rate increase is attributable to a decline in the exit rate from unemployment. For the most recent 2007-09 recession the drastic unemployment rate increase has to be attributed to an exceptional decline of the unemployment exit rate, whereas the up tick in the unemployment entry rate was not exceptional compared to past recessions.

[7] Shimer (2012) calculates a very smooth trend with a Hodrick-Prescott filter using a smoothing parameter 100,000. The fact that we use a different filter to calculate the trend has only a minor impact.

The average contributions of different exit rate volatilities to the overall unemployment rate volatility are displayed in Table 1.B. We follow Shimer (2012) and write the variance components of the unemployment rate as

$$1 = \sum_i \frac{Cov\left(du^T, du_i^T\right)}{Var\left(du^T\right)} + \frac{Cov\left(du^T, r^T\right)}{Var\left(du^T\right)}.$$

For the full sample, 1950-2009, unemployment exit rate volatility accounts for eighty percent of unemployment rate volatility, and the contribution of the unemployment exit rate volatility increases steadily for more recent time periods. So far our results are essentially the same as in Shimer (2012).

1.3. Accounting for long-term unemployment

By construction our accounting procedure matches total and short-term unemployment, but not necessarily the overall duration distribution of unemployment. One way to see how well the homogeneous agent model fits the outflows from unemployment is to construct the duration distribution implied by the model and compare that hypothetical distribution with the actual duration distribution. In addition to the data on short-term unemployment that we have used above, Table A.12 of the BLS Employment and Earnings survey also provides monthly data on the number of unemployed that have been unemployed between 5 and 14 weeks, $u_{t,2}^m$, between 15 and 26 weeks, $u_{t,3}^m$, and for 27 weeks or more, $u_{t,4}^m$.[8] The sequence of duration distributions is denoted $u^m = \left\{u_{t,k}^m : k = 1, \ldots, 4 \text{ and } t = 1, \ldots, T\right\}$. The duration distribution implied by the homogeneous exit model is obtained by simple iteration using the constructed entry and exit rates

$$u_{t,1} = u_{t,1}^m \tag{1.5}$$

$$u_{t,j} = \left(1 - \bar{\lambda}_t\right) u_{t-1,j-1} \text{ for } j = 2, \ldots, J \tag{1.6}$$

The duration distribution is truncated at a sufficiently large value J. For our calculations we use a maximum unemployment duration of four years, that is, $J = 48$. We can then time-aggregate the monthly duration data and obtain the implied numbers of unemployed for the reported long-term unemployment bins, $\hat{u}_{t,j}^m$. These measurement equations are

$$\hat{u}_{t,2}^m = \sum_{j=2,3} u_{t,j},\ \hat{u}_{t,3}^m = \sum_{j=4,5,6} u_{t,j}, \text{ and } \hat{u}_{t,4}^m = \sum_{j=7,\ldots,J} u_{t,j}. \tag{1.7}$$

[8] We assume that an unemployment duration of 5 to 14 weeks represents 2 and 3 months, a duration of 15 to 26 weeks represents 4 to 6 months, and a duration of more than 26 weeks represents more than 6 months.

The duration distribution is displayed in the four panels of Figure 3. By construction the homogeneous exit rate model matches the very short-term unemployment, Panel A. The model overstates short-term and medium-term unemployment, Panels B and C, and significantly understates long-term unemployment, Panel D. Even though the model captures the qualitative features of long-term unemployment, it fails to account for the magnitude of long-term unemployment. For almost all recessions the model accounts for only one third of long-term unemployment at its peak.

Figure 3. Duration Distribution

2. Unemployment accounting with heterogeneous unemployment

We now describe a simple model of unobserved heterogeneity among unemployed workers that accounts quite well for the duration distribution of unemployment. For this model we assume that there are two types of unemployed workers: 'short-term' (ST) unemployed with a relatively high exit rate and 'long-term' (LT) unemployed with a relatively low exit rate from unemployment. An unemployed worker may start out as being of either type. Furthermore, a ST unemployed worker that does not find work may over time make a transition to LT unemployment, but the reverse does not happen. On the one hand this framework confirms Shimer's (2012) results that variations in exit rates from unemployment account for most of the unemployment rate volatility. On the other hand this framework also suggests that even though unemployment tends to be short-term on average, unemployment rate volatility is mostly driven by variations in the entry and exit rates of LT unemployed workers.

2.1. A simple model of long-term unemployment

Consider two types of unemployed workers, $i = 1, 2$, and type 2 has a lower exit rate from unemployment than does type 1, $\lambda^1(s) > \lambda^2(s)$ for all s. The transition equations for short-term and long-term unemployment are

$$\begin{aligned} \dot{u}^1(s) &= f^1(s) - \left[\lambda^1(s) + \gamma^1(s)\right] u^1(t) \\ \dot{u}^2(s) &= f^2(s) + \gamma^1(s) u^1(s) - \lambda^2(t) u^2(s). \end{aligned} \tag{2.1}$$

Similar to the model with homogeneous unemployment, we assume that the instantaneous entry and exit rates are constant for the monthly time intervals, $f_t^i = f^i(s)$, $\lambda_t^i = \lambda^i(s)$, and $\gamma_t^1 = \gamma^1(s)$ for $s \in (t-1, t]$. In the Appendix we derive for each type the expressions for the implied monthly net-entry to unemployment and the transition equations for end-of-period

unemployment by duration,

$$\begin{aligned}
u^1_{t,1} &= F^1\left(f^1_t, f^2_t, \lambda^1_t, \lambda^2_t, \gamma^1_t\right) \\
u^2_{t,1} &= F^2\left(f^1_t, f^2_t, \lambda^1_t, \lambda^2_t, \gamma^1_t\right) \\
u^1_{t,j} &= \left[1-\Lambda^1\left(\lambda^1_t\right)\right]\left[1-\Gamma^1\left(\gamma_{1t}\right)\right] u^1_{t-1,j-1} \\
u^2_{t,j} &= \left[1-\Lambda^1\left(\lambda^1_t\right)\right]\left[u^2_{t-1,j-1} + \Psi\left(\lambda^1_t, \gamma^1_t\right) u^1_{t-1,j-1}\right]
\end{aligned}$$

The measurement equations for the model are

$$\begin{aligned}
u^m_{t,1} &= \sum_{i=1,2} u^i_{t,1},\ u^m_{t,2} = \sum_{j=2,3}\sum_{i=1,2} u^i_{t,j} \\
u^m_{t,3} &= \sum_{j=4,5,6}\sum_{i=1,2} u^i_{t,j}, \text{ and } u^m_{t,4} = \sum_{j=7,\ldots,J}\sum_{i=1,2} u^i_{t,j}.
\end{aligned} \tag{2.2}$$

