Accounting for Cross-Country Differences in Intergenerational Earnings Persistence: The Impact of Taxation and Public Education Expenditure*

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Abstract

Western economies exhibit substantial differences in the degree of intergenerational earnings persistence between fathers and sons. Earnings persistence is relatively low in Northern Europe and relatively high in the US, Britain, and Southern Europe. In this chapter I first document that there is a strong negative cross-country correlation between intergenerational earnings persistence and tax progressivity, and intergenerational earnings persistence and public expenditure on tertiary education. I then develop an intergenerational life-cycle model of human capital accumulation and earnings, which features progressive taxation, public education expenditure, and borrowing constraints among the determinants of earnings persistence. I calibrate the model to US data and use it to quantify how earnings persistence in the US changes as I introduce policies from Denmark. Denmark is an interesting example because it is the country in my sample with the highest and most progressive taxes and the greatest expenditure on tertiary education, as well as the lowest earnings persistence. I find that the Danish policies would reduce earnings persistence in the US by reducing parental/individual incentives for investing in human capital, thereby creating a weaker relationship between the parent's financial resources and the child's earnings. Quantitatively, taxation is more important than education expenditure. Introducing a Danish tax policy in the US reduces the intergenerational elasticity of earnings from 0.47 to 0.35, or about 40% of the difference between the US and the Scandinavian countries, which have the lowest earnings persistence among the countries in my sample. I also find that borrowing constraints have a very limited impact on earnings persistence.

1 Introduction

In recent years, several empirical studies have been concerned with estimating and comparing the intergenerational persistence of earnings between fathers and sons in Western economies. The main finding of this literature is that intergenerational persistence is relatively high in the US, Britain, and Southern Europe, and relatively low in Northern Europe and in Canada. Table 1 below displays the results from a meta study of intergenerational earnings persistence across countries by Corak (2006) [1], supplemented with two recent studies from Italy and Spain[2]. The next question follows naturally: What are the reasons for these differences? Western economies differ greatly with respect to public expenditure on education and with respect to tax schemes. Does the cross-country variation in public institutions explain the variation in earnings persistence? Understanding why earnings mobility differs across countries is interesting, even if only for positive reasons. However, the question of whether economic fate is predetermined or whether it is influenced by public institutions may also have important policy implications. For instance, if the pattern we observe occurs because poor parents in some countries are borrowing constrained and cannot invest optimally in their children's human capital, it may call for policy intervention.

Several explanations that could contribute to the observed cross-country pattern in intergenerational earnings persistence have been proposed in the economic literature, but there is little quantitative work in the area. To the best of my knowledge there are no previous papers studying the impact of cross-country differences in policies on earnings persistence. I start by documenting that there is a strong negative cross-country correlation between earnings persistence and tax progressivity, and earnings persistence and public expenditure on tertiary education. I then provide an intergenerational life-cycle model of human capital accumulation and earnings to

[1] See also Blanden (2009) for an extensive summary of the empirical literature.

[2] There are many difficulties with comparing different studies of earnings persistence; see Appendix 10.1. Table 1 is to be interpreted as a stylized fact.

Table 1: Intergenerational Earnings Elasticity Across Countries

Country	Estimated Earnings Elasticity
Denmark	0.15
Norway	0.17
Finland	0.18
Canada	0.19
Sweden	0.27
Germany	0.32
Spain**	0.40
France	0.41
Italy*	0.43
USA	0.47
UK	0.50

This table displays the results from a meta study by Heinz Corak (2006). *Taken from Piraino (2007). **Taken from Pla (2009). Pla estimates one earnings elasticity using sons aged 30-40, and one earnings elasticity using sons aged 40-50. The number listed is the average of the two.

separate and quantify the determinants of earnings persistence. The model contains key elements that have been proposed as determinants of earnings persistence in the literature, namely, progressive taxation, the efficiency of human capital investments, public education expenditure, borrowing constraints, partially inheritable abilities, inter vivos transfers from parents to children, and idiosyncratic wage shocks. I calibrate the model to US data and decompose the contributions of the different model elements. Next I study how earnings persistence in the US changes as I introduce policies from Denmark into the model. Denmark is an interesting example because it is the country in my sample with the highest and most progressive taxes and greatest expenditure on tertiary education, as well as the lowest earnings persistence. I find that taxation and public education expenditure have a significant impact on earnings persistence and therefore are important contributors to the cross-country patterns that empirical researchers have found. More government expenditure on education and higher taxes reduce earnings persistence by reducing parental/individual incentives for investing in human capital, which leads to a weaker relationship between the parents financial resources and the childs earnings. The impact of taxation is quan-

titatively greater than the impact of education expenditure. Introducing a Danish tax system in the US reduces the intergenerational elasticity of earnings from 0.47 to 0.35, or about 40% of the difference between the US and the Scandinavian countries, which have the lowest earnings persistence among the countries in my sample. I also study the quantitative importance of borrowing constraints in the model and conclude that they have very little impact on earnings persistence.

