

# Buying the Right to Harm: The Economics of Buyouts

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## Abstract

Many injurers now offer to buy out victims' property to reduce harm. This paper examines the efficiency of buyouts negotiated in the shadow of regulation and liability rules which require injurers to take efficient precautions. It shows that if the social benefit from precautions is increasing with victims' expected harm, buyouts reduce social welfare. Because buyouts allow injurers to take fewer precautions, a buyout of one victim produces a negative externality for the remaining victims. The injurer can thus exploit victims through a "divide-and-conquer" strategy: making simultaneous, discriminatory take-it-or-leave-it buyout offers. The injurer's profit from buyouts is greater if offers are sequential. Perhaps most surprisingly, buyouts reduce social welfare and victims' joint profits even if victims make simultaneous or sequential take-it-or-leave-it buyout demands to the injurer.

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# 1 Introduction

The *James M. Gavin* plant is a major electricity facility in Ohio surrounded by residential neighborhoods. In 2002, following allegations by neighboring residents that the plant had failed to properly prevent the emission of toxic particles, the plant's owner—American Electric Power Company (AEP)—offered to buy out nearby residents' properties. AEP's offer, however, was restricted to residents who lived on the northern side of the plant. Other residents, although similarly, or even more severely, exposed to the alleged pollution, were not offered to sell their properties. Consequently, the dispute between the remaining residents and AEP was taken to court (Hunt, 2004).

Although a recent phenomenon, buyout programs have become increasingly widespread. Oil refineries, utility companies, mining companies and airports, among others, have publicly announced "voluntary property purchase" programs. Under these programs, victims who have suffered, or are likely to suffer, harm from the offeror's activity are invited to sell their properties. Recent reports show that large-scale injurers spend millions of dollars annually in buying out properties of potential victims (Schneider, 1990, Capiello & Feldstein, 2005).

Commentators and policymakers have highlighted the efficiency advantage of buyout programs (Lerner, 2005). First, because they set apart injurers and their victims, buyouts reduce the risk of litigation. If litigation costs exceed negotiation costs, buyouts' net effect increases social welfare. Second, buyout programs facilitate a more efficient allocation of resources. Because victims are free to accept or reject the injurer's offer, a buyout must result in a Pareto improvement. Acknowledging these benefits, federal and state agencies have stimulated buyout programs by offering them public funding as well as political and administrative assistance.<sup>1</sup>

Against this background, this paper argues that this favorable view of buyouts fails to consider their interplay with injurers' incentives to prevent harm. Rules and regulations governing harmful activities often require injurers to take efficient precautions. For example, the Environmental Protection Agency sets regulatory standards which oblige polluters to invest in pollution-preventing technologies so long as the resulting benefit is greater than the required investment.<sup>2</sup> Similarly, courts regularly consider the costs and benefits of precautions in deciding whether polluters should bear liability for the harm they have caused.<sup>3</sup> We show that injurers subject to such cost-benefit standards can increase their profits by strategically bargaining with victims. By offering to buy out

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<sup>1</sup>A combination of payments from nearby polluting factories and federal and state funding, for example, supported the buyout of nearly 100 homes in Wagner's Point—a neighborhood located in the vicinity of Baltimore's petrochemical industry (Mathews, 1998). Federal and state grants also support local airports' buyout programs (Janofsky, 1999).

<sup>2</sup>The Supreme Court has recently upheld the EPA's reliance on cost-benefit analysis in *Entergy Corp. v. Riverkeeper, Inc* (2009). According to the court's ruling, unless explicitly stated otherwise by Congress, the EPA can exempt polluters from investing in precautions if the burden on the polluter exceeds the environmental benefit.

<sup>3</sup>Negligence and nuisance are the two major causes of action against polluters. In negligence, courts apply the "Hand formula"—a cost-benefit rule (Henderson et al., 1999; p. 179). While the standard in nuisance is murkier, liability for lawful activities often depends on the magnitude of victims' harm (Dobbs, 2000; p. 1321). In particular, under the "locality rule" the level of precautions required to avoid liability for nuisance is higher in more densely populated areas.

victims, injurers can engage in harmful activities without taking efficient precautions.<sup>4</sup>

Our analysis examines the consequences of buyouts in cases in which the marginal social gain from precautions increases with victims' expected harm. It thus applies to a wide range of safety devices such as smokestacks, acoustic walls and dust collectors, as well as safety-enhancing techniques such as the use of non-toxic materials and the employment of highly-qualified workers. To illustrate the nature of these precautions, consider a polluting factory which can elevate its smokestack to reduce the probability of harm for nearby residents. The social benefit from elevating the smokestack depends on residents' harm from the factory's pollution: the greater the harm, the greater the benefit from a higher smokestack. The cost of elevating the smokestack, in contrast, is independent of the residents' harm. Thus, as victims' aggregate harm increases, the socially-optimal precaution costs increase as well.

This paper's main argument is that in cases involving such precautions, buyouts result in social loss. Because buyouts reduce victim's expected harm, they allow injurers—subject to a cost-benefit standard—to take fewer precautions. An injurer can thus profit from buying out one victim if his saving in precaution costs exceeds his buyout payments. However, because the injurer takes fewer precautions, the probability of harm for the remaining victims increases. The resulting increase in the expected harm for the remaining victims must be greater than the injurer's profit from the buyout, or otherwise it would be optimal to require the injurer to take fewer precautions in the first place. Consequently, if victims cannot make side payments to each other, injurers could use buyouts to avoid taking efficient precautions.

To demonstrate this argument, consider a simple example involving one injurer and three victims. Suppose that each victim gains 2.5 from his activity. Suppose further that if the injurer does not take precautions, victims lose their entire gain. The injurer can spend 2 or 5 (low or high) on precautions, thereby preventing harm to the victims with probability .5 or 1, respectively. Since the additional benefit from a high level of precaution ( $3.75 = .5 \times 7.5$ ) is greater than the additional cost ( $3 = 5 - 2$ ), the injurer should take high precautions.

Suppose the injurer can only negotiate with one victim.<sup>5</sup> Rather than spending 5 on precaution, the injurer can make a take-it-or-leave buyout offer of 2.5 (or slightly more) to one of the victims. As this offer equals the victim's gain, he will accept it. In the presence of only two victims, the additional benefit from taking a high level of precaution ( $2.5 = .5 \times 5$ ) is now lower than the additional cost (3). Under a cost-benefit standard, therefore, the injurer is no longer required to take a high level of precaution; instead, he is required to take a low level. The injurer's profit thus increases by 0.5 (because he buys out one victim at 2.5 and saves 3 in precaution costs), but the remaining victims' expected profit decreases by 2.5 (because each remaining victim's expected profit decreases from 2.5 to 1.25). Social welfare accordingly decreases by 2.

While this example illustrates injurers' incentives—as in the *Gavin* plant case—to

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<sup>4</sup>According to the Restatement (Second) of Torts, § 871, cmt.d., “sound public policy demands that the land in each locality be used for purposes suited to the character of that locality” and that “the suitability of the particular use or enjoyment invaded must be determined as of the time of the invasion rather than the time when the use or enjoyment began.”

<sup>5</sup>We later consider the case in which the injurer can negotiate with all victims.

buy out victims, they may not always find it profitable to do so. Whether a buyout produces gains from trade depends, as we later show, on the efficiency of the injurer's precautions and on victims' aggregate as well as individual harm. In particular, for a buyout to take place, the injurer's precaution costs must not be too high (in a sense made precise later) and the magnitude of the selling victim's individual harm must be sufficiently small relative to victims' aggregate harm.

We further show that injurers' incentives to buy out victims, and the resulting loss of social welfare, increase when injurers can negotiate with many victims. We begin by examining the case in which the injurer makes simultaneous take-it-or-leave-it offers to victims. Our solution concept is subgame-perfect equilibrium in coalition-proof strategies (see Bernheim et al., 1987). The coalition-proof refinement ensures that, when making their acceptance decisions, any subset of victims can coordinate on their most-preferred Pareto outcome by entering into self-enforcing agreements. The only restriction imposed on victims' ability to act collectively is that they may not make side payments among themselves. We show that the unique equilibrium involves a divide-and-conquer strategy in which the injurer extends high buyout offers to some victims and low buyout offers to others.

We then explore the case in which the injurer makes a sequence of take-it-or-leave-it offers to victims. Each victim decides whether to accept or reject the injurer's offer after having observed the acceptance decisions of all previous victims. We show that, relative to the simultaneous-offers case, injurers who can approach victims sequentially have greater incentives to exploit buyouts. Sequential negotiations enable the injurer to credibly threaten to buy out subsequent victims if a victim rejects his offer. Each victim is thus induced to accept the injurer's offer out of fear of ending up with a lower profit should he decline the offer. With sequential offers, therefore, injurers may buy out victims at even lower costs.

The conclusion that buyouts reduce social welfare does not hinge on the injurer's ability to make take-it-or-leave-it offers to victims. Interestingly, the loss of social welfare from buyouts is independent of the distribution of bargaining power between the injurer and the victims. Even when victims possess the entire bargaining power, buyouts result in inefficiency as injurers still take too few precautions.

We examine both the cases of victims making simultaneous and sequential demands. Under simultaneous bargaining, victims make take-it-or-leave-it buyout demands to the injurer without observing other victims' demands. Under sequential bargaining, each victim makes a take-it-or-leave-it buyout demand after having observed the injurer's decisions to accept or reject the previous demands. We show that the coalition-proof equilibrium of the simultaneous-negotiations game involves a race-to-the bottom: In the unique equilibrium all victims make the same low buyout demands, and the injurer accepts all demands except one. Under sequential bargaining, in contrast, the equilibrium outcome involves exploitation among victims: Some victims make higher demands, and collect more, than other victims. Although the distribution of payoffs between victims and the injurer, as well as among victims themselves, differs for simultaneous and sequential demands, both forms of negotiations nevertheless result in an equal loss of social welfare.

We extend the analysis by relaxing the assumption that only the injurer can take

precautions. Instead we assume that victims as well can take precautions so that their precautions are substitutes to the injurer's in the prevention of harm. As opposed to the injurer's precautions which reduce the probability of harm globally, victims' precautions usually reduce this probability only locally. We accordingly assume that by taking precautions each victim reduces his own risk of harm, but not other victims' risk. Because the injurer's and victims' precautions are substitutes, each victim has to take more precautions as the number of victims, as well as victims' aggregate harm, decreases. As a result, injurers might use buyouts to shift the burden of taking precautions on to the remaining victims. More generally, we show that buyouts may produce greater gains from trade and thereby cause a greater loss in social welfare when victims as well can take precautions.

■ This paper is inspired by the burgeoning literature on contracting with externalities. This literature, initiated by Segal (1999) and Segal and Whinston (2000), examines the nature and efficiency consequence of strategic contracting between a central player (principal) and a group of affiliated players (agents). The essential feature of this contracting environment is that bilateral trade between the principal and one agent produces externalities for the other agents. Segal's (1999) main insight is that if the principal possesses the entire bargaining power, he can coordinate agents on his most preferred equilibrium by discriminating among them: offering different contracts to different agents.

The notion that a central player can employ a 'divide-and-conquer' strategy to exploit a class of affiliated players was also applied to concrete settings. Segal and Whinston (2000), following Rasmusen et al. (1991), show that an incumbent monopoly can deter entry by offering a sufficient number of customers to sign exclusionary contracts in exchange for a reduced price.<sup>6</sup> Che and Spier (2008) more recently show that a defendant can reduce his overall settlement payments by making different settlement offers to different plaintiffs who share fixed litigation costs. Both papers show, as does this paper, that the central player can further exploit the group players by making sequential offers.

The scope of inefficiency in this paper, however, is different in two respects. Unlike Segal and Whinston's analysis, the result that buyouts lead to inefficiency does not depend on a critical number of victims contracting with the injurer. While Che and Spier's results do not hinge on the defendant's ability to settle with a sufficient number of plaintiffs, they are nevertheless susceptible to the distribution of bargaining power between the parties. As shown by Stremitzer (2010), if plaintiffs can make sequential take-it-or-leave-it settlement offers, the defendant's settlement payment is equal to his payment at trial. Because the defendant's incentive to comply remains the same, social welfare is unaffected by plaintiffs' strategic bargaining. Here, in contrast, the loss of social welfare occurs even if victims make sequential take-or-leave-it buyout offers. Thus, the loss of social welfare from strategic bargaining between the injurer and the victims seems particularly likely.

Previous studies have also suggested that victims who suffer from collective-action problems can be exploited by injurers. Hamilton (1993, 1998) and Brooks and Sethi (1997), using voter turnout as a proxy for individuals' willingness to act collectively, empirically found that polluters tend to locate their activities in communities in which

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<sup>6</sup>More recent papers examined the likelihood of exclusion experimentally (Landeo and Spier, 2009) and when buyers are Bertrand competitors, rather than final consumers (Fumagalli and Motta 2006; Simpson and Wickelgren, 2007).

they are less likely to face collective opposition from local residents. While these studies underscore the importance of collective action among victims, they focus only on victims' failure to mount legal or public opposition to the injurer. This paper shows, however, that injurers may exploit victims even if they are able to take their case to court or to otherwise collectively act to protect their rights.

This paper also contributes to the extensive literature on the efficiency of liability rules. Following Brown (1973), scholars have pointed out the economic advantage of legal rules which impose liability only for harm caused by failures to take cost-effective precautions (as opposed to strict liability rules) (Shavell, 2007; pp. 164-7). As this literature has shown, the negligence rule provides incentives for injurers and victims to take socially-optimal precautions when efficiency requires bilateral harm prevention (precautions by both injurers and victims). Shavell (1987) and Kornhauser and Revesz (1994) later highlighted the advantage of the negligence rule in providing incentives to take socially-optimal precautions when injurers jointly cause harm. This paper shows, however, that in cases involving multiple victims, the negligence rule—and, more generally, cost-benefit standards—is vulnerable to strategic behavior through buyouts.