We find the entry and exit rates for the two types by solving a nonlinear least squares problem. For the algorithm we first specify (1) an initial distribution for unemployment by monthly duration, $x_1 = \left\{u^i_{1,j} : j = 1, \ldots, J, \text{ and } i = 1, 2\right\}$, and (2) a sequence of transition rates for both types, $x_2 = \left\{f^1_t, f^2_t, \lambda^1_t, \lambda^2_t, \gamma^1_t : t = 1, \ldots, T\right\}$. This allows us to construct the sequence of end-of-period duration distributions for unemployment $\{u^i_{t,j} : i = 1, 2,\ j = 1, \ldots, J, \text{ and } t = 1, \ldots, T\}$, which we then time-aggregate using the measurement equations to get the implied measured duration distribution, $\hat{u}^m = \left\{\hat{u}^m_{t,k} : k = 1, \ldots, 4 \text{ and } t = 1, \ldots, T\right\}$. We choose the vector of unknowns, (x_1, x_2), to minimize the criterion function $\sum_{t=1}^T \sum_{k=1}^4 \left(\hat{u}^m_{t,k} - u^m_{t,k}\right)^2$. We also impose a penalty on month to month changes in the transition rate from type 1 to type 2, and the relative exit rate of type 2 workers, $\kappa_t = \lambda^2_t/\lambda^1_t$. The smoothness restrictions are imposed for two reasons. First, the type-to-type transition rate tends to be excessively volatile without this restriction. Second, standard MPH model estimates of unemployment exit rates impose a constant relative hazard rate.

Our algorithm estimates current transition rates based on their implications for current and future duration distributions. This means that at the end of the sample the restrictions imposed by data on the transition rates are quite loose. Estimating transition rates for truncated samples suggests that at least a half year of data is required to obtain transition rates that remain invariant to an extension of the truncated sample. In the following we therefore report only transition rates up to December 2009 even though we use data up to March 2011 to estimate the transition rates.

2.1.1. Discussion

Our framework captures the two explanations for negative duration dependence in unemployment data that have been proposed in the literature: 'true duration dependence' and 'unobserved heterogeneity,' for example, Machin and Manning (1999). The case of 'true duration dependence' is represented by the assumption that all new entrants to unemployment are ST unemployed, $f^2(s) = 0$, and over time ST unemployed make a random transition to LT unemployment, $\gamma^1(s) \geq 0$. The case of 'unobserved heterogeneity' is represented by the assumption that at the time of entry into unemployment a worker's type is determined as either ST or LT, $f^i(s) \geq 0$, and the unemployed worker will not switch type before he exits the unemployment pool, $\gamma^1(s) = 0$. In the following we will refer to the 'unobserved heterogeneity' case as ex-ante heterogeneity and to the 'true duration dependence' case as ex-post heterogeneity.

The model with ex-ante heterogeneity can potentially account for changes in unemployment and the duration distribution of unemployment through changes in the inflow rates and the exit rates of the two types. For such a model, Darby et al (1985) suggest that changes in the relative inflow rates could represent a major source of changes in the duration distribution. In this context it is tempting to assume that 'structural' unemployment is mainly reflected in LT unemployment, that is, an increase (decline) in the relative inflow (exit) rate of LT unemployed is mainly caused by structural factors. One can then use this framework to evaluate the relative contributions of ST and LT unemployment to overall unemployment volatility.

2.2. Exit and entry

The model with heterogeneous unemployment matches the duration distribution of unemployment quite well, Figure 3. The lines for the actual duration distribution (black) and the constructed duration distribution (red) are almost on top of each other. Most of the inflow into unemployment consists of ST unemployed workers who exit unemployment rapidly. Even though LT unemployed workers account for only a small share of unemployment inflows, due to their very low exit rate they constitute close to one half of total unemployment.

The unemployment exit rates for the model with heterogeneous unemployment bracket the exit rate from the homogeneous agent model, Figure 4.A. For the sample, the monthly exit probability for ST unemployed workers fluctuates between fifty and eighty percent, with no clear trend and an average of about sixty-five percent. Thus the average duration of ST unemployment is less than one month, which is less than half the unemployment duration predicted by the homogeneous agent model. On the other hand, the monthly exit rate of the LT unemployed fluctuates between ten and thirty percent, with an average exit rate of

fifteen percent and a clear downward trend. Thus for LT unemployed workers the average unemployment duration is about seven months, more than three times the average duration of unemployment in the model with homogeneous unemployment.

Figure 4. Heterogeneous Unemployment, 2-Types

The unemployement entry probabilities of LT and ST unemployed workers roughly sum to the entry probabilities of the model with homogeneous unemployment. Most of unemployment entry is ST, and LT unemployed workers contribute a relatively small share to the unemployment inflow, between ten and twenty percent, Figure 4. B and C. Because of their low exit rate LT unemployed workers, however, account for a substantial share of total unemployment, usually between thirty and sixty percent, Figure 4.C. Furthermore, LT unemployed make up essentially all of measured long-term unemployed that is, those unemployed for more than 26 weeks. In this sense, long-term unemployed are indeed different from the overall pool of unemployed.

'Pure duration dependence' appears to play a rather limited role in the determination of unemployment. The monthly transition probability from type ST to type LT fluctuates around 1.5 percent, 4.B. Given the high exit rates from unemployment for ST unemployed workers the probability that such a worker makes the transition to type LT before finding a job is negligible, about 1.5 percent. The low type transition rate in the general model reflects that the general model is actually quite close to the special case with ex-ante heterogeneity only.[9]

Prior to the 2007-09 recession unemployment declined, but the share of workers that were unemployed for more than 26 weeks stayed higher than in previous expansion phases, Figure 1. The model accounts for this secular increase in the share of long-term unemployment through a decline of the inflow rates for ST unemployed workers and relatively constant inflow of LT unemployed, Figure 4.B. The apparent trend increase in the inflow share of LT unemployed workers started already in the 1990s. Whereas the inflow of LT unemployed almost never contributed more than 20 percent to total inflow before the mid-1990s, LT unemployed have been contributing close to twenty percent or more to total inflow since the mid-1990s.

The empirical labor literature on duration dependence usually estimates a multiplicative proportional hazard model (MPH) for unemployment exit rates, for example Machin and

[9] The estimation of the model with ex-ante and ex-post heterogeneity discussed in this section takes as an initial starting point the results from the model with ex-ante heterogeneity only. We also estimate the model starting with the results from the model with ex-post heterogeneity only. The least-sqaure procedure converges to two different local minimia that share the properties of their respective initial starting values. The model discussed in this section provides a better fit, that is, it has the smaller sum of squares. We discuss the alternative estimate in B.