Determinants of Earnings Persistence

In classical human capital theory, it is usually assumed that the earnings of individuals depend on their level of human capital and on market luck, or random shocks. Two factors go into human capital formation. One is a fixed endowment, imperfectly inherited by children from parents, and the other is investments in human capital, which can be made both by the parents and by the government; see Becker and Tomes (1979), Becker and Tomes (1986), and Solon (2004). Endowments here refer to everything from genetically inherited ability to knowledge acquired from the parents, family culture, and the parents social connections. In my model below I will refer to the family endowment as ability. The narrowest definition of human capital investment is investment in education, but many authors use broader definitions. It is also commonly assumed that parents care about their childrens utility and that utility depends only on the consumption of goods that cannot be considered as investments in human capital; see, for instance, Becker and Tomes (1986). This way, the only reason to invest in childrens human capital is to increase their future consumption through higher earnings. If there are diminishing returns to investment, there will be an optimal level of investment for each child.

From this theory, several explanations for cross-country differences in earnings persistence emerge. One possibility is that the inheritability of family endowments is stronger in some countries. There could be many underlying reasons for this. The degree of assortative mating does, for instance, differ across countries. In some

countries, couples are more similar with respect to their education and family background, and since almost all research studies the correlation between fathers and sons, this will cause the sons to be more similar to their fathers. Indeed, there seems to be a somewhat higher correlation in spousal education in the US and Italy than in Northern Europe, but Britain, which has relatively high earnings persistence, has a relatively low correlation in spousal education.[3]

Another possibility is that countries just differ in the returns to human capital or the cost of acquiring it. In standard intergenerational models of earnings formation, earnings persistence increases with the returns to human capital investments; see, for instance, Restuccia and Urrutia (2004). Depending on modeling choices, there are several channels through which this may work, but I will mention just a common one: Optimal human capital investments are usually increasing in parental financial resources, as altruistic parents face a tradeoff between their own consumption today and their children's future consumption. If human capital investments become more efficient, then for a given inequality of investments in children of high and low earners, the inequality of earnings outcomes will increase. This results in higher intergenerational earnings persistence. In Section 3 below, I illustrate this mechanism with a simple model. Tax codes are also plausible explanations for the cross-country differences in earnings persistence, as they affect the incentives to invest in human capital. If taxes are progressive, it will have the effect that human capital investments become less attractive, particularly for someone with high ability. This will shrink the dispersion of human capital investments and cause smaller earnings persistence. In Section 2, I document a negative correlation between tax progressivity and earnings persistence.

If there are diminishing returns to human capital investments, and investments made by parents and the government are substitutes, then a parent's incentive to

[3] See Fernandez, Guner, and Knowles (2005)

invest will be falling as the government invests more. As the government invests more, the difference between how much is invested in rich and poor children becomes smaller and earnings persistence will fall. Western economies differ with respect to public education expenditure. As I document in Section 2, the countries with low earnings persistence tend to spend more on public investments in education relative to GDP per capita. The difference is particularly large when it comes to spending on tertiary education.

Finally, one potential cause of earnings persistence that has received much attention in the literature is the presence of credit constraints. As mentioned above, there will usually be a direct relationship between parents' and childrens earnings. This will be true even if the parents are not credit-constrained with respect to their own resources, and if human capital investments are risky it may also be true even if they are not credit constrained with respect to their children's future earnings. A stronger relationship may, however, occur if low earners with high ability/endowment children face binding credit-constraints with respect to investing in their childrens human capital. One potential source of cross-country differences in earnings persistence is the degree of credit market completeness. I do not have any good measure of credit market completeness across countries, but if the government heavily subsidizes education, it should reduce the number of credit-constrained parents. In my structural model below, I do, however, find that increasing or decreasing borrowing limits has very little quantitative impact on earnings persistence in the US.