The paper proceeds as follows. Section 2 illustrates the inefficiency of buyouts using a simple example. Section 3 sets up the model. Section 4 examines the conditions under which the injurer and one victim could gain from a buyout and presents the divergence between the private incentives to enter a buyout agreement and social efficiency. Building on this analysis, Section 5 considers the case in which the injurer can negotiate with multiple victims. It shows that, irrespective of the bargaining structure and the distribution of bargaining power, buyouts reduce social welfare and victims' joint profits. Section 6 extends the analysis to cases in which not only the injurer but also victims can take precautions. Section 7 concludes.

## 2 Numerical Example

In this section, we present a simple numerical example in which an injurer subject to a cost-benefit standard can negotiate with many victims. This example illustrates the equilibrium outcomes under different bargaining structures (sequential versus simultaneous) and distributions of bargaining power.

Consider again the basic example discussed above with one injurer and three identical victims. Recall that each victim's activity yields gains of 2.5, but these gains are lost if the injurer does not take precautions. The injurer can prevent harm to the victims with probability .5 at a cost of 2, and with probability 1 at a cost of 5. Observe that, if the injurer takes socially-optimal precautions, social welfare is 2.5 ( $7.5 - 5$ ). In the presence of two victims, however, the injurer should only take a low level of precautions; social welfare is accordingly .5 ( $2.5 - 2$ ). Finally, in the presence of one victim, the injurer should take no precautions; social welfare is therefore 0.

First, suppose that the injurer makes simultaneous take-it-or-leave-it buyout offers to the victims (for simplicity, assume that each victim accepts the injurer's offer if he is indifferent between accepting and rejecting it). In the unique, coalition-proof, subgame-perfect equilibrium, the injurer makes take-it-or-leave it offers of 2.5, 1.25, and 0 to the

three victims, and the victims accept their respective offers. To see why, note that, given the injurer's offers, the only equilibrium that survives the iterated elimination of dominated strategies is for all the victims to accept their respective offers. The victim who is offered 2.5 obtains his no-buyout profit and therefore accepts the injurer's offer irrespective of the other victims' acceptance decisions. Given that one victim accepts the injurer's offer, the victim who is offered 1.25 is better off accepting than rejecting the injurer's offer irrespective of the other victim's acceptance decision. Finally, given that two victims accept the injurer's offers, the last victim obtains 0 irrespective of whether he accepts or rejects the injurer's offer; he therefore accepts it. This equilibrium is coalition-proof because, in any coalition of victims who deviate from their equilibrium strategies, there is at least one member who is better off accepting than rejecting the injurer's offer. Relative to the case in which the injurer does not buy out any victim, the injurer's equilibrium profit increases by 1.25 (he pays victims 3.75 and saves 5 on precautions), one victim's expected profit decreases by 1.25, and another victim's expected profit decreases by 2.5 (the profit of the victim who is bought out for 2.5 remains unchanged).<sup>7</sup>

Now assume that the injurer approaches the victims sequentially. Suppose the injurer first approaches victim 1, then victim 2, and finally victim 3. In the unique subgame-perfect equilibrium, the injurer offers each of the victims 0, and each victim accepts the injurer's offer. This equilibrium outcome is supported by the injurer's threat to buy out victim 2 for 1.25 if victim 1 declines the injurer's offer. This threat, in turn, is supported by the injurer's threat to buy out victim 3 for 2.5 if both victim 1 and victim 2 refuse the injurer's offers. Given that victim 1 accepts the injurer's offer, victim 2's equilibrium-strategy is similarly supported by the injurer's threat to buy out victim 3 for 1.25.

Next, consider the case in which the victims possess the entire bargaining power (again, for simplicity, assume that the injurer would rather accept than reject a victim's demand if he is indifferent between accepting and rejecting it). When the victims make simultaneous buyout demands, the equilibrium outcome involves a race-to-the-bottom. To see why, observe that there does not exist a no-buyout equilibrium, because one of the victims could profitably deviate by making the injurer a buyout demand of 3—the injurer's additional precaution costs as the number of victims increases from 2 to 3. Moreover, an equilibrium in which only one victim is bought out also does not exist, because one of the remaining victims could then profitably deviate by making the injurer a buyout demand of 2—the injurer's additional precaution costs as the number of victims increases from 1 to 2. In any equilibrium of victims' simultaneous-demand game, therefore, exactly two victims are bought out. But then all the victims must make buyout demands of 0, or otherwise the victim whose offer is declined could profitably deviate by making the injurer a lower demand than the highest demand that the injurer accepts.<sup>8</sup>

Last, when the victims approach the injurer sequentially, earlier victims exploit later victims. To help understand this equilibrium outcome, suppose victim 1 approaches

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<sup>7</sup>It is straightforward to show that the injurer cannot profitably deviate from these buyout offers without violating the coalition proof refinement.

<sup>8</sup>If all victims made the injurer the same buyout demand  $x$ , where  $0 < x \leq 2$ , the injurer would randomly accept one demand. But then there exists  $\epsilon$  sufficiently small such that any one victim could profitably deviate by making the injurer a demand of  $x - \epsilon$ , which the injurer accepts with probability 1. (Because the injurer accepts at least two buyout demands, all buyout demands must be smaller than 2; for otherwise the injurer could profitably deviate by rejecting a demand strictly greater than 2.)

the injurer first, then victim 2, and finally victim 3. In the unique subgame-perfect equilibrium, victim 1 and victim 2 make take-it-or-leave-it buyout demands of 3 and 2, respectively, victim 3 makes a buyout demand of 0, and the injurer accepts all demands. This equilibrium outcome is supported by victim 2's off-equilibrium buyout demand of 3 if the injurer declines victim 1's demand, which is supported, in turn, by victim 3's off-equilibrium buyout demand of 3 if the injurer declines both victim 1's and victim 2's buyout demands. Given that the injurer accepts victim 1's demand, the injurer's equilibrium strategy to accept victim 2's demand is similarly supported by victim 3's off-equilibrium demand of 2.

This example illustrates that, irrespective of the bargaining structure and the parties' relative bargaining power, trade between the injurer and the victims is socially inefficient. Thus, because under all combinations of bargaining protocols and distribution of bargaining power the injurer does not take precautions, social welfare is reduced (compared to the first best) by 2.5. The parties' equilibrium profits, however, vary in each case. For the injurer, buyouts are most profitable either if he possesses the entire bargaining power and makes sequential offers, or if the victims possess the entire bargaining power and make simultaneous demands. The injurer's profit is intermediate if he possesses the entire bargaining power and makes simultaneous offers, and lowest if the victims possess the entire bargaining power and make sequential demands. The reverse order, naturally, applies to the victims' joint profits. Finally, for each victim buyouts can result in a profit higher than, equal to, or lower than his profit in the no-buyout case.

### 3 Model

Consider one injurer and  $n \geq 2$  potential victims ( "victims"). All players are risk-neutral. Each victim engages in a socially-valuable activity in his property, which yields him a private benefit. Let  $h$  be victims' aggregate benefits and  $\Delta h \equiv h/n$  be each victim's private benefit. The injurer engages in a socially-valuable activity, which interferes with victims' benefits from their activities. For simplicity, assume that the injurer's activity fully destroys victims' benefits. Thus, victims' aggregate *harm* from the injurer's activity is  $h$ . We use the term 'harm' for victims' benefits because these benefits are lost if the injurer does not take precautions. We further assume that the injurer attaches no value to victims' properties.

The injurer can incur costs to increase the probability that victims do *not* suffer harm; we call this probability the 'probability of no-harm.' For example, the injurer may scale back his activity or take precautions to render it safer. We assume that, to prevent victims' harm with probability  $p \in [0, 1]$ , the injurer must spend  $c(p)$ .<sup>9</sup> We also assume that  $c(p)$  is thrice continuously differentiable and that  $c(0) = 0$ ,  $c'(0) = 0$ ,  $c'(p) > 0$ , and  $c''(p) > 0$ . The assumptions on the first- and second derivative of the injurer's precaution-cost function imply that the marginal cost of increasing the probability of no-harm is strictly increasing. The assumption that  $c(p)$  is differentiable at  $p = 0$  implies that increasing the probability of no-harm involves no fixed cost.

We further assume that the injurer's cost of increasing the probability of no-harm

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<sup>9</sup>Alternatively, to prevent a fraction  $p$  of victims' harm, the injurer must spend  $c(p)$ .



is not affected by victims' aggregate harm. This assumption implies that the marginal social net benefit from increasing the probability of no-harm is increasing with victims' expected harm.<sup>10</sup> As mentioned in the Introduction, this is a natural assumption for many harm-preventing measures and technologies. For example, the injurer's cost of raising a plant smokestack is not affected by victims' harm. Rather, this cost is a function of the probability of no-harm. The higher this probability is, the greater the corresponding precaution costs will be (i.e., a higher smokestack).

Social welfare is equal to victims' expected profits—the probability that they do not suffer harm times their aggregate benefits from their activities—less the injurer's precaution costs:

$$ph - c(p). \quad (1)$$

The first-order condition for maximum is:

$$c'(p) = h. \quad (2)$$

(2) implicitly defines the socially-optimal probability of no-harm as a function of victims' aggregate harm.<sup>11</sup> We accordingly let  $P(h) : [0, \infty) \rightarrow [0, 1]$  map victims' aggregate harm to the socially-optimal probability of no-harm (as we show momentarily,  $P(h)$  is strictly increasing).

Under a cost-benefit standard, the injurer is required to take precautions so long as the marginal cost of his precautions is lower than the marginal benefit. In the regulatory context, an injurer must comply with the standard to be allowed to engage in his activity. In the context of liability rules, an injurer must comply with the standard to avoid liability for victims' harm. We can use  $P(h)$  to define the required level of precautions under a cost-benefit standard.

**Definition 1** *Under a cost-benefit standard, the injurer is required to spend  $c(P(h))$  on precautions to engage in the regulated activity or to avoid liability for victims' harm.*

In the context of regulation, because the cost-benefit standard serves as a prerequisite to carrying out the regulated activity, injurers will take efficient precautions. To see the effect of a cost-benefit standard on the injurer's incentive to take precautions under threat of liability, note that the injurer never chooses  $p > P(h)$ , because he can increase his profit by spending less on precautions. The injurer's optimal choice of  $p$  thus solves  $\max_p [ph - c(p)]$ , which implies that  $c'(p) = h$ . The injurer therefore maximizes his profit by choosing to prevent harm with the socially-optimal probability,  $P(h)$ .

Now, implicit differentiating of (2) gives

$$P'(h) = \frac{1}{c''(P(h))} > 0. \quad (3)$$

(3) implies that the socially-optimal probability of no-harm is strictly increasing with victims' expected harm. Because  $P(h)$  is strictly increasing, it admits an inverse function.

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<sup>10</sup>Note that social welfare is  $ph - c(p)$  (see (2)) and that, because  $\frac{\partial c(p)}{\partial h} = 0$ ,  $\frac{\partial (ph - c(p))}{\partial p \partial h} = 1$ .

<sup>11</sup>The assumption that  $c''(p) > 0$  ensures that the value of  $p$  that solves (2) maximizes social welfare.

Let  $H(p) : [0, 1] \rightarrow [0, \infty)$  map the socially-optimal probability of no-harm to victims' aggregate harm. Note that, because the socially-optimal probability of no-harm satisfies  $c'(p) = h$  (see (2)),  $H(p)$  maps the socially-optimal probability of no-harm to the injurer's marginal cost of precautions. Lemma 1 considers the relations between  $P(h)$  and  $H(p)$  and provides further insight into their economic interpretation.

**Lemma 1** *Let  $P(h) : [0, \infty) \rightarrow [0, 1]$  map victims' aggregate harm to the socially-optimal probability of no-harm and let  $H(p) : [0, 1] \rightarrow [0, \infty)$  be the inverse function of  $P(h)$ . Then:*

- (a) *The socially-optimal precaution costs are  $\int_0^{P(h)} H(x)dx$ .*
- (b) *The maximum social welfare is  $\int_0^h P(x)dx = P(h)h - \int_0^{P(h)} H(x)dx$ .*
- (c) *If victims' aggregate benefit from their activities (i.e., their harm in the absence of precautions) decreases from  $h_2$  to  $h_1$  ( $h_2 > h_1 \geq 0$ ), then the maximum social welfare decreases by  $\int_{h_1}^{h_2} P(x)dx = [P(h_2)h_2 - P(h_1)h_1] - \int_{P(h_1)}^{P(h_2)} H(x)dx$ .*

PROOF. Here we prove parts (a) and (b). We relegate to the appendix the proof of part (c). Consider part (a). The injurer's socially-optimal precaution costs given that victims' harm is  $h$  can be obtained by integrating the injurer's marginal precaution-cost function from 0 to  $P(h)$ :  $\int_0^{P(h)} c'(x)dx$ . Because, from (2), for all  $p \in [0, P(h)]$ ,  $c'(p) = H(p)$ , we can substitute  $H(x)$  for  $c'(x)$  to get  $\int_0^{P(h)} H(x)dx$ .