Manning (1999) or van den Berg and van der Klaauw (2001). In the MPH model the exit rate from unemployment is the product of a function of known demographic characteristics, a function of observed unemployment duration ('true duration dependence'), and a fixed effect ('unobserved heterogeneity'). This multiplicative structure then implies that the relative exit rates of workers with different fixed effects are constant. Our simple two-type model of unobserved heterogeneity does not impose constant relative exit rates. As we can see from Figure 4.D, even though the relative exit rate of LT unemployed workers is quite stable it nevertheless exhibits a downward trend over the sample period. Furthermore, the exit rate of LT unemployed appears to decline more in recessions than does the exit rate of ST unemployed, especially in the 2007-09 recession. Another way to see that the relative exit rate is not constant is to look at the cross-correlation between the trend deviations of the two unemployment exit rates. For the full sample that cross correlation is 0.66 and it increases to 0.73 for later periods.

2.3. Contributions to unemployment rate volatility

The model with heterogeneous unemployment suggests a reassessment of the sources of unemployment rate volatility. Associated with the heterogeneity of unemployment we find that LT unemployment alone accounts for three fourths of unemployment rate volatility. Similar to Shimer (2012), we find that overall exit rate volatility of ST and LT unemployed workers accounts for about two thirds of unemployment rate volatility.

In Figure 5 we plot the contributions of different transition rates to the trend deviations of the unemployment rate. The contributions are calculated as described in Section 1.2. We can see that a decline of the unemployment exit rate for LT unemployed workers makes a substantial contribution to every increase of the unemployment rate. Furthermore, increased inflow of LT unemployed and/or a reduction of the exit rate of ST unemployed make substantial contributions to most increases of the unemployment rate. Fluctuations of the entry rate of ST unemployed and the transition rate from type ST to type LT make only small contributions to unemployment rate volatility. The visual impression is confirmed by the variance decomposition in Table 1.C. For the full sample, 1950-2009, exit and entry rate volatility of the LT unemployed accounts for about one third each of overall unemployment rate volatility, and exit rate volatility of ST unemployed accounts for another one fourth of unemployment rate volatility. There is no big difference between more recent sample periods, except for an increased contribution to volatility from the LT unemployment exit rate.

Figure 5. Contributions to Unemployment Volatility

The model with heterogeneous unemployment allows a characterization of the unemployment rate increase in the 2007-09 recession that is consistent with structural reallocation as

an important source of unemployment. In the introduction we suggested that unemployment due to structural change in the economy is likely to show up as an increased inflow and reduced exit rate of LT unemployment. As we can see from Figure 5, increased entry and reduced exit of LT unemployment are indeed the major drivers of the unemployment increase in 2009. With respect to the behavior of LT unemployment, the 2007-09 recession is similar to other previous recessions in 1957-58, 1981-82, and 2001. Exceptions to this pattern are the recessions of 1953-54, 1969-70, 1973-75 and 1990-91 where declining exit rates for both, ST and LT unemployment, are important sources of increased unemployment. We should note that our results on LT unemployment as an important driver of the unemployment rate depend on the fact that we do not restrict the relative exit rates from unemployment. As noted above the cross-correlation between the ST and LT unemployment exit rates is positive, but not large, about 0.65 for the full sample. If we impose a constant relative exit rate from unemployment, then the model will attribute the unemployment increase in the 2007-09 recession to a general decline in exit rates from unemployment.

3. Unobserved heterogeneity for demographic groups

The unemployment experience can differ substantially across identifiable demographic groups. For example, the average unemployment rate among college graduates is less than one third the unemployment rate among workers with less than a high school degree, and the average unemployment duration of older male workers is about twice that of younger male workers. Applying our accounting framework to aggregate unemployment may then mistakenly attribute changes in the composition of the unemployment pool to unobserved heterogeneity. To evaluate this possibility we now perform our accounting exercise for different age groups of unemployed males, and for industry and occupation classifications of unemployed workers. We find that the results we obtain for aggregate unemployment are broadly consistent with the results from the more detailed demographic groups. This result should not be too surprising, after all micro studies have found significant evidence for duration dependence of unemployment exit rates, for example, Machin and Manning (1999), and composition effects have not found to be important for aggregate unemployment rates and average unemployment duration, for example, Baker (1992) or Aaronson et al (2010).

3.1. Unemployment for male age groups

Unemployment rates for older workers tend to be lower than for younger workers, yet once being unemployed older workers tend to remain unemployed for longer than younger workers, Table 2.A. Despite these differences between older and younger workers, the driving sources of unemployment in terms of exit and entry rates for short-term and long-term unemploy-

ment are broadly comparable with those for aggregate unemployment. Unemployment rate volatility for each age group is mainly driven by variations in the entry and exit rates from long-term unemployment.

We use monthly data from Table A.35 of the BLS Employment and Earnings Survey for male workers from 1976 to the present to estimate (un)employment transition rates for the age groups 20-24, 25-34, 35-44, 45-44, 55-64 and 65 years and older. Unemployment duration distribution data are available for five bins: less than 5 weeks, 5 to 14 weeks, 15 to 26 weeks, 26 weeks to 51 weeks, and 51 weeks or more.[10] For each age group we estimate transition rates for the model with unobserved heterogeneity, merging the last two bins. The results of the accounting exercise are displayed in Table 2.

The results are displayed in Table 2.B. For each age group we display the average sample values for transition rates in the first column and the contributions of transition rates to unemployment rate volatility in the second column. The model accounts for the increasing average unemployment duration for older workers mostly through a declining exit rate from unemployment for both types. An exception is the age group of 45 to 54 year old males where the average exit rate declines because relatively more LT unemployed enter the unemployment pool. Unemployment rates are declining with age, despite the average unemployment duration being increasing with age, because entry rates to unemployment decline with age. For almost all age groups variations in the LT transition rates remain the most important source of unemployment variations, and variations in exit rates from unemployment tend to be more important than variations of entry rates.[11] Only for workers aged 34 or younger are variations in the entry rate of LT unemployed relatively less important. On the other hand, for workers older than 45 years the role of variations in the ST exit rate as a source of unemployment volatility is significantly diminished. For no age group, except the workers aged 55 to 64 years do variations in the transition rate from ST to LT unemployment play a role for unemployment volatility, and even for this group the contribution is small.