Empirical Literature

The most commonly used measure of earnings persistence is the coefficient, often denoted , from the regression of the logarithm of the sons earnings on the logarithm of the fathers earnings and a constant, also called the intergenerational elasticity of earnings:

$$\log(y_{son}) = \alpha + \beta \log(y_{father}) + \epsilon \tag{1}$$

The relevant measure of earnings is lifetime or permanent earnings, but as this measure is rarely available, the best a researcher can do is often to average several years of earnings and control for the age at which the earnings were observed. What β tells us, in a purely statistical sense, is what percentage of a fathers earnings advantage, relative to the mean in his generation, that is on average transferred to the son. A β of 0 would represent the case in which the earnings of fathers and sons are completely unrelated, while a β of 1 would represent the case in which the earnings advantage of the father is perfectly transferred to the son. Hypothetically, one can also imagine β smaller than 0 or greater than 1. In practice, however, empirical studies have found β between 0 and 1, which implies that earnings tend to revert to the mean over generations.

The statistical literature, which estimates and compares the intergenerational elasticity of earnings for different countries, is by now quite large. Blanden (2009) provides a thorough discussion. There are some difficulties related to methodology and data, which makes it harder to compare different studies (see Appendix 10.1). It is, however, clear that there are substantial differences between countries. Corak (2006) provides a meta study based on previous empirical studies of earnings persistence in different countries and current knowledge of data and methodological issues. Table 1 reproduces the main findings of his study, supplemented with two recent studies from Italy and Spain. It documents the pattern with relatively high earnings persistence in the US, Britain, and Southern Europe, and relatively low earnings persistence in Northern Europe and in Canada.

Quantitative Literature

In addition to the empirical work, there is also a theoretical literature, pioneered by Becker and Tomes, which gives us a framework for understanding the factors that may affect the correlation of childrens and parents earnings. The quantitative/structural literature, which takes models to the data, is, however, very sparse. I will briefly

mention the two papers that are closest in spirit to the work I am undertaking.

Han and Mulligan (2001) develop a very simple two-period/two-generation model in which parents care about their children and have the opportunity to invest in their human capital and to give them monetary bequests. They calibrate their model to fit characteristics of the US economy, including the intergenerational elasticity of earnings, β, which they take to be 0.4. They then study how β changes as they eliminate intergenerational borrowing constraints and increase the variance of shocks to ability. The authors conclude that eliminating borrowing constraints reduces β by up to 0.1, but they also find that β increases as the heterogeneity of family endowments increases. They suggest that if there is a greater variance of family endowments in the US and Britain, perhaps because those countries are more racially and culturally diverse, then this result could be used to explain higher earnings persistence in those countries. However, it is not an obvious result or theoretical implication that a larger variance of family endowments should lead to larger and not smaller persistence. This is something that comes out of their specific model for specific parameter values. It should also be noted that in their model agents experience the same shocks to human capital and financial assets. It is therefore no insurance in holding both assets. An individual will invest in human capital until the return equals the return on financial assets, and if needed borrow financial assets to achieve this level of human capital investment. This may increase the importance of borrowing constraints.

Restuccia and Urrutia (2004) develop a model with infinite dynasties in which agents live for four periods: two as children and two as adults. Parents decide how much to invest in their childrens elementary education and whether to send them to college. There is also a government that imposes taxes, runs a balanced budget, and invests the tax revenues in education. The focus of the paper is to determine whether investments in early or college education are quantitatively more

important for earnings persistence. They find that early education matters more and that government investments in early education have a much greater impact than government investments in college education.

My paper is the first to study the impact of cross-country differences in policies on β. It turns out that across countries there is greater variation in spending on tertiary education than on early education. Tertiary education spending therefore seems like a more likely explanation for cross-country differences in β. My paper also offers a richer, more realistic model, combining some elements that are present in each of the two papers above. In Section 5, I discuss the different model elements in detail and why they are important in a study of earnings persistence.

The remainder of the paper is organized as follows: In Section 2, I document the correlation between and tax progressivity and between and spending on tertiary education. Section 3 studies the impact of taxation and public investment in education on in a simple analytical model. Section 4 presents the quantitative model. In Section 5, I discuss and justify some of the modeling choices. Section 6 discusses data and calibration. Section 7 decomposes the contributions to earnings persistence from the different model elements. Section 8 presents results from policy experiments. Section 9 concludes.