Now, the maximum social welfare is  $P(h)h - c(P(h))$ . By the Envelope Theorem, the derivative of the maximum social welfare with respect to  $h$  is  $P(h)$ . Social welfare can thus be obtained by integrating  $P(x)$  with respect to  $x$  from 0 to  $h$ :  $\int_0^h P(x)dx$ . Now, because  $P(h)h$  is victims' expected profit given the socially-optimal probability of no-harm and  $\int_0^{P(h)} H(x)dx$  is, from part (a), the socially-optimal precaution costs, the maximum social welfare is also equal to  $P(h)h - \int_0^{P(h)} H(x)dx$ . This completes the proof of part (b).<sup>12</sup> ■

Figure 1 presents the relation between  $P(h)$  and  $H(p)$  graphically. Because  $P(h)$  and  $H(p)$  are inverse functions,  $H(p)$  can be obtained by reflecting the graph of  $P(h)$  across the line from the origin to the point  $(h, P(h))$ . Victims' expected profits are equal to  $P(h)h$ , the area bounded by the rectangle. The injurer's socially-optimal precaution costs is the area bounded by the rectangle and  $P(h)$  (and is also equal to the area bounded by  $H(p)$  and the  $p$  axis after switching the positions of the axes). The maximum social welfare is equal to the difference between victims' expected profits and the injurer's socially-optimal precaution costs; it is thus equal to the area bounded by  $P(h)$  and the  $h$  axis.

## 4 Negotiation with One Victim

Suppose the injurer may negotiate with only one victim. If the injurer buys out one victim, victims' aggregate harm is reduced by  $\Delta h$ . The socially-optimal probability of

<sup>12</sup>In the Appendix we provide a direct proof of Lemma 1(b), which utilizes the fact the  $P(h)$  and  $H(p)$  are inverse functions.

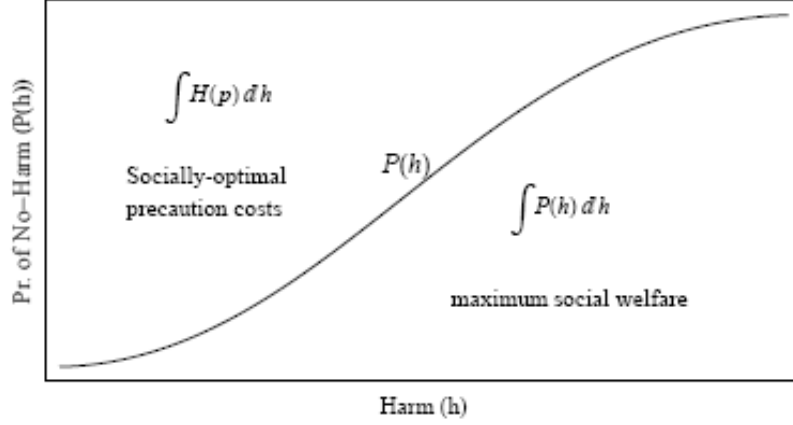


Figure 1: Maximum social welfare and socially-optimal precaution costs

no-harm accordingly decreases to  $P(h - \Delta h)$ . The injurer's *maximum* buyout payment is therefore equal to his saving in precaution costs from the buyout:  $c(P(h)) - c(P(h - \Delta h))$ . By Lemma 1(a), this expression is equal to  $\int_0^{P(h)} H(x) dx - \int_0^{P(h - \Delta h)} H(x) dx$ , which can be written as  $\int_{P(h - \Delta h)}^{P(h)} H(x) dx$ .

If the injurer does not buy out any victim, he prevents harm with probability  $P(h)$ . A victim's *minimum* buyout amount is therefore his expected profit given the socially-optimal probability of no-harm:  $P(h)\Delta h$ . Proposition 1 uses these expressions to state a necessary and sufficient condition for a buyout to produce gains from trade.

**Proposition 1** *There are gains from trade between the injurer and one victim iff the injurer's saving in precaution costs following the buyout is (weakly) greater than the victim's expected profit before the buyout:  $\int_{P(h - \Delta h)}^{P(h)} H(x) dx \geq P(h)\Delta h$ .<sup>13</sup>*

Figure 2 shows the injurer's maximum buyout payment and a victim's minimum buyout amount. The injurer's maximum buyout payment—i.e., his saving in precautions costs from buying out one victim—is the area  $A + B$  ( $\int_{P(h - \Delta h)}^{P(h)} H(x) dx$ ). A victim's minimum buyout amount—his expected profit given the socially-optimal precautions—is the area  $B + C$  ( $P(h)\Delta h$ ). There are gains from trade from a buyout iff  $B + C \geq A + B$ .

To see the different effects of a buyout on social welfare, suppose the injurer buys out one victim so that the number of victims decreases from  $n$  to  $n - 1$ . Let the bought-out victim be victim  $n$ . The buyout of victim  $n$  has three effects on social welfare, which can be illustrated by the different areas in Figure 2. First, because the injurer is required to take fewer precautions, the injurer's expected profit increases (area  $A + B$ ). Second, because the injurer is required to take fewer precautions, the probability of harm is higher and therefore the expected profits of the other victims decreases (area  $A$ ). Finally, victim  $n$  no longer derives benefit from his activity (area  $B + C$ ). The net decrease in social welfare (area  $C$ ) is equal to the sum of these three effects ( $-C = (A + B) - A - (B + C)$ ).

<sup>13</sup>To simplify the exposition, we assume in what follows that both the injurer and the victims prefer to enter a buyout agreement if their no-buyout profit is identical to their buyout profit.

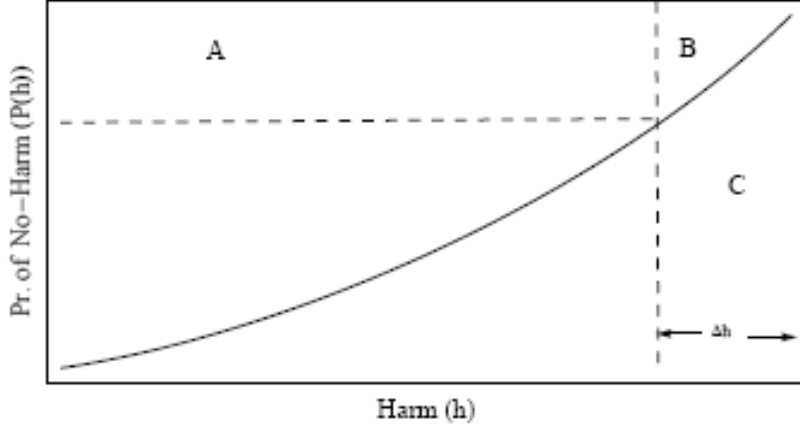


Figure 2: Gains to Trade from a Buyout

Now, recall that a buyout produces gains from trade iff  $B + C \geq A + B$ . Subtracting  $B$  from both sides implies that there are gains from trade iff  $A \geq C$ . Note that  $A$  is the decrease in the joint profits of  $n - 1$  victims following a buyout of one victim, and  $C$  is the corresponding decrease in social welfare. This in turn implies that a buyout produces gains from trade iff the external effect of a buyout of one victim on the other victims' joint profits is (weakly) greater than the corresponding decrease in social welfare:

$$(h - \Delta h)[P(h) - P(h - \Delta h)] \geq \int_{h-\Delta h}^h P(x)dx. \quad (4)$$

The left-hand side is the decrease in the joint profits of  $n - 1$  victims from decreasing the probability of no-harm from  $P(h)$  to  $P(h - \Delta h)$ . It is equal to the aggregate harm of  $n - 1$  victims,  $h - \Delta h$ , times the difference between the socially-optimal probability of no-harm when there are  $n$  and  $n - 1$  victims. The right-hand side is, by Lemma 1(c), the decrease in social welfare from decreasing the number of victims from  $n - 1$  to  $n$ .<sup>14</sup>

We can now exploit (4), as we show in the Appendix, to derive a necessary and sufficient condition for a buyout to produce gains from trade in the extreme case in which each victim's harm approaches zero, but victims' aggregate harm remains fixed. This in turn provides a more general insight into the condition under which a buyout produces gains from trade.

**Proposition 2(a)** *As each victim's harm approaches zero (i.e.,  $\Delta h \rightarrow 0$ ), there are gains from trade between the injurer and one victim iff the point elasticity of the optimal probability of no-harm with respect to victims' aggregate harm is greater than 1:  $P'(h) \frac{h}{P(h)} \geq 1$ .*

PROOF. See the Appendix.

<sup>14</sup>To see it formally, substitute  $P(h)h - P(h - \Delta h)(h - \Delta h) - \int_{h-\Delta h}^h P(x)dx$  for  $\int_{P(h-\Delta h)}^{P(h)} H(x)dx$  (by Lemma 1(c)) in the inequality given in Proposition 1 to get  $P(h)h - P(h - \Delta h)(h - \Delta h) - \int_{h-\Delta h}^h P(x)dx \geq P(h)\Delta h$ . Rearranging gives the inequality in (4).

Proposition 2(a) implies that there are gains from trade between the injurer and one victim as each victim's harm becomes infinitesimally small iff increasing victims' aggregate harm (i.e., their lost benefits) by one percent increases the socially-optimal probability of no-harm by more than one percent. Because  $P(h)$  is the derivative of the maximum social welfare with respect to victims' aggregate harm—as shown in the proof of Lemma 1(b)—the condition in Proposition 2(a) means that for a buyout to produce gains from trade, the percentage increase in the rate of increase in the maximum social welfare must be higher than the corresponding percentage increase in victims' aggregate harm.

To gain intuition into this condition, suppose that victims' aggregate harm is  $h$  and that the corresponding socially-optimal probability of no-harm is  $p$ . Because each victim's harm is  $\Delta h$ , a victim's minimum buyout amount is  $p\Delta h$ . Suppose further that a decrease of  $\Delta h$  in victims' aggregate harm reduces the socially-optimal probability of no-harm by  $\Delta p$ . Then, the corresponding decrease in the injurer's socially-optimal precaution costs,  $\Delta c$ , is roughly  $\Delta ph$  (because, from (2),  $\frac{\Delta c}{\Delta p} \approx h$ ). A buyout produces gains from trade, therefore, iff  $\Delta ph \gtrsim p\Delta h$ , which implies that  $\frac{\Delta p}{p} \gtrsim \frac{\Delta h}{h}$ .

We can restate the condition in Proposition 2(a) by substituting  $\frac{1}{c'(\hat{p})}$  for  $P'(h)$  and  $c'(\hat{p})$  for  $h$  in  $P'(h)\frac{h}{P(h)}$ , where  $\hat{p} = P(h)$ . This provides a necessary and sufficient condition for there to be gains from trade between the injurer and one victim, which is directly related to the efficiency of the injurer's precautions.

**Proposition 2(b)** *As each victim's harm approaches zero (i.e.,  $\Delta h \rightarrow 0$ ), there are gains from trade between the injurer and one victim iff the point elasticity of the marginal cost function with respect to the socially-optimal probability of no-harm is smaller than one:  $c''(\hat{p})\frac{\hat{p}}{c'(\hat{p})} \leq 1$ , where  $\hat{p} = P(h)$ .*

Proposition 2(b) implies that there are gains from trade between the injurer and one victim as each victim's harm becomes infinitesimally small iff increasing the socially-optimal probability of no-harm by one percent increases the injurer's *marginal* precaution cost by less than one percent. For a buyout to produce gains from trade, therefore, the injurer's precaution costs must not be too expensive.

To gain intuition into this condition, recall from Proposition 2(a) that for a buyout to produce gains from trade it must be that  $\frac{\Delta p}{p} \gtrsim \frac{\Delta h}{h}$ . But from (2)  $\frac{\Delta c}{\Delta p} \approx h$  and therefore  $\Delta h$  is roughly equal to the change in  $\frac{\Delta c}{\Delta p}$  as victims' aggregate harm increases from  $h - \Delta h$  to  $h$  (call it  $\Delta \frac{\Delta c}{\Delta p}$ ). Substituting  $\frac{\Delta c}{\Delta p}$  for  $h$  and  $\Delta \frac{\Delta c}{\Delta p}$  for  $\Delta h$  implies that a buyout produces gains from trade iff  $\frac{\Delta p}{p} \gtrsim \frac{\Delta \frac{\Delta c}{\Delta p}}{\frac{\Delta c}{\Delta p}}$ .<sup>15</sup>

The following example illustrates the conditions in Propositions 2(a) and 2(b):

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<sup>15</sup>The condition in Proposition 2(b) can be restated in terms of  $p(c)$ , the probability of no-harm as a function of the injurer's precaution costs (the inverse function of  $c(p)$ ). Specifically, the condition in Corollary 1A is satisfied iff the point elasticity of the marginal socially-optimal probability of no-harm with respect to the socially-optimal probability of no-harm is smaller than 1 ( $\frac{dp'(\hat{c})}{dp} \frac{\hat{p}}{p'(\hat{c})} \leq 1$ , where  $\hat{c} = c(\hat{p})$ ). This implies that, as each victim's harm approaches 0, a buyout produces gains from trade iff increasing the socially-optimal probability of no-harm by one percent increases the marginal socially-optimal probability of no-harm by less than one percent.

**Example 1** Suppose the injurer can prevent harm with probability  $\frac{1}{2}$  or 1 by spending 2 or 5, respectively. Then the socially-optimal probability of no-harm is

$$P(h) = \begin{cases} 0 & \text{if } h \in [0, 4) \\ 0.5 & \text{if } h \in [4, 6) \\ 1 & \text{if } h \in [6, \infty) \end{cases}.$$

If  $h = 6$  and  $\Delta h = 2$ , then there are gains from trade between the injurer and one victim.

Here, because  $P(h)$  is not strictly increasing,  $H(p)$  is not well-defined. Instead, let  $\overline{H}(p)$  be a function that maps each optimal probability of no-harm to the *minimum* of victims' aggregate harm. Then  $\overline{H}(0) = 0$ ,  $\overline{H}(0.5) = 4$ , and  $\overline{H}(1) = 6$ . Note that, for  $p > 0$ ,  $\overline{H}(p)$  is equal to the additional precaution costs divided by the additional probability of no-harm:  $\overline{H}(0.5) = \frac{2}{.5} = 4$  and  $\overline{H}(1) = \frac{3}{0.5} = 6$ .