3.2. Unemployment across industries and occupations

The data for aggregate unemployment and the different age groups of male unemployment indicate that the volatility of exit rates rather than entry rates and the volatility of LT transition rates rather than ST transition rates are the major sources of unemployment

[10] The data are not seasonally adjusted. Again, we use Watson's (1996) version of X-11 to deseasonalize the data.

[11] For the different male age groups the residual part of unemployment rate volatility is quite large at 15 percent. Thus, relative to overall unemployment the accounting framework does not as nicely separate out the contributions of the different transition rates to unemployment rate volatility. Part of this seems due to the fact that our X-11 procedure does not completely remove all the seasonal components in the duration distribution of unemployment.

rate volatility. Unemployment data by industry and occupation confirm the latter, and the former for most but not all industries and occupations. We use monthly data from Table A.36 of the BLS Employment and Earnings Survey for industries and occupations from 2000 to the present to estimate unemployment transition rates for the model with unobserved heterogeneity using unemployment duration distribution data for four bins: less than 5 weeks, 5 to 14 weeks, 15 to 26 weeks, and 27 weeks or more.

The results for industry data are displayed in Table 3. For the majority of industries variations in the unemployment exit rate are the main source of unemployment rate volatility, but there are four industries–construction, the production of durable and nondurable goods, and leisure and hospitality–for which variations in the unemployment entry rate are the main source of unemployment rate volatility. In either case, variations in the LT transition rates make the biggest contribution to unemployment volatility. Variations in the type transition rate tend to have a negligible effect on unemployment volatility, with the exception of nondurable goods production and public administration. But even for these two industries the contribution of transition rate volatility is small.

The results for occupational data are displayed in Table 4. The results for occupational unemployment are very similar to the ones for industry unemployment. With the exception of production and professional occupations, variations in exit rates from unemployment are the most important source of unemployment rate volatility, and variations in LT transition rates are more important than variations in ST transition rates.[12]

4. Robustness

4.1. Misclassification of labor market states

[To be written]

4.2. Misreporting unemployment durations

[To be written]

5. Why long-term unemployment matters

Most quantitative macro-theory work on the search and matching model of the labor market relies on a characterization of labor market transitions that emphasizes the short-term nature

[12] The only exception to this pattern is the farming related occupation, but for this occupation the framework we use for the decomposition of unemployment volatility performs quite badly. The main reason is again that our X-11 procedure does not remove the seasonal components completely.

of unemployment. For example, the literature on the search and matching model's ability to account for unemployment rate volatility is concerned with matching the cyclicality of the measured exit rate for short-term unemployment.[13] In this context we should emphasize again, that our results are in line with Shimer (2012) in the sense that on average, variations in the exit rate for both, short-term and long-term unemployed contribute the most to overall unemployment rate volatility. Allowing for unobserved unemployment heterogeneity does suggest, however, that it is mainly the volatility of long-term unemployment that contributes to overall unemployment volatility. The following 'back of the envelope' calculation suggests that this feature might have consequences for the welfare costs of income fluctuations associated with unemployment.

Suppose that all employed workers get a wage, normalized at one, $w = 1$, and that the flow value of unemployment is sixty percent of the wage, $b = 0.6$. Suppose that workers are risk-neutral and discount the future at a constant annual interest rate, $r = 0.05$. This is a standard calibration for quantitative macro models of the labor market. We now calculate the capital values of being (un)employed for the estimated (un)employment transition rates of the model with homogeneous and heterogeneous unemployment. For example, for the simple model with two types of unemployment we define the capital value equation for being employed as

$$\begin{aligned} rW_t &= w + \sigma_{1,t}\left(U_{1,t} - W_t\right) + \sigma_{2,t}\left(U_{2,t} - W_t\right) + \theta\left(\bar{W} - W_t\right) \\ r\bar{W} &= w + \bar{\sigma}_1\left(\bar{U}_1 - \bar{W}\right) + \bar{\sigma}_2\left(\bar{U}_2 - \bar{W}\right) + \theta\left(W_t - \bar{W}\right) \end{aligned}$$

The first equation defines the return on being employed in period t, given the current exit rates from employment, and the possibility that the transition rates change to the sample average transition rates at a rate θ. The second equation is the converse of the first equation if the current state is defined by the sample average transition rates. If we set the aggregate transition rate to zero, $\theta = 0$, we get the steady state capital values if the current transition rates were to persist forever. The larger is θ the less variation there is in capital values. The capital value equations for current unemployment are defined as

$$\begin{aligned} rU_{1,t} &= b + \lambda_{1,t}\left(W_t - U_{1,t}\right) + \gamma_{1,t}\left(U_{2,t} - U_{1,t}\right) + \theta\left(\bar{U}_1 - U_{1,t}\right) \\ rU_{2,t} &= b + \lambda_{2,t}\left(W_t - U_{2,t}\right) + \theta\left(\bar{U}_2 - U_{2,t}\right) \end{aligned}$$

Implicit in these capital value definitions is that an employed worker may end up as ST or LT unemployed on separation, and that the unemployment type affects the rate at which the worker finds new employment, but it does not affect the type of employment.

[13] See, e.g., Shimer (2005), Hall (2005), and Hagedorn and Manovski (2008)

Figure 7. Capital Value of Employment

In Figure 7 we plot the percentage deviation of the capital value of employment from its mean for the two models of unemployment when $1/\theta = 3$ years. The qualitative features of the capital values of employment are very similar across the two models: they exhibit sharp declines in recessions. The capital values do differ in the magnitude of their decline during and after recessions. For the model with homogeneous unemployment matched to data on short-term unemployment the declines in the capital value of employment are relatively small: the biggest declines represent about one percent of the mean sample value and occur in the early 1980s and after the 2007-09 recession. The capital value of employment declines much more with heterogeneous unemployment, especially following the 2007-09 recession. For this model the employment value declines by ten percent relative to its mean. The capital values of unemployment are closely correlated with the capital values of employment, but more volatile with the volatility of ST unemployment being lower than for LT unemployment.