2 Correlations Between Earnings Persistence and Tax Progressivity and Earnings Persistence and Public Spending on Tertiary Education

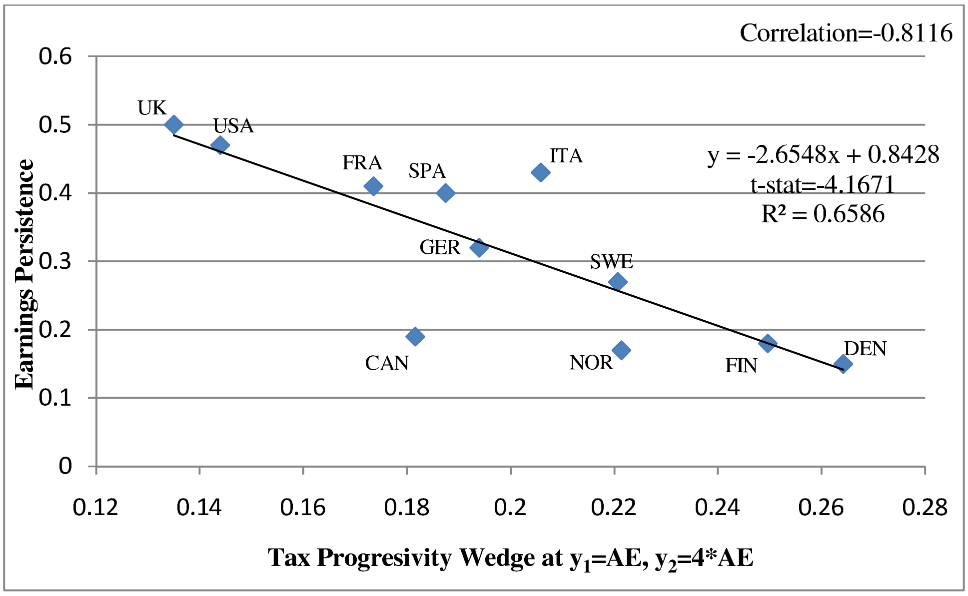
It is difficult to summarize the tax system in a country with just one number. A commonly used measure of tax progressivity is so-called progressivity wedges; see,

for instance, Guvenen, Kuruscu, and Ozkan (2009):

$$PW(y_1, y_2) = 1 - \frac{1 - \tau(y_2)}{1 - \tau(y_1)} \tag{2}$$

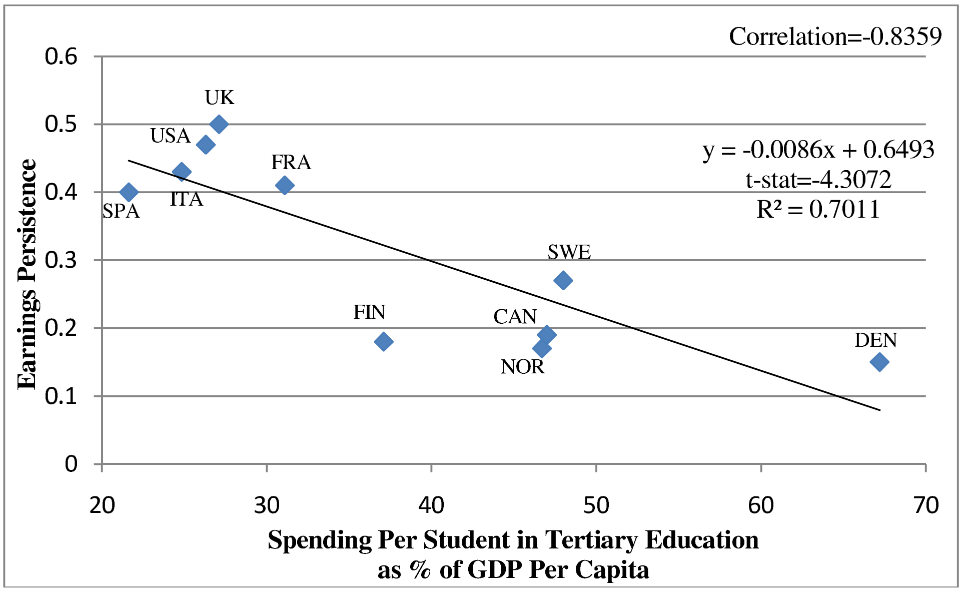
This measure says something about how fast the tax rate increases as earnings increase from y_1 to y_2. If there is a flat tax, then the progressivity wedge would be zero for all levels of y_1 and y_2. For each country in Table 1, I use labor income tax data from the OECD tax database to fit a tax function; see Appendix 10.2 for a detailed description. I then construct progressivity wedges using the average tax rate. I use the average earnings, AE, in each country for y_1 and four times average earnings for y_1. In Figure 1, I plot earnings persistence on the y-axis against this measure of tax progressivity on the x-axis. The correlation between the two quantities is -0.81 and the regression coefficient is highly significant when earnings persistence is regressed on the progressivity wedges. A strong correlation between two variables need not imply, of course, that one has a causal effect on the other. However, this empirical observation motivates a further investigation of the impact of taxes on earnings persistence in a structural model with careful modeling of the tax systems. In Figure 2, I plot the correlation between earnings persistence and public expenditure per student in tertiary education as a fraction of GDP per capita. The correlation between the two variables is -0.84, and the regression coefficient is highly significant when earnings persistence is regressed on education expenditure.