If  $\Delta h = 2$  and the number of victims increases from 2 and 3 (so that victims' aggregate harm increases from 4 to 6), the socially-optimal probability of no-harm increases from 0.5 to 1. The percentage increase in the socially-optimal probability of no-harm (1) is therefore greater than the corresponding percentage increase in victims' aggregate harm (0.5) (or, equivalently, the corresponding percentage increase in the additional precaution costs). Accordingly, there are gains from trade between the injurer and one victim.<sup>16</sup>

The next example considers a continuous precaution-cost function.

**Example 2** Let  $c(p) = \alpha p^{1.5}$  for  $\alpha \geq (2/3)h$ . Then  $P'(h) \frac{h}{P(h)} = 2$ ;  $\frac{c''(p)}{c'(p)}p = 0.5$  for all  $p \in [0, 1]$ ; and there are gains from trade between the injurer and one victim for any number of victims.

In this example the (equivalent) conditions in Propositions 2(a) and 2(b) hold for any value of  $h$ . Thus there exists a sufficiently small  $\Delta h$  such that a buyout produces gains from trade. To find the range of  $\Delta h$  for which a buyout produces gains from trade, consider the inequality in (4). Note that the socially-optimal probability of no-harm as a function of victims' aggregate harm is  $P(h) = \left(\frac{2h}{3\alpha}\right)^2$ .<sup>17</sup> Plugging  $\left(\frac{2h}{3\alpha}\right)^2$  for  $P(h)$  in (4) gives  $(h - \Delta h) \left( \left(\frac{2h}{3\alpha}\right)^2 - \left(\frac{2(h-\Delta h)}{3\alpha}\right)^2 \right) \geq \int_{h-\Delta h}^h \left(\frac{2x}{3\alpha}\right)^2 dx$ . Simplifying and solving for the values of  $\Delta h$  as a function of  $h$  which satisfy this inequality gives  $\Delta h \leq \frac{3-\sqrt{3}}{2}h$ . But  $\frac{3-\sqrt{3}}{2} > \frac{1}{2}$  and  $\Delta h \equiv \frac{h}{n} \leq \frac{1}{2}h$  (because  $n \geq 2$ ) and therefore  $\Delta h \leq \frac{3-\sqrt{3}}{2}h$  always holds. Thus a buyout produces gains from trade for any number of victims.

The next Lemma generalizes the previous example by providing a sufficient, but not necessary, condition for there to be gains from trade from a buyout. This condition will

<sup>16</sup>Note that if  $\Delta h = 3$  (so that there are three victims), the percentage increase in the socially-optimal probability of no-harm as the number of victims increases from 1 to 2 (1) is identical to the corresponding percentage increase in victims' aggregate harm (1). In this case the inequality in (4) is satisfied as an equality: the increase in social welfare as the number of victims increases from 1 and 2 is equal to the increase in one victim's expected profit.

<sup>17</sup>Recall from (2) that the socially-optimal probability of no-harm solves  $c'(p) = h$ . Here  $c'(p) = 1.5\alpha p^{0.5}$  and therefore  $P(h) = \left(\frac{2h}{3\alpha}\right)^2$ .

become useful in the next Section, which considers the case in which the injurer may negotiate with more than one victim.

**Lemma 2** *There are gains from trade between the injurer and one victim for any value of  $\Delta h$  (i.e.,  $\Delta h \leq h/2$ ) or for a sufficiently small  $\Delta h$  if, as victims' lost benefits increase, the rate of increase in the maximum social welfare is increasing ( $P''(x) > 0$  for  $x \in [0, h]$ ) or, equivalently, the rate of increase in the socially-optimal precaution costs is decreasing ( $c'''(p) < 0$  for  $p \in [0, P(h)]$ ).*

PROOF. See the Appendix.

Lemma 2 provides a sufficient condition for a buyout to produce gains from trade for any number of victims (i.e.,  $n \geq 2 \rightarrow \Delta h \leq h/2$ ) or for sufficiently many victims (i.e.,  $n \geq \tilde{n}$ , where  $\tilde{n} > 2$ ,  $\rightarrow \Delta h < h/2$ ): the marginal (maximum) social welfare is increasing at an increasing rate with victims' aggregate harm<sup>18</sup>; or, equivalently, the marginal precaution costs are increasing at a decreasing rate with the socially-optimal probability of no-harm. The former condition means that, for any level of victims' harm, the socially-optimal precaution costs are sufficiently greater than the maximum social welfare. The latter condition means that the injurer's precaution technology is relatively efficient in that, for any probability of no-harm, the marginal cost of precaution increases at a sufficiently slow rate.<sup>19</sup>

We now turn to consider buyout negotiations involving more than one victim.

## 5 Multi-Victim Negotiations

This Section extends the analysis in the previous Section by considering buyout negotiations involving more than one victim. We first examine the case in which the injurer possesses the entire bargaining power and makes either simultaneous or sequential buyout offers. We then explore the case in which bargaining power is switched to the victims. Before proceeding to the analysis, we introduce a condition on the relations between the injurer's maximum buyout payment and each victim's minimum buyout amount for different numbers of victims. To simplify notation, let

$$\underline{S}_i \equiv P(i \cdot \Delta h) \Delta h$$

and

$$\overline{S}_i \equiv \int_{P[(i-1) \cdot \Delta h]}^{P(i \cdot \Delta h)} H(x) dx$$

be, respectively, a victim's minimum buyout amount and the injurer's maximum buyout payment in the presence of  $i$  victims, where  $i \in \{1, \dots, n\}$ . More specifically,  $\underline{S}_i$  is one victim's expected profit in the presence of  $i$  victims, given that the injurer invests in the

<sup>18</sup>Because  $P(h)$  is the derivative of the maximum social welfare with respect to victims' expected harm,  $P''(h)$  is the derivative of the marginal (maximum) social welfare with respect to victims' expected harm.

<sup>19</sup>Lemma 2 implies that if  $P''(x) < 0$  for  $x \in [0, h]$  or, equivalently,  $c'''(p) > 0$  for  $p \in [0, P(h)]$ , then a buyout does not produce gains from trade for any  $\Delta h > 0$ .

socially-optimal precautions. Note that, because  $P(\cdot)$  is strictly increasing (see (3)),  $\underline{S}_i$  is strictly increasing with  $i$ .  $\bar{S}_i$  is the injurer's *additional* precaution costs as the number of victims increases from  $i - 1$  to  $i$ . Note that, because  $P(\cdot)$  is not necessarily convex,  $\bar{S}_i$  is not necessarily increasing with  $i$ .<sup>20</sup>

**Condition 1**  $\bar{S}_i \geq \underline{S}_i$  for  $i = 2, \dots, n$ .

Condition 1 states that there are gains from trade between the injurer and one victim for any number of victims (holding fix each victim's individual harm). Observe that, for precaution-cost functions with  $c'''(p) < 0$  for  $p \in [0, P(h)]$ , there always exists  $\Delta h$  sufficiency small such that Condition 1 holds. Note also that in the presence of only one victim, the injurer's precaution costs are lower than the victim's expected profit. Therefore,  $\bar{S}_1 < \underline{S}_1$ .

It might be instructive to consider two examples of precaution-cost functions which, for all values of  $\Delta h$  (i.e.,  $\Delta h \leq h/2$ ) or for a sufficiently small  $\Delta h$ , satisfy Condition 1:

$$c(p) = \alpha p^\beta \text{ for } \alpha \geq \frac{h}{\beta} \text{ and } \beta \in (1, 2), \quad (5)$$

and

$$c(p) = \alpha(p + 1)(\ln(p + 1) - 1) \text{ for } \alpha > 0. \quad (6)$$

In both functions, the *marginal* cost of increasing the probability of no-harm is increasing at a decreasing rate throughout the domain of the function.<sup>21</sup> In addition, in both function the increase in the injurer's precaution costs is increasing in the number of victims. Thus in both function  $\bar{S}_i$  is strictly increasing with  $i$ .<sup>22</sup>

■ **Simultaneous offers by the Injurer.** Suppose the injurer makes simultaneous buyout offers to victims. In the first stage, the injurer makes publicly-observed, take-it-or-leave-it offers to victims; the injurer thus commits to buy out any victim who accepts his offer. In the second stage, each victim decides simultaneously, but in a coalition-proof manner, whether to accept or reject the injurer's offer. In the third stage, the injurer chooses a probability of no-harm depending on the number of remaining victims. The coalition-proof refinement requires that, given the injurer's equilibrium offers, no coalition of victims could profitably deviate from their equilibrium strategies so that at least one victim in the coalition is better off and no victim in the coalition is worse off.

**Proposition 3** *Suppose the injurer makes simultaneous take-it-or-leave-it buyout offers to victims and that Condition 1 holds. Then there exists a unique, subgame-perfect, coalition-proof equilibrium in which the injurer makes  $n-1$  discriminatory offers  $\underline{S}_2, \dots, \underline{S}_n$  to  $n-1$  victims and each victim accepts his respective offer.*

<sup>20</sup>More specifically, if  $P(\cdot)$  is concave in part of its domain then  $P(i \cdot \Delta h) - P((i-1) \cdot \Delta h)$ , as well as  $\bar{S}_i$ , may be decreasing with  $i$ .

<sup>21</sup>For the precaution-cost function in (6),  $c'(p) = \alpha \beta p^{\beta-1}$ ,  $c''(p) = \alpha \beta(\beta-1)p^{\beta-2}$ , and  $c'''(p) = \alpha \beta(\beta-1)(\beta-2)p^{\beta-3}$ . For the precaution-cost function in (7),  $c'(p) = \alpha \ln(p+1)$ ,  $c''(p) = \frac{\alpha}{p+1}$ , and  $c'''(p) = -\frac{\alpha}{(p+1)^2}$ .

<sup>22</sup>This follows because  $H(p)$  is an increasing function and because, given that  $P(\cdot)$  is convex,  $P(i \Delta h) - P((i-1) \Delta h)$  is increasing with  $i$ .



PROOF. Suppose, without loss of generality, that the injurer makes victim  $i = 2, \dots, n$  a buyout offer of  $\underline{S}_i$ . We proceed to show by induction that, given the injurer's buyout offers, the unique equilibrium that survives the iterated elimination of strictly dominated strategies is for all victims to accept their respective offers. Because the unique equilibrium of a dominance solvable game is the unique coalition-proof Nash equilibrium (see Moreno and Wooders, 1996), it follows that victims' equilibrium strategy profile is coalition proof.

To start, observe that victim  $n$ 's maximum profit from rejecting the injurer's offer is  $\underline{S}_n$ , which he obtains if victims  $2, \dots, n-1$  reject their respective offers. But victim  $n$  is offered  $\underline{S}_n$  by the injurer and prefers to accept the injurer's offer if he is indifferent between accepting and rejecting it. Thus it is a dominant strategy for victim  $n$  to accept the injurer's offer. For the induction step, let  $j \geq 3$  and suppose that victims  $j+1, \dots, n$  accept their respective offers. Then victim  $j$  is better off accepting than rejecting the injurer's offer irrespective of the acceptance decisions of victims  $2, \dots, j-1$ . This follows because victim  $j$ 's maximum profit from rejecting the injurer's offer is  $\underline{S}_j$ , which he obtains if victims  $2, \dots, j-1$  reject their respective offers. But victim  $j$  is offered  $\underline{S}_j$  by the injurer and would rather accept the injurer's offer if he is indifferent between accepting and rejecting it. This proves the induction step and thereby that it is a dominant strategy for victims  $3, \dots, n$  to accept their respective offers. This also implies that victim 2 obtains  $\underline{S}_2$  if he accepts or rejects the injurer's offer; it is therefore a dominant strategy for victim 2 as well to accept the injurer offer.

We now show that the buyout offers  $\underline{S}_2, \dots, \underline{S}_n$  are the smallest—and therefore the injurer's most profitable—buyout offers that victims accept in equilibrium without violating the coalition-proof refinement. First, observe that, in the presence of only one victim, the socially-optimal precaution costs are necessarily lower than the victim's expected harm, for otherwise it would be efficient to require the injurer to take fewer precautions. Therefore, the injurer does not buy out all victims. Next, suppose the injurer makes victim  $i = 2, \dots, n$  a buyout offer of  $\tilde{\underline{S}}_i$ , where  $\tilde{\underline{S}}_i \leq \underline{S}_i$ , with strict inequality for at least one  $i$ . This implies that there exists  $j \in \{2, \dots, n\}$  such that  $\tilde{\underline{S}}_j < \underline{S}_j$ . Consider a coalition  $J$  of  $j-1$  victims consisting of victims  $2, \dots, j$ . If all victims in  $J$  reject their respective offers, then  $j$  victims remain (recall the injurer makes buyout offers to  $n-1$  victims); each victim in  $J$  accordingly obtains an expected profit of  $\underline{S}_j$ . But  $\underline{S}_i$  is increasing in  $i$  by construction and  $\tilde{\underline{S}}_j < \underline{S}_j$  by assumption and therefore  $\tilde{\underline{S}}_i < \underline{S}_j$  for  $i = 2, \dots, j$ . This implies that, if all victims in  $J$  reject their respective offers, each obtains a higher profit than his profit from accepting the injurer's offer. Thus a deviation of the injurer from the equilibrium buyout offers in Proposition 3 does not survive the coalition-proof refinement. ■

The intuition behind Proposition 3 is as follows. To buy out the “first” victim (victim  $n$ ), the injurer's offer must equal that victim's expected profit given that the injurer takes the socially-optimal precautions (the adjective “first” is illustrative, rather than literal, in a simultaneous-move game). The buyout of the first victim reduces the injurer's required precaution costs, thereby raising the probability of harm for the other victims. Consequently, the offer necessary to buy out the “next” victim (victim  $n-1$ ) is lower. More generally, the more victims are bought out, the lower the offer required to buying out an additional victim. The injurer can thus maximize his profit by making different buyout offers to different victims.