6. Conclusion

Relative to other OECD countries the labor market of the United States is usually characterized as being subject to a high degree of turnover, for example, Elsby, Hobijn and Sahin (2008). Workers in the U.S. economy are more likely to lose their job, but they are also more likely to find a job. Shimer (2012) provides an unemployment accounting scheme that focuses on this feature of short-term unemployment. Shimer's characterization of transition rates, in particular, the importance of high and volatile exit rates from unemployment has become very influential in the way macro economists quantitatively account for labor markets in search and matching models of unemployment. In view of the increasing share of long-term unemployment since the 2007-09 recession we provide an extension of the Shimer (2012) unemployment accounting scheme that allows for both, short-term and long-term unemployment based on data on the duration distribution of unemployment. We find that long-term unemployment is important and that the share of long-term unemployment has increased since the 1990s. Whereas before 1990 short-term unemployed workers with an average unemployment duration of less than one month accounted for the majority of total unemployment, after 1990 long-term unemployed with an average unemployment duration of eight months account for the majority of unemployment. Furthermore, it appears that most of unemployment volatility is not due to short-term unemployment but due to variations in the entry and exit rates of long-term unemployed.

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Appendix

A. A hybrid model with ex-ante and ex-post unobserved heterogeneity

This Appendix develops the continuous time version of the model with two types of unemployed agents, $i = 1, 2$. Let $u^i(t, \tau)$ denote the number of type i agents that at time t have been unemployed for duration τ, both t and τ are potentially real valued. Agents become newly unemployed at the instantaneous inflow rates f^i, unemployed agents exit the unemployment state at the instantaneous rate λ^i, and type one agents make the transition to type two at the instantaneous rate γ^1. Let the unit of time be one month. We observe the number of unemployed agents at discrete time intervals. We assume that the instantaneous transition rates are constant for a given time interval, e.g. $\lambda^i(s) = \lambda^i_t$ for $s \in (t-1, t]$ where t is integer valued. Unemployed workers make the following transitions between unit time intervals:

$$
\begin{aligned}
u^1(t,\tau) &\rightarrow u^1(t+1,\tau+1), u^2(t+1,\tau+1), \text{ or employed} \\
u^2(t,\tau) &\rightarrow u^2(t+1,\tau+1) \text{ or employed}
\end{aligned}
$$

Between discrete time intervals the measures of agents that have been unemployed for longer than one unit period, that is, for $\tau > 1$, evolve as follows. For type one agents, the measure of agents that remain unemployed is

$$
\begin{aligned}
u^1(t+1,\tau+1) &= u^1(t,\tau)\, e^{-\left(\lambda^1_{t+1}+\gamma^1_{t+1}\right)} \\
&= u^1(t,\tau)\left(1-\bar{\lambda}^1_{t+1}\right)\left(1-\bar{\gamma}^1_{t+1}\right)
\end{aligned}
$$

Note that the law of motion is independent of the duration the agent has been unemployed. Similarly, for type two agents the measure of agents that remain unemployed evolves according to

$$
u^2(t+1,\tau+1) = u^2(t,\tau)\, e^{-\lambda^2_{t+1}} + u^1(t,\tau)\int_0^1 e^{-\lambda^1_{t+1}s}\left[\gamma^1_{t+1}e^{-\gamma^1_{t+1}s}\right]e^{-\lambda^2_{t+1}(1-s)}ds
$$

where the product in the integral represents the probability that a type one agent does not exit unemployment within duration s from time t, makes a transition to type two at duration s, and does not exit unemployment in the remaining time up to $t+1$. This law of motion

can be rewritten as

$$
\begin{aligned}
u^2(t+1,\tau+1) &= u^2(t,\tau)\, e^{-\lambda^2} + u^1(t,\tau)\, e^{-\lambda^2}\gamma^1 \int_0^1 e^{-\left(\lambda^1+\gamma^1-\lambda^2\right)s} ds \\
&= u^2(t,\tau)\, e^{-\lambda^2} + u^1(t,\tau)\, e^{-\lambda^2} \frac{\gamma^1}{\gamma^1+\lambda^1-\lambda^2}\left[1-e^{-\left(\gamma+^1\lambda^1-\lambda^2\right)}\right] \\
&= \left(1-\bar{\lambda}_{t+1}^2\right)\left\{u^2(t,\tau)+u^1(t,\tau)\,\bar{\psi}_{t+1}^1\right\}
\end{aligned}
$$

The number of type i unemployed workers that have been unemployed for s weeks at time t is

$$
u_t^i(s) \equiv \int_s^{s+1} u^i(t,\tau)\, d\tau.
$$

Using the above defined transition equations for $u^i(t,\tau)$ we get the following expressions for the law of motion of the measured unemployment stocks

$$
\begin{aligned}
u_{t+1,s+1}^1 &= \left(1-\bar{\lambda}_{t+1}^1\right)\left(1-\bar{\gamma}_{t+1}^1\right) u_t^1(s) \\
u_{t+1,s+1}^2 &= \left(1-\bar{\lambda}_{t+1}^2\right)\left[u_{t,s}^2+u_{t,s}^1\bar{\psi}_{t+1}^1\right]
\end{aligned}
$$

The measures of agents that have been unemployed for less than unit of time are defined as follows. For type one agents, the number of unemployed agents at time t is the cumulative entry into unemployment in the preceding time interval that has not exited again

$$
\begin{aligned}
u_{t,1}^1 &= \int_0^1 f^1(t-\tau)\, e^{-\left(\lambda_t^1+\gamma_t^1\right)\tau} d\tau = f_t^1 \int_0^1 e^{-\left(\lambda_t^1+\gamma_t^1\right)\tau} d\tau \\
&= f_t^1 \frac{1-e^{-\left(\lambda_t^1+\gamma_t^1\right)}}{\lambda_t^1+\gamma_t^1} = f_t^1\bar{\phi}_t^1
\end{aligned}
$$

And for type two agents that measure is the cumulative entry into unemployment in the preceding time interval that has not exited again plus the entry of type one agents that have made the transition to type two and also have not exited unemployment

$$
\begin{aligned}
u_{t,1}^2 &= \int_0^1 f^2(t-\tau)\, e^{-\lambda_t^2\tau} d\tau + \int_0^1 f^1(t-\tau)\left[\int_\tau^1 e^{-\lambda_t^1(s-\tau)}\gamma_t^1 e^{-\gamma_t^1(s-\tau)} e^{-\lambda_t^2(1-s)} ds\right] d\tau \\
&= f_t^2 \frac{1-e^{-\lambda^2}}{\lambda^2} + f_t^1 \frac{\gamma_t^1}{\lambda_t^1-\lambda_t^2+\gamma_t^1}\left\{\frac{1-e^{-\lambda_t^2}}{\lambda_t^2} - \frac{1-e^{-\left(\lambda_t^1+\gamma_t^1\right)}}{\lambda_t^1+\gamma_t^1}\right\} \\
&= f_t^2\bar{\phi}_t^2 + f_t^1 \frac{\gamma_t^1}{\lambda_t^1-\lambda_t^2+\gamma_t^1}\left(\bar{\phi}_t^2-\bar{\phi}_t^1\right)
\end{aligned}
$$

We can obtain estimates of the transition rates and entry rates as follows.