Figure 1: Correlation Between Tax Progressivity and Earnings Persistence



Earnings persistence from Table 1. The tax data is an average of the years 2001-2005, taken from the OECD Tax and Benefit Calculator and the OECD Tax Database. The regression coefficient is significant at the 1% level.

Figure 2: Correlation Between Public Expenditure on Tertiary Education and Earnings Persistence



Earnings persistence from Table 1. The education spending data are an average of the years 1999-2005, taken from the UNESCO Institute for Statistics. The regression coefficient is significant at the 1% level.

3 Gaining Intuition: The Impact of Taxation and Public Education Expenditure on Intergenerational Earnings Persistence in a Simple Model

To obtain an intuitive understanding of how taxation and public education expenditure qualitatively affect earnings persistence, it may be helpful to start with a simple model. The model is a slight modification of Solon (2004), where I have changed the wage function and the process for inheritance of abilities to be similar to the wage function and the process for inheritance of abilities in the quantitative model of Section 4. Assume that there is a continuum of infinitely lived single individual dynasties. Each individual lives for two periods: one as a child and one as an adult. Parents decide how much to consume and how much to invest in their children's human capital, while children do not make any economic decisions. A parent's utility is a function of today's consumption, c_t , and his child's future earnings, y_{t+1}:

$$U_t(c_t, y_{t+1}) = \log(c_t) + \alpha \log(y_{t+1}) \tag{3}$$

The parameter α measures how altruistic parents are with respect to their children. The earnings of the child are determined by his level of human capital. Human capital is a function of investments made by the parents, I_t, investments made by the government, I_g, and of the childs ability or family endowment, A_t:

$$y_{t+1} = \gamma h_{t+1} \tag{4}$$

$$h_{t+1} = A_{t+1}(I_t + I_g)^{\psi} \tag{5}$$

Abilities are imperfectly transmitted from parent to child. I assume them to be log-normally distributed, and follow an AR(1)-process:

$$\log(A_{t+1}) = \theta \log(A_t) + \nu, \qquad \nu \sim N(0, \sigma_\nu^2) \tag{6}$$

Assuming that labor income is taxed at rate τ, the utility maximization problem of a parent can now be written as:

$$\begin{aligned} \max_{c_t>0, I_t\geq 0} \log(c_t) &+ \alpha \log(y_{t+1}) \\ s.t.: \quad c_t + I_t &= y_t(1-\tau) \\ y_{t+1} &= \gamma A_{t+1}(I_t + I_g)^\psi \end{aligned} \tag{7}$$

Substituting for c_t, and y_{t+1}, gives a maximization problem in I_t:

$$\max_{0\leq I_t<y_t} \log(y_t(1-\tau) - I_t) + \alpha\psi \log(I_t + I_g) + \alpha \log(A_{t+1}) + \alpha \log(\gamma) \tag{8}$$

The first-order condition is:

$$\begin{aligned} &\frac{-1}{y_t(1-\tau) - I_t} + \frac{\alpha\psi}{I_t + I_g} \leq 0 \\ &\frac{-1}{y_t(1-\tau) - I_t} + \frac{\alpha\psi}{I_t + I_g} = 0, \quad if \quad I_t > 0 \end{aligned} \tag{9}$$