The next corollary considers the equilibrium buyout payments and profits as each victim's harm approaches zero (but victim's aggregate harm remains fixed).

**Corollary 1** *Suppose the injurer makes simultaneous, take-it-or-leave-it buyout offers to victims and that Condition 1 holds. Then in equilibrium, as  $\Delta h \rightarrow 0$ : (a) the injurer's buyout payments approach the maximum social welfare; (b) victims' aggregate loss, relative to the no-buyout case, approaches the socially-optimal precaution costs.*

PROOF. See the Appendix.

As each victim's harm becomes infinitesimally small, the injurer pays victims in aggregate an amount equal to the maximum social welfare instead of taking the socially-optimal precautions: by reducing victims' expected harm by an amount equal to the maximum social welfare, the injurer fully escapes the duty to take precautions. To see why, note that the injurer's aggregate buyout payments in equilibrium is  $\sum_{i=2}^n \underline{S}_i$ . Plugging  $P(i \cdot \Delta h) \Delta h$  for  $\underline{S}_i$  and recalling that  $\Delta h \equiv h/n$ , the injurer's aggregate buyout payments as  $\Delta h \rightarrow 0$  approach

$$\begin{aligned} \lim_{n \rightarrow \infty} \sum_{i=2}^n P(i \cdot \Delta h) \Delta h &= \lim_{n \rightarrow \infty} \sum_{i=1}^n P(i \cdot \Delta h) \Delta h - \lim_{n \rightarrow \infty} P(\Delta h) \Delta h \\ &= \int_0^h P(x) dx, \end{aligned} \tag{7}$$

which, by Lemma 1(b), equals the maximum social welfare. (7) follows because the first limit term on the right-hand side of the first equality is the left Riemann sum of  $\int_0^h P(x) dx$  and the second limit term is equal to 0.

Victims' aggregate loss from buyouts as  $\Delta h \rightarrow 0$  equals the injurer's socially-optimal precaution costs: instead of obtaining an expected profit equal to the socially-optimal probability of no-harm times their harm, victims obtain in aggregate an amount equal to the maximum social welfare. The difference between victims' joint profits with and without buyouts is equal to the socially-optimal precaution costs. Figure 3 in p. 25 provides a graphical intuition for this result. It shows that, as each victim's harm becomes infinitesimally small, the injurer's buyout payments approach the area bounded by  $P(h)$  and the  $x$ -axis, which is equal to the maximum social welfare.

Polluters seem to be negotiating simultaneously with multiple victims and thereby exploit the latter's collective action problem. For example, in response to a lawsuit by residents living nearby its large facility in Freeport, Texas, Dow Chemical offered to purchase adjacent properties, but negotiated with residents individually. As one of the residents explained, the strategy of extending individual offers—rather than the same offer to all property owners—has proven successful for Dow: “A lot of people got scared that they were not going to get anything. Some families took a buyout . . . They [Dow] try to divide and conquer.” (Cappiello & Feldstein, 2005).

## ■ Sequential offers by the Injurer.

Suppose now that the injurer approaches victims sequentially. The game under sequential bargaining proceeds in  $n + 1$  stages. We number the stages in which the injurer

makes take-it-or-leave-it offers to victims in reverse order so that the stage number indicates the number of victims whom the injurer has not yet approached. Thus, at stage  $i$  there are  $i$  victims left. We number victims accordingly: the injurer first approaches victim  $n$  in stage  $n$ , victim  $n - 1$  in stage  $n - 1$ , and so on. In stage 1, with one victim left, the injurer approaches victim 1. Each victim decides whether to accept or reject the injurer's offer after having observed all previous victims' acceptance decisions. In stage 0, after all victims have been approached, the injurer chooses a probability of no-harm depending on the number of remaining victims. The next Proposition shows that, relative to a simultaneous-offer game, the injurer's profit from buyouts, as well as his ability to exploit victims, is further increased.

**Proposition 4** *Suppose the injurer makes sequential take-it-or-leave-it-buyout offers to victims and that Condition 1 holds. Then there exists a unique subgame-perfect equilibrium in which the injurer makes each of victims  $n, \dots, 2$  (i.e., all victims except the last victim) a buyout offer of  $\underline{S}_1$  and each of these victim accepts his respective offer.*

PROOF. Let  $k_i \in \{0, \dots, n - i\}$  be the number of victims who rejected the injurer's offer prior to stage  $i = n, \dots, 1$ . We first show, using backward induction, that the injurer makes victim  $i = n, \dots, 2$  a buyout offer of  $\underline{S}_{k_i+1}$ , which victim  $i$  accepts. We then show, again by induction, that the injurer makes each of victims  $n, \dots, 2$  a buyout offer of  $\underline{S}_1$ , which each of these victims accepts.

We begin by showing that the injurer makes victim 2 (the second-to-last victim) a buyout offer of  $\underline{S}_{k_2+1}$ , which victim 2 accepts. If victim 2 rejects the injurer's offer, then  $k_1 = k_2 + 1$ . The injurer thus has a credible threat to buy out victim 1 for  $\underline{S}_{k_2+2}$ .<sup>23</sup> If victim 2 rejects the injurer's offer, therefore, there are  $k_2 + 1$  victims left in stage 0:  $k_2$  victims prior to stage 2 and victim 2. Each of these victims—including victim 2—obtains an expected profit of  $\underline{S}_{k_2+1}$ . Victim 2 is thus indifferent between accepting and rejecting an offer of  $\underline{S}_{k_2+1}$ ; hence, he accepts it.

For the backward-induction step, suppose the injurer has reached stage  $j \geq 3$ . Then, if the injurer will make victim  $i = j - 1, \dots, 2$  a buyout offer of  $\underline{S}_{k_i+1}$ , which victim  $i$  accepts, then the injurer makes victim  $j$  a buyout offer of  $\underline{S}_{k_j+1}$ , which victim  $j$  accepts. Suppose that victim  $j$  rejects the injurer's offer. Then, by the induction hypothesis, the injurer will buy out victim  $j - 1$  through 2. This implies that  $k_1 = k_{j-1} \geq 1$  and therefore that the injurer will buy out victim 1 as well. If victim  $j$  rejects the injurer's offer, therefore,  $k_j + 1$  victims are left in stage 0. Each of these victims—including victim  $j$ —obtains an expected profit of  $\underline{S}_{k_j+1}$ . Victim  $j$  is thus indifferent between accepting and rejecting an offer of  $\underline{S}_{k_j+1}$  and therefore he accepts it. This proves the backward-induction step and completes the backward induction argument.

Now, because  $k_n = 0$ , the backward-induction argument implies that the injurer makes victim  $n$ , the first victim, a buyout offer of  $\underline{S}_1$ , which victim  $n$  accepts. We now show by induction that the injurer makes victim  $i = n - 1, \dots, 2$  a buyout offer of  $\underline{S}_1$ , which victim  $i$  accepts. For the induction step, suppose the injurer has reached stage  $j \geq 2$ . Then if the injurer bought out all previous victims (i.e., victims  $n, \dots, j + 1$ ), then the

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<sup>23</sup>Recall that, under Condition 1, if  $k_1 \geq 1$ , there are gains from trade between the injurer and victim 1.

injuror will make victim  $j$  a buyout offer of  $\underline{S}_1$ , which victim  $j$  will accept. The induction step follows because the fact that the injurer bought out victims  $n, \dots, j+1$  implies that  $k_j = 0$  and because, by the backward-induction argument, the injurer makes victim  $j$  a buyout offer of  $\underline{S}_{k_j+1}$ , which victim  $j$  accepts. This proves the induction step and the second induction argument and concludes the proof. ■

Observe that, when the injurer makes sequential take-it-or-leave-it offers, Condition 1 is sufficient, but not necessary, for the injurer to buy out  $n-1$  victims. Rather, the injurer might find it profitable to buy out early victims—even if his buyout payments exceed his saving in precaution costs for these victims—if he can buy out subsequent victims for a low price. As the next Example illustrates, by sinking some of his buyout payments in early stages, the injurer can credibly threaten a victim to buy out subsequent victims if his offer is declined.

**Example 3** *Suppose the injurer can prevent harm with probability .5 or 1 by spending 2 or 5, respectively, and that the injurer makes sequential take-it-or-leave-it buyout offers. Suppose further that  $n = 4$  and  $\Delta h = 2.5$ . In the unique-subgame perfect equilibrium, the injurer offers to buy out victim 1 for 2.5, and each of the subsequent three victims for 0. All victims accept their respective offers.*

When victims' aggregate harm is 10, there are no gains from trade between the injurer and one victim: The injurer does not gain from buying out one victim, because even in the presence of three victims he is still required to spend 5 on precautions.<sup>24</sup> Thus, if negotiations were simultaneous, the injurer would have to buy out two victims for 2.5 each to reduce his precaution costs by 3 (from 5 to 2). Under sequential bargaining, however, buying out victim 4 for 2.5 provides the injurer a credible threat to buy out an additional victim for 2.5: Because the remaining victims' aggregate harm is now 7.5, the injurer's saving in precaution costs from one buyout (3) is greater than his buyout payment (2.5). As a result of the buyout of the first victim, the injurer can buy out victims 3, 2, and 1 at no cost. Although the injurer saves 2.5 in precaution costs, victims lose (collectively) 7.5. Social welfare is consequently reduced by 5 (as compared to the first best).

More generally, the maximum buyout payments that the injurer would be willing to pay early victims so as to sink his buyout costs are equal to the socially-optimal precaution costs. For instance, consider the precaution-cost function in Example 3 and assume that  $n = 11$  and  $\Delta h = 1$ . Then, in equilibrium, the injurer buys out each of the first five victims for 1, and the remaining six victims at no cost.

This analysis demonstrates that injurers' ability to exploit buyouts is greater when they approach victims sequentially. Some support for this conclusion can be found in actual buyout strategies employed by injurers. For example, a senior Exxon Mobile adviser recently admitted that rather than negotiating with all victims at once, "we grow it (the company's buyout program) a bit by bit. We go to the community closest to the existing plant first." (Cappiello and Feldstein, 2005).

■ **Simultaneous offers by Victims.** We now turn to examine the equilibrium outcomes when victims possess the entire bargaining power. We begin with the case in which

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<sup>24</sup>Recall that, if victims' aggregate harm is greater than 6, the socially-optimal precautions cost is 5.

victims make simultaneous buyout demands. In the first stage, victims make simultaneous, publicly-observed, demands to the injurer. In the second stage, the injurer decides whether to accept or reject the victims' demands. In the third stage, the injurer chooses a probability of no-harm depending on the number of remaining victims.

**Proposition 5** *Suppose victims make simultaneous take-it-or-leave-it buyout demands to the injurer and that Condition 1 holds. Then there exists a unique subgame-perfect, coalition-proof equilibrium in which all  $n$  victims make the injurer a buyout demand of  $\underline{S}_1$  and the injurer accepts  $n - 1$  demands and rejects one.*

PROOF. The proof proceeds in three steps. We first show that in equilibrium  $n - 1$  victims are bought out by the injurer. We then show that all  $n$  victims make the injurer the same buyout demand. Finally, we show that all  $n$  victims make the injurer a buyout demand of  $\underline{S}_1$ .

First, observe that, if there is only one victim, there are no gains from trade between the injurer and that victim. Therefore the injurer never buys out in equilibrium all  $n$  victims. Next, suppose that in equilibrium the injurer does not buy out  $k$  victims, where  $k \geq 2$ . Each of these victims thus obtain an expected profit of  $\underline{S}_k$ . But from Condition 1, each of these victims could profitably deviate by making the injurer a buyout demand of  $\bar{S}_k$ , where  $\bar{S}_k > \underline{S}_k$ , which the injurer accepts. Thus in equilibrium the injurer buys out exactly  $n - 1$  victims. This in turn implies that no buyout demand is strictly greater than  $\bar{S}_2$ ; for otherwise the injurer could profitably deviate from accepting  $n - 1$  demands by rejecting a buyout demand which is strictly greater than  $\bar{S}_2$ .<sup>25</sup>

Next, suppose that  $k$  victims, where  $1 \leq k \leq n - 1$ , make the injurer a buyout demand of  $\bar{S}$ , where  $\bar{S}_1 \leq \bar{S} \leq \bar{S}_2$ , and  $n - k$  victims make the injurer buyout demands which are strictly lower than  $\bar{S}$ , but greater or equal to  $\bar{S}_1$ . Because in equilibrium the injurer only accepts  $n - 1$  demands, the injurer must reject each demand of  $\bar{S}$  with positive probability. But then there exists  $\epsilon > 0$  such that each victim who makes a buyout demand of  $\bar{S}$  could profitably deviate by making a buyout demand of  $\bar{S} - \epsilon$ , where  $\bar{S} - \epsilon > \bar{S}_1$ , which the injurer accepts with probability 1. All  $n$  victims must therefore make the injurer the same buyout demand.

Finally, each victim must obtain  $\underline{S}_1$  or more in equilibrium, for each victim can secure  $\underline{S}_1$  if he is the only remaining victim. Consequently, all  $n$  victims must make a buyout demand of  $\underline{S}_1$ . Suppose that all  $n$  victims' equilibrium demands are strictly greater than  $\underline{S}_1$ . Then, because the injurer only accepts  $n - 1$  demands, the injurer rejects each of the  $n$  victims' demands with positive probability. Each victim can thus profitably deviate to making a slightly lower demand, which the injurer accepts with probability 1. The second and third steps of the proof also imply that no coalition of victims can deviate from their equilibrium strategies in a self-enforcing fashion, because any such deviation is vulnerable to further deviation. Thus the equilibrium in Proposition 5 is coalition proof. ■

The equilibrium in Proposition 5 is the unique equilibrium that survives the iterated elimination of weakly-dominated strategies. The dynamics that underlies it is competition among victims, driven by victims' fear of being left out as the only remaining victim.