- First, specify a time path for the instantaneous transition rates and the total entry rate of type one agents, $y = \left\{\lambda_t^1, \lambda_t^2, \gamma_t^1, u_t^1\right\}$.

- Then construct the discrete time equivalents of the transition rates and the total inflow for type two agents, $\left\{\bar{\lambda}_t^1, \bar{\lambda}_t^2, \bar{\gamma}_t^1, \bar{\psi}_t^1, u_t^2\right\}$.

- Use the laws of motion to construct the end-of-period unemployment stocks by duration, $\left\{u_{t,s}^i\right\}$ and calculate the difference to the observed duration distribution. Find a sequence y that minimizes the difference.

- Conditional on a solution for y recover the instantaneous entry rates $\{f_t^i\}$

B. Pure duration dependence

If we were to restrict the model to ex-post heterogeneity only, transitions from type ST to type LT play a more important role. For this special case of the model with heterogeneous unemployment the type transition rate fluctuates around 7.5 percent before the 1980s and around 12 percent starting in the 1990s.

[TO BE WRITTEN]

Table 1. Accounting for Unemployment

Sample	1950-2009	1967-2009	1976-2009	1987-2009
	A. Aggregate Statistics			
u	5.7	6.0	6.2	5.6
D	13.8	14.5	15.7	16.2
	B. Homogeneous Unemployment			
σ	0.17	0.14	0.12	0.11
λ	0.80	0.84	0.86	0.89
Residual	0.04	0.01	0.01	0.01
	C. Heterogeneous Unemployment			
σ^1	0.04	0.03	0.01	-0.02
σ^2	0.33	0.32	0.33	0.32
λ^1	0.24	0.24	0.22	0.21
λ^2	0.34	0.39	0.42	0.48
γ^1	0.00	0.00	0.00	0.00
Residual	0.05	0.02	0.02	0.00

Note: For each sample period, Part A displays the average unemployment rate in percent, u, and the average mean duration of unemployment in weeks, D; Part B displays the contributions of exit rates from employment, σ, and unemployment, λ, to unemployment rate volatility for the model with homogeneous unemployment; Parts C respectively D display the contributions of exit rates for different types to unemployment rate volatility for the two-type respectively three-type model of unobserved heterogeneity.

Table 2. Male Age Groups, June 1976-Dec 2009

Age	20-24		25-34		35-44		45-54		55-64		65+	
						A. Aggregate Statistics						
u	10.6		5.9		4.4		4.0		3.9		3.5	
D	14.2		17.1		19.8		22.6		24.0		22.5	
						B. Transition Probabilities						
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
σ^1	0.063	0.04	0.026	0.10	0.017	0.08	0.013	0.06	0.014	0.05	0.010	0.07
σ^2	0.009	0.17	0.005	0.18	0.003	0.33	0.004	0.37	0.004	0.30	0.006	0.09
λ^1	0.621	0.31	0.580	0.23	0.565	0.17	0.588	0.04	0.612	0.04	0.717	0.07
λ^2	0.168	0.33	0.150	0.34	0.130	0.26	0.132	0.34	0.126	0.40	0.257	0.63
γ^1	0.005	-0.02	0.004	0.00	0.010	0.00	0.005	0.01	0.021	0.06	0.005	0.00
Res		0.17		0.15		0.16		0.18		0.15		0.14

Note: For each age group Part A displays the average unemployment rate in percent, u, and the average mean duration of unemployment in weeks, D; Part B displays the properties of transition rates for the model with unobserved heterogeneity and two types, where column (1) is the sample average for the monthly transition probability and column (2) is the average contribution of the transition rate to the unemployment rate volatility.

Table 3. Industry Groups, January 2001-December 2009

	MIN		CON		DUR		NDR		WRT		TRU	
	A. Aggregate Statistics											
u	4.9		7.4		5.7		5.7		5.4		4.0	
D	17.0		15.1		20.2		20.4		17.4		18.4	
	B. Transition Probabilities											
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
σ^1	0.04	-0.03	0.04	-0.01	0.03	0.07	0.03	-0.01	0.03	-0.07	0.02	0.01
σ^2	0.01	-0.08	0.01	0.40	0.01	0.53	0.01	0.41	0.01	0.08	0.01	0.36
λ^1	0.78	0.36	0.70	0.05	0.73	0.00	0.74	0.04	0.67	0.28	0.77	0.24
λ^2	0.23	0.50	0.17	0.23	0.15	0.29	0.15	0.34	0.16	0.74	0.16	0.27
γ^1	0.00	0.01	0.00	0.00	0.00	0.00	0.02	0.07	0.00	0.00	0.01	-0.01
Res		0.24		0.33		0.11		0.15		-0.03		0.12

	IT		FAC		PBS		EHS		LHO		PAD	
	A. Aggregate Statistics											
u	5.0		3.2		5.9		2.8		7.8		2.1	
D	21.0		19.0		17.9		16.9		15.1		20.3	
	B. Transition Probabilities											
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
σ_1	0.03	-0.05	0.01	-0.11	0.03	-0.05	0.02	-0.06	0.05	-0.14	0.01	0.00
σ_2	0.01	0.38	0.00	0.21	0.01	0.25	0.00	0.21	0.01	0.69	0.00	0.10
λ_1	0.79	0.21	0.66	0.27	0.68	0.25	0.67	0.23	0.71	0.06	0.71	0.20
λ_2	0.15	0.34	0.15	0.62	0.16	0.49	0.15	0.59	0.17	0.41	0.15	0.42
γ_1	0.04	-0.01	0.01	0.01	0.00	0.00	0.00	-0.01	0.00	-0.01	0.01	0.05
Res		0.13		0.01		0.05		0.04		0.00		0.13

Note: Industries are Mining (MIN), Construction (CON), Durable Goods Manufacturing (DUR), Nondurable Goods Manufacturing (NDR), Wholesale and Retail Trade (WRT), Transportation and Utilities (TRU), Information (IT), Financial Activities(FAC), Professional and Business Services (PBS), Educational and Health Services (EHS), Leisure and Hospitality (LHO), Public Administration (PAD). Parts A and B are as defined in Table 2.