Rearranging this expression we get the following solution for I_t:

$$I_t = \begin{cases} \frac{\alpha\psi}{1+\alpha\psi} y_t(1-\tau) - \frac{1}{1+\alpha\psi} I_g, & if \quad y_t > \frac{I_g}{\alpha\psi(1-\tau)} \\ 0, & else \end{cases} \tag{10}$$

As long as there is an interior solution, I_t is decreasing in the tax rate, τ, decreasing in government investment, I_g, increasing with the altruism parameter, α, and increasing in the human capital production function parameter, ψ. Substituting for I_t in 5 and

taking the log of 4, we get an equation relating the log of the earnings of children to the earnings of their parents:

$$\log(y_{t+1}) = \begin{cases} \psi \log(y_t(1-\tau) + I_g) + \log(\theta A_t + \nu) + \log\left(\gamma(\frac{\alpha\psi}{1+\alpha\psi})^{\psi}\right), & if \quad y_t > \frac{I_g}{\alpha\psi(1-\tau)} \\ \psi \log(I_g) + \log(\theta A_t + \nu) + \log\left(\gamma(\frac{\alpha\psi}{1+\alpha\psi})^{\psi}\right), & else \end{cases} \tag{11}$$

Proposition 3.1.

$$If \quad y_t > \frac{I_g}{\alpha\psi(1-\tau)} \quad and \quad I_g > 0 \quad then$$
$$\frac{\partial^2 \log(y_{t+1})}{\partial \log(y_t)\partial\tau} < 0, \quad \frac{\partial^2 \log(y_{t+1})}{\partial \log(y_t)\partial I_g} < 0, \quad \frac{\partial^2 \log(y_{t+1})}{\partial \log(y_t)\partial\psi} > 0 \tag{12}$$

Proof: See Appendix 10.3

Proposition 3.1 states that as long as both the parental investment and the government investment are positive, the impact of the parent's earnings on the child's earnings become smaller when there is a higher tax level or more government investment, or when human capital production is more efficient. In the case of the tax, this happens because a smaller share of the parent's earnings can be devoted to investing in human capital when the tax is higher. Government investment, which is equal for all children, then accounts for a larger share of the total human capital investment, and a given percentage change, or a change in the log, of parental earnings will have a smaller impact on the log of the child's earnings. However, if government investments were zero, then the flat tax could be separated out as a constant term. When government investment increases, it has the same effect as when the tax increases. The relative importance of parental earnings is decreasing when I_g increases. The impact of parental earnings on the child's earnings is increasing in the human capital production function parameter, ψ. This is simply because an increase

in ψ increases the effect of parental investments. The equation usually estimated by empirical researchers studying intergenerational earnings persistence is:

$$\log(y_{it+1}) = \alpha + \beta \log(y_{it}) + \epsilon_{it+1} \tag{13}$$

where i denotes the family or dynasty. If we assume that the government invests a constant fraction of average earnings in education, $I_g = \tilde{I}_g \bar{y}$, and that $y_{it} > \frac{I_g}{\alpha\psi(1-\tau)}$ $\forall$ i, which implies that all parents invest a positive amount in their child's human capital, we only have to consider the first part of equation 11. Let us also assume that the economy is in steady state; i.e., the cross-sectional distributions of $\log(y_{it+1})$ and $\log(y_{it})$ are identical. With the purpose of obtaining an analytical solution for the regression coefficient, β, we can log-linearize the first part of 11 around average earnings, $\bar{y}$, and average ability, $\bar{A}$:

$$\begin{aligned} \log(y_{it+1}) &= \alpha^* + \frac{\psi(1-\tau)}{(1-\tau)+\tilde{I}_g} \log(y_{it}) + \log(A_{it+1}) \\ \textit{where}\ \ \alpha^* &= \psi \log\Big(\bar{y}\big((1-\tau)+\tilde{I}_g\big)\Big) + \log\left(\gamma(\frac{\alpha\psi}{1+\alpha\psi})^{\psi}\right) - \frac{\psi(1-\tau)}{(1-\tau)+\tilde{I}_g}\bar{y} \end{aligned} \tag{14}$$