<sup>25</sup>Recall that  $\bar{S}_2$  represents the additional precaution costs imposed on the injurer by the second victim.

Because the injurer only accepts  $n - 1$  demands—recall that there are no gains from trade between the injurer and the final victim left—the expected profit of the victim whose demand is rejected is necessarily lower than the buyout amount paid to each of the other victims. Anticipating that other victims will reduce their buyout demands to avoid the risk that their demand is rejected, each victim decreases further his own buyout demand. As a result, victims engage in a race-to-the-bottom: each victim makes a buyout demand equals to the expected profit of the last remaining victim.

■ **Sequential offers by Victims.** We now assume that victims negotiate with the injurer sequentially. We will show that, even under these favorable conditions for victims, the resulting loss of social welfare from buyouts is identical to the other cases examined before. The game under sequential bargaining proceeds in  $n + 1$  stages. We number the stages in this game in reverse order so that in stage  $i$  there are  $i$  victims left. We number victims accordingly: in stage  $i = n, \dots, 1$  victim  $i$  makes a take-it-or-leave-it buyout demand to the injurer, after having observed the injurer's acceptance decisions in all previous stages. In stage 0, after all victims made demands to the injurer, the injurer chooses a probability of no-harm depending on the number of remaining victims.

**Proposition 6** *Suppose that victims make the injurer sequential take-it-or-leave-it buyout demands and that Condition 1 holds. Then there exists a unique subgame-perfect equilibrium in which victim  $i = n, \dots, 2$  makes the injurer a buyout demand of  $\bar{S}_i$ , and the injurer accepts all victims' demands.*

PROOF. Let  $k_i \in \{0, \dots, n - i\}$  be the number of demands which were rejected by the injurer prior to stage  $i = n, \dots, 1$ . We first show, using backward induction, that victim  $i = n, \dots, 2$  makes the injurer a buyout demand of  $\bar{S}_{k_i+i}$ , which the injurer accepts. We then show, again by induction, that victim  $i = n, \dots, 2$  makes the injurer a buyout demand of  $\bar{S}_i$ , which the injurer accepts.

We begin by showing that victim 2, the second-to-last-victim, makes the injurer a buyout demand of  $\bar{S}_{k_2+2}$ , which the injurer accepts. If the injurer rejects victim 2's demand, then  $k_1 = k_2 + 1 \geq 1$ . Victim 1 will thus make the injurer a buyout demand of  $\bar{S}_{k_2+2}$ , which the injurer will accept.<sup>26</sup> The injurer's continuation payment if he rejects victim 2's demand is therefore

$$\bar{S}_{k_2+2} + \sum_{i=1}^{k_2+1} \bar{S}_i,$$

where the first term is the buyout payment to victim 1 and the second term is the injurer's precaution costs given that  $k_2+1$  victims are left. If the injurer accepts victim 2's demand, then  $k_1 = k_2$ . If  $k_2 \geq 1$ , victim 1 will make the injurer a buyout demand of  $\bar{S}_{k_2+1}$ , which the injurer accepts; if  $k_2 = 0$ , the injurer will incur  $\bar{S}_1$  in precaution costs. Thus, if the injurer accepts victim 2's demand, his payment to victim 2 plus his continuation payment is

$$\bar{S}_{k_2+2} + \bar{S}_{k_2+1} + \sum_{i=1}^{k_2} \bar{S}_i,$$

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<sup>26</sup>Recall that, under Condition 1, if  $k_1 \geq 1$ , there are gains from trade between the injurer and victim 1.

where the first term is the injurer's buyout payment to victim 2, the second term is the injurer's buyout payment to victim 1 if  $k_2 \geq 1$  or the injurer's precaution costs if  $k_2 = 0$ , and the third term is the injurer's precaution costs if  $k_2 \geq 1$ . But the last two terms are equal to  $\sum_{i=1}^{k_2+1} \bar{S}_i$  and therefore the injurer's profit is identical if he accepts or rejects victim 2's demand. Hence, the injurer accepts the demand.

For the backward-induction step, suppose the injurer has reached stage  $j$ , where  $j \geq 3$ . Then, if victim  $i = j - 1, \dots, 2$  makes the injurer a buyout demand of  $\bar{S}_{k_{j-1}+i}$ , which the injurer accepts, then victim  $j$  makes the injurer a buyout demand of  $\bar{S}_{k_j+j}$ , which the injurer accepts. Before proceeding, we present a Lemma which provides an equivalent condition to the induction hypothesis:

**Lemma 3** *If the injurer buys out victim  $i = j - 1, \dots, 2$  for  $\bar{S}_{k_{j-1}+i}$ , then the injurer's continuation payment in stage  $j$  is  $\sum_{i=1}^{k_{j-1}+(j-1)} \bar{S}_i$ .*

Proof. See the Appendix.

If the injurer rejects victim  $j$ 's demand, then  $k_{j-1} = k_j + 1$ . By the induction hypothesis and Lemma 3, the injurer's continuation payment in stage  $j$  is  $\sum_{i=1}^{(k_j+1)+(j-1)} \bar{S}_i$ , which simplifies to  $\sum_{i=1}^{k_j+j} \bar{S}_i$ . In contrast, if the injurer accepts victim  $n$ 's demand, then  $k_{j-1} = k_j$ . By the induction hypothesis and Lemma 3, the injurer's continuation payment in stage  $n$  is  $\sum_{i=1}^{k_j+(j-1)} \bar{S}_i$ . Adding up the injurer's buyout payment of  $\bar{S}_{k_j+j}$  to victim  $j$  gives  $\bar{S}_{k_j+j} + \sum_{i=1}^{k_j+(j-1)} \bar{S}_i$ , which simplifies to  $\sum_{i=1}^{k_j+j} \bar{S}_i$ . But this is identical to the injurer's profit if he rejects victim  $j$ 's demand. Because the injurer's profit is identical if he accepts or rejects victim  $j$ 's demand, the injurer accepts the demand. This proves the backward-induction step and completes the backward-induction argument.

Now, because  $k_n = 0$ , the backward-induction argument implies that victim  $n$ , the first victim, makes the injurer a buyout demand of  $\underline{S}_n$ , which the injurer accepts. We now show by induction that victim  $i = n - 1, \dots, 2$  makes the injurer a buyout demand of  $\bar{S}_i$ , which the injurer accepts. For the induction step, suppose the injurer has reached stage  $j \geq 2$ . Then, if the injurer bought out victim  $i = n, \dots, j + 1$  for  $\bar{S}_i$ , then victim  $j$  will make the injurer a buyout demand of  $\bar{S}_j$ , which the injurer will accept. The induction step follows because the fact that the injurer bought out victim  $i = n, \dots, j + 1$  implies that  $k_j = 0$  and because, by the backward-induction argument, victim  $j$  makes the injurer a buyout demand of  $\bar{S}_{k_j+j}$ , which the injurer accepts. This proves the induction step and the second induction argument and concludes the proof. ■

The equilibrium outcome in Proposition 5 is driven by the off-equilibrium play that follows the injurer's rejection of a victim's demand: If the injurer rejects an earlier victim's demand, he is bound to face the same demand by a later victim. To see this, suppose the injurer rejects the first victim's (victim  $n$ ) demand. This rejection now places the subsequent victim (victim  $n - 1$ ) in the same bargaining position as the first victim. Any additional rejection of a victim's demand will only replace this victim with the next victim in line. If the injurer reaches the last victim after having rejected all previous demands, the last victim will make the injurer the same buyout demand the first victim made, which the injurer (from Condition 1) will accept. Anticipating this, the injurer accepts the first victim's buyout offer. The same reasoning applies to any subsequent

victim. Each victim therefore extracts the injurer's saving in precaution costs resulting from his own buyout.

Proposition 6 also implies that some victims collect more in buyout payments than others. Recall that under Condition 1 there are gains from trade between the injurer and any one victim (except the last victim). The buyout payment to the first victim (victim  $n$ ) must therefore be greater than this victim's expected profit given that the injurer takes the socially-optimal precautions. But because buyouts result in the injurer taking fewer precautions than optimal, victims' joint profits must be lower. The first victim's (high) buyout payment thus comes at the expense of other victims. Although the first victim (and possibly other victims) collects more than his no-buyout profit, other victims collect less. In particular, for precaution-cost functions with  $c'''(p) < 0$  (such as those given in (6) and (7)), early victims are able to extract a higher buyout payment than later victims, because for such functions the injurer's additional precaution costs are increasing in the number of victims.

The next corollary considers the equilibrium buyout payments and profits as each victim's harm approaches zero (but victim's aggregate harm remains fixed).

**Corollary 2** *Suppose victims makes the injurer sequential, take-it-or-leave-it buyout demands and that Condition 1 holds. Then, in equilibrium, as  $\Delta h \rightarrow 0$ : (a) The injurer buyout payments to victims approach the socially-optimal precaution costs; (b) victims' aggregate loss, relative to the no-buyout case, approaches the maximum social welfare.*

PROOF. See the Appendix.

As each victim's harm becomes infinitesimally small, the injurer pays victims in aggregate an amount equal to the socially-optimal precaution costs. Instead of having the injurer take the socially-optimal precautions, the injurer pays the same amount to victims in buyout payments. To see why, note that the injurer's aggregate buyout payment in equilibrium is  $\sum_{i=2}^n \bar{S}_i$ . Plugging  $\int_{P((i-1)\cdot\Delta h)}^{P(i\cdot\Delta h)} H(x)dx$  for  $\bar{S}_i$  and recalling that  $\Delta h \equiv h/n$ , the injurer's aggregate buyout payments as  $\Delta h \rightarrow 0$  approach

$$\begin{aligned} \lim_{n \rightarrow \infty} \sum_{i=2}^n \int_{P((i-1)\cdot\Delta h)}^{P(i\cdot\Delta h)} H(x)dx &= \int_{P(0)}^{P(h)} H(x)dx - \lim_{n \rightarrow \infty} \int_0^{P(\Delta h)} H(x)dx \\ &= \int_0^{P(h)} H(x)dx \end{aligned} \tag{8}$$

Victims' aggregate loss from buyouts as  $\Delta h \rightarrow 0$  equals the maximum social welfare: instead of obtaining an expected profit equal to the socially-optimal probability of no-harm times their harm, victims obtain in aggregate an amount equal to the socially-optimal precaution costs. The difference between victims' joint profits with and without buyouts is equal to the socially-optimal precaution costs. Figure 3 provides further intuition into this result. It shows that, as each victim's harm becomes infinitesimally small, the injurer's buyout payments to victims approach the area bounded below by  $P(h)$ , which is equal to the socially-optimal precaution costs.



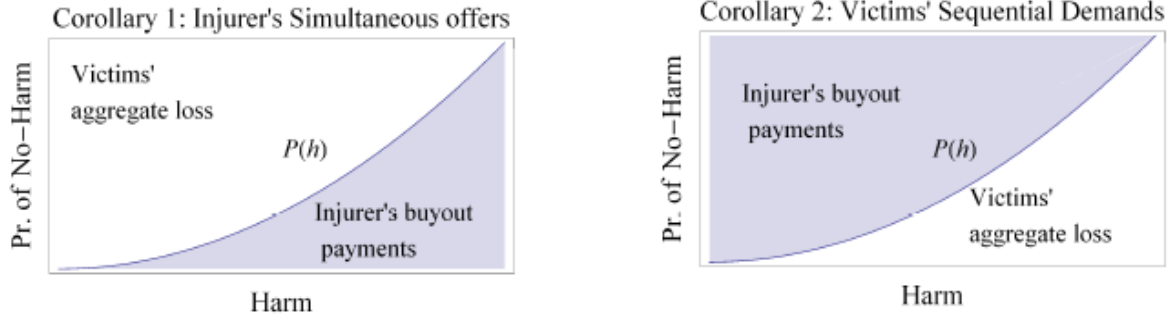


Figure 3: Buyout payments and victims' aggregate loss as  $\Delta h \rightarrow 0$ .

Corollary 2 illustrates that, although the resulting loss of social welfare is the same as in the case in which victims make simultaneous demands, victims as a group are better off if they make sequential demands. It also illustrates, however, that each victim cannibalizes subsequent victims: By extracting a greater profit than his no-buyout profit, each victim imposes a negative externality on subsequent victims.

## 6 Victims' Precautions

We have assumed so far a unilateral-care model in which only the injurer can take precautions. This section extends the analysis by considering a general example in which not only the injurer, but also victims, can take precautions. We show that, relative to the unilateral-care case, the incentives to enter buyout agreements in the joint-care case may further increase.

Our analysis rests on two assumptions. The first is that injurers' precautions affect the probability of harm globally, whereas victims' precautions affect it only locally. The second is that injurers and victims' precautions are substitutes in the prevention of harm. To illustrate these assumptions, consider a typical pollution case in which the injurer can reduce harm by elevating his factory's smokestack, and victims can reduce harm by installing residential venting systems. Whereas the reduction in the probability of harm from a higher smokestack benefits all victims (global precaution), the benefit from installing a venting system is limited to each household (local precaution). In addition, because the marginal benefit from residential venting systems decreases with the height of the smokestack, the injurer and victims' precautions are substitutes.

Victims' precautions may help to facilitate buyouts by further decreasing the injurer's required investment in care following a buyout. Because buyouts reduce victims' aggregate harm, they render local precautions more efficient and global precautions less so. By buying out victims, injurers subject to a cost-benefit standard can thus shift part of the burden of taking precautions onto the remaining victims. Consequently, compared to the case in which victims cannot take care, buyouts may produce even greater gains from trade.