Table 4. Occupation Groups, January 2001 to December 2009

	MBFO		PR		SVC		S		OADM	
	A. Aggregate Statistics									
u	2.6		2.7		6.6		10.7		4.9	
D	20.6		18.4		16.3		17.5		18.3	
	B. Transition Probabilities									
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
σ^1	0.01	-0.03	0.01	0.00	0.04	-0.12	0.05	-0.09	0.02	-0.07
σ^2	0.00	0.34	0.00	0.40	0.01	0.40	0.01	0.20	0.01	0.21
λ^1	0.67	0.11	0.68	0.12	0.69	0.15	0.68	0.13	0.69	0.10
λ^2	0.14	0.50	0.15	0.29	0.16	0.51	0.16	0.77	0.16	0.74
γ^1	0.01	-0.01	0.00	0.00	0.00	-0.02	0.00	0.01	0.00	0.00
Res		0.10		0.18		0.07		-0.02		0.02

	FFF		CE		IMR		PROD		TMM	
	A. Aggregate Statistics									
u	11.4		9.2		4.4		7.4		7.2	
D	14.1		15.3		19.0		19.3		17.4	
	B. Transition Rates									
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
σ_1	0.09	0.00	0.06	0.01	0.03	-0.10	0.03	0.02	0.04	-0.08
σ_2	0.02	0.18	0.01	0.41	0.01	0.17	0.01	0.52	0.01	0.34
λ_1	0.76	0.13	0.71	0.06	0.77	0.15	0.72	-0.01	0.69	0.20
λ_2	0.23	0.10	0.16	0.21	0.16	0.52	0.14	0.35	0.15	0.34
γ_1	0.00	0.00	0.00	0.00	0.02	0.09	0.00	0.01	0.01	0.02
Res		0.58		0.31		0.16		0.11		0.19

Note: Occupations are Management, Business and Finance Operations (MBFO), Professional and related occupations (PR), Services (SVC), Sales and related (S), Office and administrative support (OADM), Farming, Forestry, and Fisheries (FFF): Construction and Extraction (CE), Installation, Maintenance, and Repair (IMR), Production (PROD), Transportation and Material Moving (TMM). Parts A and B are as defined in Table 2.