To illustrate this notion, consider first an alternative-care case, in which it is efficient

for either the injurer or victims to take precautions (Posner & Landes, 1987). Suppose it is efficient for only victims to take (local) precautions if victims' aggregate harm is smaller than a threshold level, but otherwise it is efficient for only the injurer to take (global) precautions. Suppose further that initially victims' aggregate harm is higher than this threshold level so that it is efficient for only the injurer to take precautions. In this case, even if there are no gains from trade between the injurer and victims, an injurer who possesses all the bargaining power may still find it profitable to buy out victims. By buying out sufficiently many victims, the injurer can shift the entire burden of taking care onto the remaining victims, thereby reducing his precautions costs to zero.

We now turn to demonstrate this notion in a joint-care example. Suppose that the injurer can prevent harm to victims with probability  $p$ . Suppose further that, given that the probability of no-harm is  $p$ , each victim can prevent the residual probability of harm with probability  $q$ . Thus, the overall probability of no-harm is  $p + (1-p)q$  (or, equivalently,  $q + (1-q)p$ ). Accordingly, the greater  $p$  is, the lower will be the marginal benefit to each victim of increasing  $q$  (and vice versa). The injurer's and victims' precautions are therefore substitutes.<sup>27</sup>

To simplify the analysis, assume that  $\Delta h = 1$  so that  $h = n$ . Each victim's precaution costs are  $q^2$ . For tractability and to facilitate the comparison between a unilateral-care and bilateral-care case, we assume that the injurer's precaution costs are  $kp^2$ , for some constant  $k \geq h/2$ .<sup>28</sup> Note that in this example there are no gains from trade between the injurer and any victim.<sup>29</sup> This outcome, however, changes if victims as well can take precautions.

The social problem when both the injurer and the victims can take precautions is to choose probabilities of no-harm  $p \in [0, 1]$  and  $q \in [0, 1]$  to maximize

$$[p + (1-p)q]h - kp^2 - hq^2. \quad (9)$$

The first term is the probability of no-harm multiplied by victims' aggregate harm. The second and third terms are the injurer's and victims' precaution costs, respectively. Note that, because  $h = n$ , victims' aggregate precaution costs are  $hq^2$ .

Partially differentiating (9) with respect to  $p$  and  $q$ , equating to zero, and solving for the socially-optimal values of  $p$  and  $q$  gives

$$P(h) = \frac{h}{4k - h}$$

and

$$Q(h) = \frac{2k - h}{4k - h},$$

where  $P(h)$  and  $Q(h)$  are the injurer's and victims' socially-optimal probabilities of no-harm, respectively.

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<sup>27</sup>To see this formally, note that the cross-derivative of the total probability of no-harm with respect to  $p$  and  $q$  is  $-1$ .

<sup>28</sup>We assume that  $k \geq h/2$  to ensure an interior solutions for the injurer's and victim's socially-optimal probabilities of no-harm.

<sup>29</sup>Because  $c'''(p) = 0$  for all  $p \in [0, 1]$ . See also the proof of Proposition 7(a).

It is easy to verify that the injurer's socially-optimal probability of no-harm is increasing with  $h$ , whereas each victim's socially-optimal probability of no-harm is decreasing with  $h$ ; the injurer's and victims' socially-optimal precautions are substitutes. In addition, the injurer's socially-optimal probability of no-harm is decreasing with  $k$ , whereas each victim's socially-optimal probability of no-harm is increasing with  $k$ . Thus, as the injurer's precaution costs increase, it is efficient for the injurer to take less care and for victims to take more care.<sup>30</sup>

Under a cost-benefit standard, the injurer should prevent harm with probability  $P(h)$ : the injurer should spend  $k \left(\frac{h}{4k-h}\right)^2$  on precautions, so that the probability of no-harm is  $\frac{h}{4k-h}$ . If the injurer prevents harm with probability  $P(h)$ , then each victim chooses to prevent the residual harm with the socially-optimal probability of  $Q(h)$ .

As in the unilateral-care case, the injurer's maximum buyout payment in the presence of  $i$  victims equals the difference between his precaution costs when there are  $i$  and  $i - 1$  victims. Because  $\Delta h = 1$ , victims' aggregate harm is simply  $i$ , the number of victims. The injurer's maximum buyout payment in the presence of  $i$  victims is therefore

$$\bar{S}_i \equiv k \cdot P(i)^2 - k \cdot P(i-1)^2. \quad (10)$$

A victim's minimum buyout amount given that there are  $i$  victims is equal to his expected profit (i.e., probability of no-harm less his precaution costs) given the socially-optimal probability of no-harm:

$$\begin{aligned} \underline{S}_i &\equiv [P(i) + (1 - P(i))Q(i)] - Q(i)^2 \\ &= [1 - Q(i)] \cdot [P(i) + Q(i)]. \end{aligned} \quad (11)$$

Proposition 7 considers the range of victims' aggregate harm for which buyouts produce gains from trade.

**Proposition 7** *Suppose that each victim's harm is  $\Delta h = 1$  and that  $n \geq 3$ . Suppose further that the injurer can prevent harm to victims with probability  $p$  at a cost  $kp^2$ , where  $k \geq n/2$ .*

(a) *If only the injurer can take precautions, buyouts do not produce gains from trade.*

(b) *If each victim can prevent the residual probability of harm to himself with probability  $q$  at a cost  $q^2$ , then buyouts produce gains from trade if the number of victims is sufficiently large. Specifically, if the number of victims is greater than or equal to  $i^*(k) = \lceil \frac{1}{3}(2 + 8k - \sqrt{1 - 4k + 16k^2}) \rceil$ , there are gains from trade between the injurer and one victim, where  $n \geq i^*(k) \geq \frac{2}{3}n$  is weakly increasing in  $k$ .*

Proof. See the Appendix.

Proposition 7 implies that the fact that victims as well can take precautions might increase the incentives to enter buyout agreements: If the socially-optimal probability of no-harm is less than 1—which follows from the assumption that  $k \geq n/2$ —and the number

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<sup>30</sup>Because  $\frac{\partial(\frac{h}{4k-h})}{\partial k} = \frac{-4h}{(h-4k)^2} < 0$  and  $\frac{\partial(\frac{2k-h}{4k-h})}{\partial k} = 2\frac{h}{(h-4k)^2} > 0$ .

of victims is sufficiently large—specifically, greater than  $i^*(k)$ —then the injurer will buy out some victims irrespective of the bargaining structure and the distribution of bargaining power. If victims cannot take precautions, in contrast, the injurer does not buy out any victim.

To illustrate the result in Proposition 7, we consider a concrete example. This example shows that if the injurer’s socially-optimal precaution costs are sufficiently high, social welfare and victims’ joint profit are lower when victims as well can take precautions.

**Example 4** *Suppose that  $k = 50$  and  $\Delta h = 1$ . If the injurer’s and the victims’ precaution costs are those given in Proposition 7, then there are gains from trade between the injurer and one victim if the number of victims is between 68 and 100. Moreover, if the number of victims is between 71 and 100, then social welfare is higher if only the injurer can take precautions.*

This example shows that, although social welfare in the absence of buyouts is greater if victims as well can take precautions, the decrease in social welfare caused by buyouts renders victims’ precautions socially undesirable. Because buyouts produce gains from trade, they allow the injurer and a subset of victims to impose a negative externality on the remaining victims. If the initial socially-optimal probability of no-harm is sufficiently high, this negative externality outweighs the social benefit from victims’ ability to take precautions. The overall effect, therefore, is that social welfare is lower if victims as well can take precautions.<sup>31</sup>

## 7 Conclusion

Cost-benefit standards, which are extensively used to regulate harmful activities, require injurers to take efficient precautions. This paper shows that, in the presence of multiple victims, these standards are vulnerable to strategic bargaining. Specifically, injurers subject to a cost-benefit standard can avoid taking efficient precautions by buying out victims’ properties. Thus, although buyouts increase the joint profits of the bargaining parties, they reduce social welfare.

The result that buyouts are socially inefficient is robust to alternative assumptions on victims’ ability to cooperate, the allocation of bargaining power, the bargaining structure, and the distribution of optimal precautions between the injurer and victims. First, an injurer will be able to buy out victims and thereby avoid taking efficient precautions even if victims can coordinate on their most preferred Pareto equilibrium. Second, buyouts reduce social welfare irrespective of whether the injurer or victims possess the bargaining power and whether his offers or their demands are sequential or simultaneous. Finally, if victims can also take precautions, the incentives to enter buyout agreements may be greater.

The presence of high transaction costs is often invoked to justify cost-benefit regulation of harmful activities. According to the conventional argument, because negotiation

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<sup>31</sup>The injurer and victims’ profits depend of course on the bargaining structure and distribution of bargaining power.

costs between injurers and victims are prohibitively high, injurers are not required to obtain victims' consent before inflicting harm but rather are required to take efficient precautions. This paper shows, however, that if injurers and victims can negotiate, a cost-benefit standard merely shifts the problem of high transaction costs to the victims. If victims cannot negotiate and make side payments among themselves, they might be exploited by the injurer or by each other through buyouts.

# Appendix

This Appendix restates and proves Lemma 1, Proposition 2(a), Lemma 2, and Proposition 6.

**Lemma 1** *Let  $P(h) : [0, \infty) \rightarrow [0, 1]$  map victims' aggregate harm to the socially-optimal probability of no-harm and let  $H(p) : [0, 1] \rightarrow [0, \infty)$  be the inverse function of  $P(h)$ . Then:*

(a) *The socially-optimal precaution costs are  $\int_0^{P(h)} H(x)dx$ .*

(b) *The maximum social welfare is  $\int_0^h P(x)dx = P(h)h - \int_0^{P(h)} H(x)dx$ .*

(c) *If victims' aggregate benefit from their activities (i.e., their harm in the absence of precautions) decreases from  $h_2$  to  $h_1$  ( $h_2 > h_1 \geq 0$ ), then the maximum social welfare decreases by  $\int_{h_1}^{h_2} P(x)dx = [P(h_2)h_2 - P(h_1)h_1] - \int_{P(h_1)}^{P(h_2)} H(x)dx$ .*

PROOF. In the text we proved part (a) directly and part (b) indirectly. Here we provide a direct proof of part (b) and prove part (c).

Recall from part (a) that social welfare is  $P(h)h - \int_0^{P(h)} H(x)dx$ . The first term is victims' expected profit given that the injurer takes socially-optimal precautions. The second term is the injurer's precaution costs, given that he takes socially-optimal precautions (see part (a)). Substituting  $P(x)$  for  $x$  and  $P'(x)dx$  for  $dx$  (because  $\frac{dP(x)}{dx} = P'(x) \Rightarrow dP(x) = P'(x)dx$ ) and changing the upper limit of integration to  $h$  gives:

$$P(h)h - \int_0^h H(P(x))P'(x)dx, \quad (\text{A1})$$

which simplifies to

$$P(h)h - \int_0^h xP'(x)dx, \quad (\text{A2})$$

because  $x = H(P(x))$ .

Finally, integrating  $\int_0^h xP'(x)dx$  by parts gives

$$\begin{aligned} & P(h)h - \left( P(h)h - \int_0^h P(x)dx \right) \\ &= \int_0^h P(x)dx. \end{aligned} \quad (\text{A3})$$

As to part (c), note that  $\int_{h_1}^{h_2} P(x)dx = \int_0^{h_2} P(x)dx - \int_0^{h_1} P(x)dx$ . Invoking part (b) and collecting terms gives

$$\begin{aligned} \int_0^{h_2} P(x)dx - \int_0^{h_1} P(x)dx &= P(h_2)h_2 - \int_0^{P(h_2)} H(x)dx - \left( P(h_1)h_1 - \int_0^{P(h_1)} H(x)dx \right) \\ &= [P(h_2)h_2 - P(h_1)h_1] - \int_{P(h_1)}^{P(h_2)} H(x)dx. \quad \blacksquare \end{aligned} \quad (\text{A4})$$

**Proposition 2(a)** *As  $\Delta h \rightarrow 0$ , there are gains from trade between the injurer and one victim iff the point elasticity of the optimal probability of no-harm with respect to victims' total harm is greater than 1:  $P'(h)\frac{h}{P} \geq 1$ .*

PROOF. Recall from (4) that there are gains from trade between the injurer and one victim iff  $(h - \Delta h)(P(h) - P(h - \Delta h)) \geq \int_{h-\Delta h}^h P(x)dx$ .