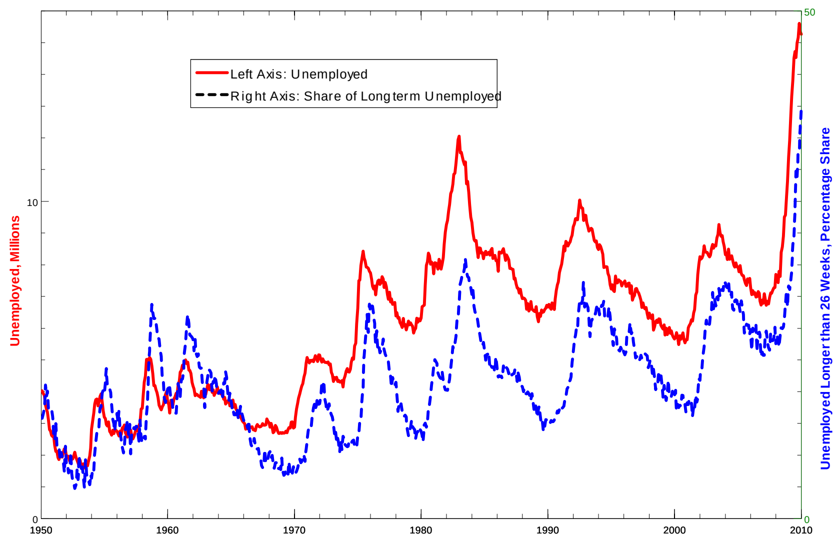

Figure 1. Long-Term Unemployment

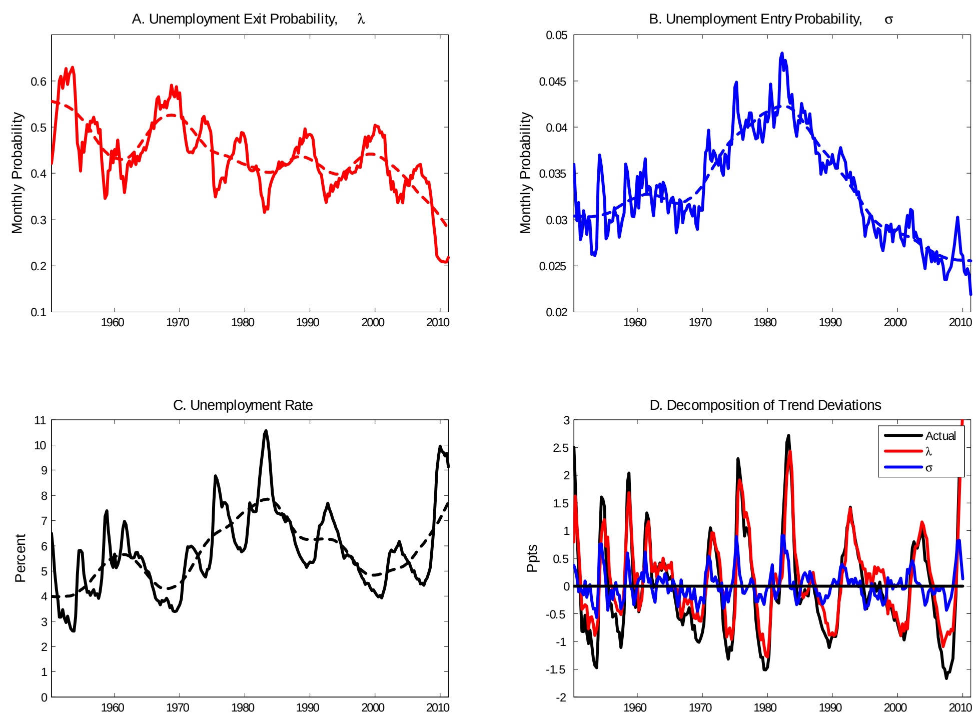

Figure 2. Homogeneous Unemployment

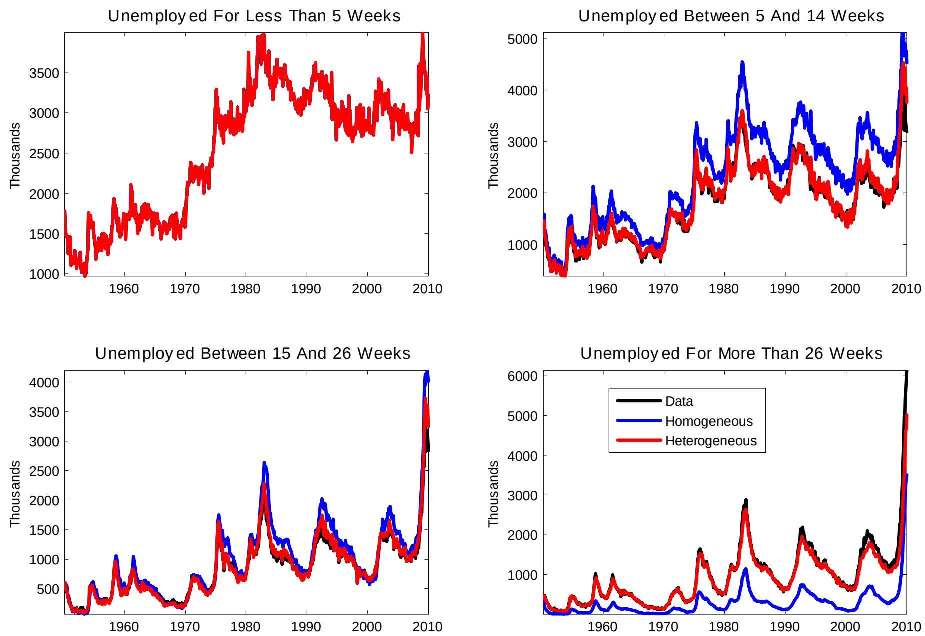

Figure 3. Duration Distribution of Unemployment

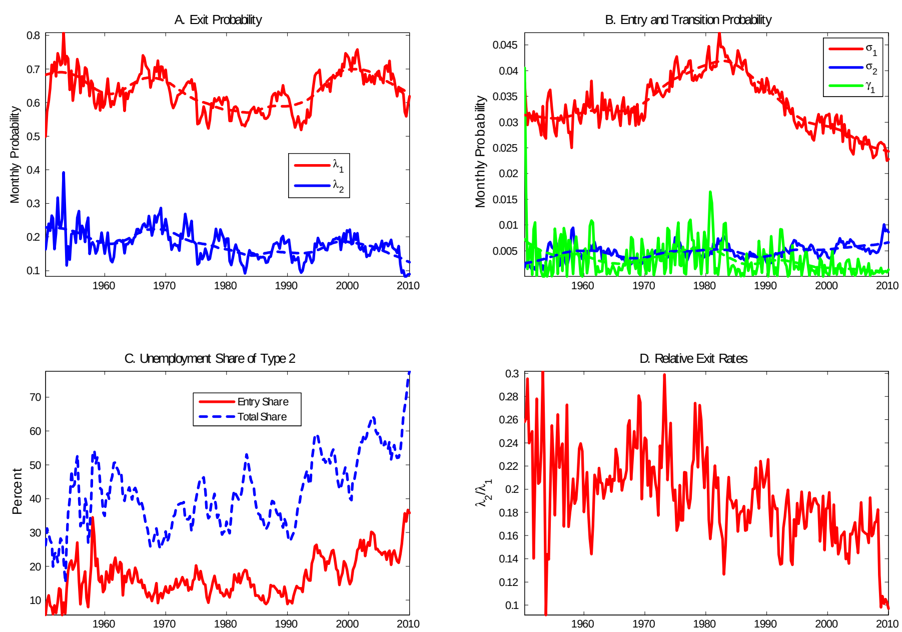

Figure 4. Heterogeneous Unemployment

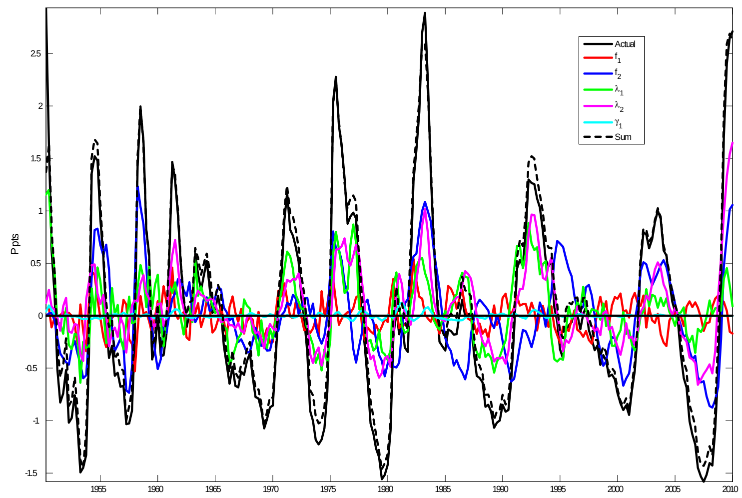

Figure 5. Contributions to Unemployment

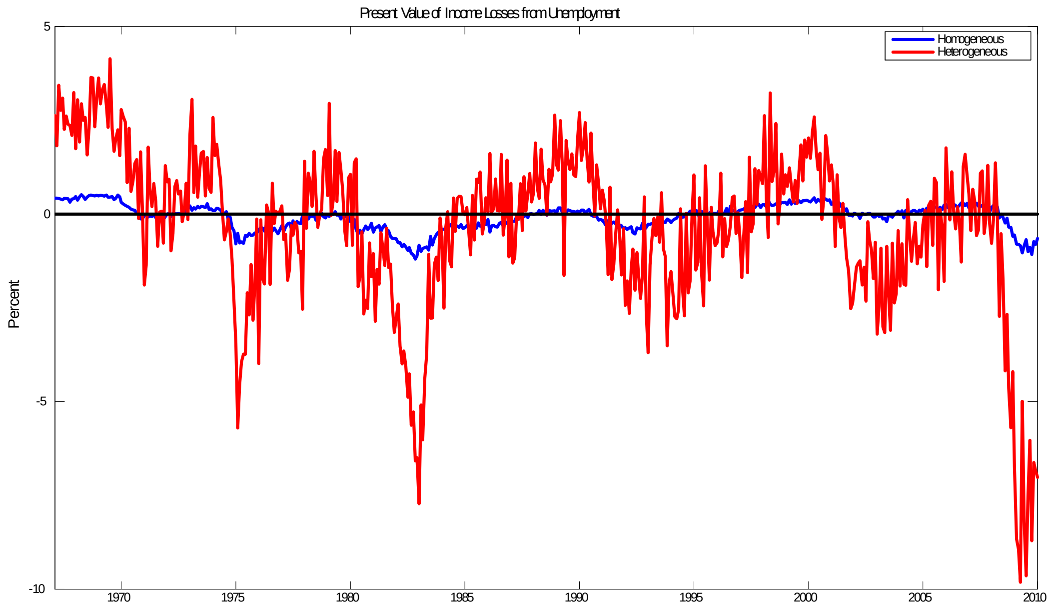

Figure 7. Capital Value of Employment