Writing  $\int_{h-\Delta h}^h P'(h)dp$  for  $P(h) - P(h - \Delta h)$ , this inequality becomes

$$(h - \Delta h) \int_{h-\Delta h}^h P'(h)dp \geq \int_{h-\Delta h}^h P(h)dp. \quad (\text{A5})$$

Dividing through by  $\Delta h$  implies that a buyout produces gains from trade iff

$$\frac{(h - \Delta h)}{\Delta h} \int_{h-\Delta h}^h P'(h)dp \geq \frac{1}{\Delta h} \int_{h-\Delta h}^h P(h)dp. \quad (\text{A6})$$

Now, a buyout produces gains from trade as  $\Delta h \rightarrow 0$  iff

$$\lim_{\Delta h \rightarrow 0} \left[ \frac{(h - \Delta h)}{\Delta h} \int_{h-\Delta h}^h P'(h)dp \right] \geq \lim_{\Delta h \rightarrow 0} \left[ \frac{1}{\Delta h} \int_{h-\Delta h}^h P(h)dp \right]. \quad (\text{A7})$$

Because the limit of a product is the product of the limits, the last inequality holds iff

$$\lim_{\Delta h \rightarrow 0} [(h - \Delta h)] \lim_{\Delta h \rightarrow 0} \left[ \frac{1}{\Delta h} \int_{h-\Delta h}^h P'(h)dp \right] \geq \lim_{\Delta h \rightarrow 0} \left[ \frac{1}{\Delta h} \int_{h-\Delta h}^h P(h)dp \right]. \quad (\text{A8})$$

The first limit on the left-hand side is  $h$ , the second limit on the left-hand side is  $P'(h)$ , and the limit on the right-hand side is  $P(h)$ . Dividing through by  $P(h)$  we get that a buyout produces gains from trade as  $\Delta h \rightarrow 0$  iff

$$P'(h)\frac{h}{P(h)} \geq 1. \quad \blacksquare \quad (\text{A9})$$

**Lemma 2** *There are gains from trade between the injurer and one victim for any value of  $\Delta h$  (i.e.,  $\Delta h \leq h/2$ ) or for a sufficiently small  $\Delta h$  if, as victims' lost benefits increase, the rate of increase in the maximum social welfare is increasing ( $P''(x) > 0$  for  $x \in [0, h]$ ) or, equivalently, the rate of increase in the socially-optimal precaution costs is decreasing ( $c'''(p) < 0$  for  $p \in [0, P(h)]$ ).*

PROOF. We first show that  $c'''(p) < 0$  for  $p \in [0, P(h)] \Leftrightarrow P''(x) > 0$  for  $x \in [0, h]$ . Recall from (3) that, for  $x \in [0, h]$ ,  $P'(x) = \frac{1}{c''(P(x))}$ . Differentiating both sides with respect to  $x$  gives  $P''(x) = \frac{c'''(P(x))}{[c''(P(x))]^2} P'(x)$ . Because  $P'(x) > 0$ , it follows that  $P''(x) > 0$  iff  $c'''(P(x)) < 0$ .

We proceed by showing that if  $c'''(p) < 0$  for  $p \in [0, P(h)]$ , then  $\frac{c'(\hat{p})}{c''(\hat{p})} > \hat{p}$ , where  $\hat{p} = P(h)$ . This implies, by Proposition 1(b), that there are gains from trade between

the injurer and one victim for a sufficiently small  $\Delta h$ . To see this, note that for all  $p \in [0, P(h)]$  we have

$$c'(p)c'''(p) < 0, \quad (\text{A10})$$

because, by assumption,  $c'(p) > 0$ .

Dividing through by  $[c''(p)]^2$  gives

$$\frac{c'(p)c'''(p)}{[c''(p)]^2} < 0. \quad (\text{A11})$$

Adding  $1 - \frac{c'(p)c'''(p)}{(c''(p))^2}$  to both sides gives

$$1 - \frac{c'(p)c'''(p)}{[c''(p)]^2} > 1. \quad (\text{A12})$$

Substituting  $\left(\frac{c'(p)}{c''(p)}\right)'$  for  $1 - \frac{c'(p)c'''(p)}{(c''(p))^2}$ , the previous inequality becomes

$$\left(\frac{c'(p)}{c''(p)}\right)' > 1. \quad (\text{A13})$$

Finally, integrating both sides from 0 to  $\hat{p}$  gives

$$\int_0^{\hat{p}} \left(\frac{c'(p)}{c''(p)}\right)' dp > \int_0^{\hat{p}} dp, \quad (\text{A14})$$

which simplifies to

$$\frac{c'(\hat{p})}{c''(\hat{p})} > \hat{p}, \quad (\text{A15})$$

because  $c'(0) = 0$ .

We now show that, if  $P''(x) > 0$  for  $x \in [0, h]$  and a buyout produces gains from trade for some  $\widetilde{\Delta h}$ , then a buyout also produces gains from trade for  $\Delta h < \widetilde{\Delta h}$ . Recall from (4) that a buyout produces gains from trade iff  $(h - \Delta h)(P(h) - P(h - \Delta h)) \geq \int_{h-\Delta h}^{\bar{h}} P(x)dx$ . We show next that the left hand side of this inequality is decreasing with  $\Delta h$  and the right-hand side is increasing with  $\Delta h$ . This implies that, if the inequality in (4) holds for  $\widetilde{\Delta h}$ , it must also hold for  $\Delta h < \widetilde{\Delta h}$ .

To see this, note that the derivative of  $\int_{h-\Delta h}^{\bar{h}} P(x)dx$  with respect to  $\Delta h$  is  $P(\bar{h} - \Delta h) > 0$ . The derivative of  $(h - \Delta h)(P(h) - P(h - \Delta h))$  with respect to  $\Delta h$  is  $-(P(h) - P(h - \Delta h)) + (h - \Delta h)P'(h - \Delta h)$ . But  $P''(x) < 0$  for  $x \in [0, h]$  implies that  $P'(h - \Delta h) < \frac{P(h) - P(h - \Delta h)}{h - \Delta h}$  and therefore that  $-(P(h) - P(h - \Delta h)) + (h - \Delta h)P'(h - \Delta h) < 0$ . ■

**Lemma 3** *If the injurer buys out each victim  $i = j-1, \dots, 2$  for  $\bar{S}_{k_{j-1}+i}$ , then the injurer's continuation payment in stage  $n$  is  $\sum_{i=1}^{k_{n-1}+(n-1)} \bar{S}_i$ .*

PROOF. Consider two cases:



Case 1:  $k_{n-1} \geq 1$ . Because the injurer buys out victim  $i = j - 1, \dots, 2$ , it follows that  $k_1 = k_{j-1}$ . Therefore, if  $k_{j-1} \geq 1$ , then  $k_1 \geq 1$ . From Condition 1, victim 1 will make the injurer a buyout demand of  $\bar{S}_{k_{j-1}+1}$ , which the injurer will accept. Because the number of remaining victims in stage 0 is  $k_0 = k_1 = k_{j-1} \geq 1$ , the injurer spends  $\sum_{i=1}^{k_{j-1}} \bar{S}_i$  on precaution costs. The injurer's continuation payment in stage  $j$  is therefore

$$\begin{aligned}
& \overbrace{\sum_{i=k_{j-1}+2}^{k_{j-1}+j-1} \bar{S}_i}^{\text{Payment to victims } j-1, \dots, 2} + \overbrace{\bar{S}_{k_{j-1}+1}}^{\text{Payment to victim 1}} + \overbrace{\sum_{i=1}^{k_{j-1}} \bar{S}_i}^{\text{Precaution costs}} \\
&= \sum_{i=1}^{k_{j-1}+j-1} \bar{S}_i. \tag{A16}
\end{aligned}$$

Case 2:  $k_{n-1} = 0$ . Because the injurer buys out each victim  $i = j - 1, \dots, 2$ , it follows that  $k_1 = k_{j-1}$ . Therefore, if  $k_{j-1} = 0$ , then  $k_1 = 0$ . Thus there are gains from trade between the injurer and victim 1. Because only victim 1 is left, the injurer spends  $\bar{S}_1$  on precaution costs. The injurer's continuation payment in stage  $j$  is therefore

$$\begin{aligned}
& \overbrace{\sum_{i=2}^{j-1} \bar{S}_i}^{\text{Payment to victims } j-1, \dots, 2} + \overbrace{\bar{S}_1}^{\text{Precaution costs}} \\
&= \sum_{i=1}^{j-1} \bar{S}_i. \tag{A17}
\end{aligned}$$

In both cases, therefore, the injurer's continuation payment in stage  $j$  is  $\sum_{i=1}^{k_{j-1}+j-1} \bar{S}_i$ . ■

**Proposition 7** *Suppose that each victim's harm is  $\Delta h = 1$  and that  $n \geq 3$ . Suppose further that the injurer can prevent harm to victims with probability  $p$  at a cost  $kp^2$ , where  $k \geq n/2$ .*

(a) *If only the injurer can take precautions, buyouts do not produce gains from trade.*

(b) *If each victim can prevent the residual probability of harm to himself with probability  $q$  at a cost  $q^2$ , then buyouts produce gains from trade if the number of victims is sufficiently large. Specifically, if the number of victims is greater than or equal to  $i^*(k) = \lceil \frac{1}{3}(2 + 8k - \sqrt{1 - 4k + 16k^2}) \rceil$ , there are gains from trade between the injurer and one victim, where  $n \geq i^*(k) \geq \frac{2}{3}n$  is weakly decreasing in  $k$ .*

PROOF. (a) If only the injurer can take precautions, the socially-optimal probability of no-harm as a function of victims' aggregate harm is  $h/2k$ . Plugging  $h/2k$  for  $P(h)$  in (4) gives  $(h - \Delta h) \left( \frac{h}{2k} - \frac{h - \Delta h}{2k} \right) \geq \int_{h - \Delta h}^h \frac{x}{2k} dx$ , which simplifies to  $\frac{1}{2k} \Delta h (h - \Delta h) \geq \frac{1}{2k} \Delta h \left( h - \frac{\Delta h}{2} \right)$ . But the left-hand side is strictly smaller than the right-hand side for any  $\Delta h > 0$ . Thus there are no gains from trade between the injurer and any one victim.

(b) Recall that the social problem is to maximize

$$[p + (1 - p)q]h - kp^2 - hq^2. \quad (\text{A18})$$

The first derivatives with respect to  $p$  and  $q$ , respectively, are  $(1 - q)h - 2kp$  and  $(1 - p)h - 2hq$ .

The Hessian matrix is

$$\begin{bmatrix} h - hq - 2kp & -2k \\ -2h & -h(p + 2q - 1) \end{bmatrix}. \quad (\text{A19})$$

The discriminant at  $p = \frac{h}{4k-h}$  and  $q = \frac{2k-h}{4k-h}$  is

$$\begin{aligned} & \left( h - h \frac{2k-h}{4k-h} - 2k \frac{h}{4k-h} \right) \times \\ & -h \left( \frac{h}{4k-h} + 2 \frac{2k-h}{4k-h} - 1 \right) - 4kh \\ & = -4hk, \end{aligned} \quad (\text{A20})$$

which implies that social welfare attains a maximum at  $P(h)$  and  $Q(h)$ .

The socially-optimal probability of no-harm is

$$\begin{aligned} \hat{P}(h) & \equiv P(h) + [1 - P(h)]Q(h) \\ & = \frac{h}{4k-h} + \left( 1 - \frac{h}{4k-h} \right) \frac{2k-h}{4k-h} \\ & = \frac{h^2 - 4hk + 8k^2}{(h - 4k)^2}. \end{aligned} \quad (\text{A21})$$

Because  $\Delta h = 1$ , victims' aggregate harm as a function of the number of victims is  $i$ . A victim's minimum buyout amount is his expected benefit given the injurer's and each victim's socially-optimal probabilities of no-harm,  $\hat{P}(i)$ , less his precaution costs,  $Q(i)^2$ :

$$\begin{aligned} \hat{P}(i) - Q(i)^2 & = \frac{1}{(4k-h)^2} (i^2 - 4ik + 8k^2) - \left( \frac{2k-i}{4k-i} \right)^2 \\ & = \frac{4k^2}{(4k-i)^2}. \end{aligned} \quad (\text{A22})$$

The injurer's maximum buyout payment is the difference between his precaution costs when victims' aggregate harm is  $i$  and when it is  $i - 1$ :

$$k \left( \frac{i}{4k-i} \right)^2 - k \left( \frac{i-1}{4k-(i-1)} \right)^2. \quad (\text{A23})$$

The injurer maximum buyout payment is greater than a victim's minimum buyout amount iff

$$k \left( \frac{i}{4k-i} \right)^2 - k \left( \frac{i-1}{4k-(i-1)} \right)^2 \geq \frac{4k^2}{(i-4k)^2}. \quad (\text{A24})$$

Tedious algebra transforms this inequality to

$$-4k[(3i^2 - 4i(1 + 4k) + 12k + 16k^2 + 1)] \geq 0, \quad (\text{A25})$$

which is a quadratic equation of  $i$ , with solutions  $\frac{1}{3}(8k + 2 \pm \sqrt{16k^2 - 4k + 1})$ . Because the quadratic coefficient is negative, the relevant solution is  $\frac{1}{3}(8k + 2 - \sqrt{16k^2 - 4k + 1})$ . Differentiating this expression with respect to  $k$  gives  $\frac{2}{3}\left(4 - \frac{8k-1}{\sqrt{16k^2-4k+1}}\right) < 0$ , which implies that  $i^*(k)$  is weakly increasing in  $k$ .

To find the minimum value of  $i^*(k)$  as  $k$  increases, we consider the value of  $i^*(k)$  when  $2k = n$  (if  $2k > n$ , then  $i^*(k)$  must be greater). Dividing  $i^*(k)$  by  $2k$  and taking the limit as  $k$  goes to infinity gives

$$\begin{aligned} & \lim_{k \rightarrow \infty} \frac{1}{3} \frac{(8k + 2 \pm \sqrt{16k^2 - 4k + 1})}{2k} \\ &= \lim_{k \rightarrow \infty} \frac{1}{3} \left( 4 + \frac{1}{k} - \sqrt{\frac{16k^2 - 4k + 1}{4k^2}} \right) \\ &= \lim_{k \rightarrow \infty} \frac{1}{3} \left( 4 + \frac{1}{k} - \sqrt{4 - \frac{1}{k} + \frac{1}{4k^2}} \right) \\ &= \frac{1}{3} \left( 4 + \lim_{k \rightarrow \infty} \frac{1}{k} - \lim_{k \rightarrow \infty} \sqrt{4 - \frac{1}{k} + \frac{1}{4k^2}} \right) \\ &= \frac{2}{3}. \end{aligned} \quad (\text{A26})$$

Therefore,  $i^*(k) \geq \frac{2}{3}n$ .

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