Equation 14 now resembles the classical linear regression equation in 13, except that the error term, $\log(A_{it+1})$, is correlated with the explanatory variable, $\log(y_{it})$. This is because both $\log(A_{it+1})$ and $\log(y_{it})$ depend on $\log(A_{it})$. OLS estimates of the slope will therefore be biased. Equation 14 is a first-order autoregression where the error term follows the AR(1)-process as in 6. It is shown in Greene (2000), pp. 534-535, that when $Var\big(\log(y_{it+1})\big) = Var\big(\log(y_{it})\big)$ the probability limit of the OLS-estimator for the slope coefficient in this equation is given by the sum of the true slope coefficient and the autoregressive parameter of the error term divided by one plus their product. Using this result we get that in the population regression

where 14 is estimated by OLS:

$$\beta = \frac{(\psi + \theta)(1 - \tau) + \theta \tilde{I}_g}{(1 + \psi\theta)(1 - \tau) + \tilde{I}_g} \tag{15}$$

Proposition 3.2.

$$\frac{\partial \beta}{\partial \tau} < 0, \quad \frac{\partial \beta}{\partial \tilde{I}_g} < 0, \quad \frac{\partial \beta}{\partial \psi} > 0, \quad \frac{\partial \beta}{\partial \theta} > 0 \tag{16}$$

Proof: See Appendix 10.3

Thus, in this simple model, we have seen that an increase in the level of taxes and/or government investment in education reduces earnings persistence by reducing the direct impact of parental earnings on the child's earnings (Proposition 3.1). The intuition behind the result is that the relative importance of parental investments compared to government investments decreases. The difference between how much is invested in rich and poor children becomes smaller in percent/log terms as taxes or government investments increase, and this leads to a fall in earnings persistence. β is, not surprisingly, increasing in the correlation of parent's and child's ability, θ. It is also increasing in the human capital production function parameter, ψ. It should be noted that the relationship between the market return to human capital, γ, and β generally is sensitive to the specification of the wage function. I have specified a constant return to a unit of human capital, and γ does not enter the expression for β. In Solons original model, an exponential return to human capital was specified and γ would then be present in the expression for β.

As long as the tax is flat in this model, there is a linear relationship between human capital investments and parental earnings. The percentage variation of private investments is the same when all parents invest a positive amount and the tax level increases; however, the percentage variation in total investments decreases because private investments are smaller compared to public investments. Introducing a pro-

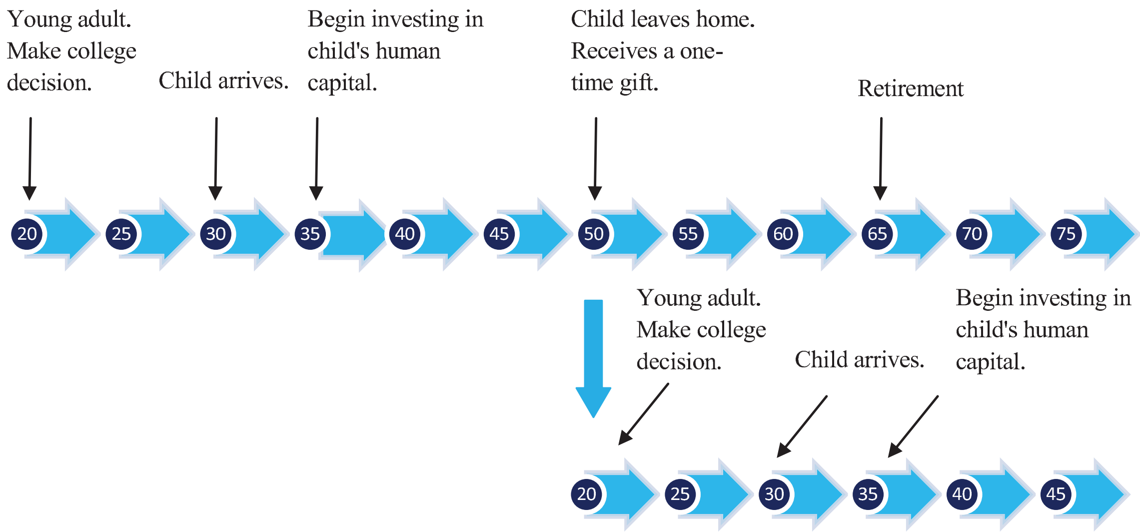
gressive tax may have had the effect of decreasing the percentage variation in private investments, and this would also reduce earnings persistence. We will now turn to the study of a more realistic model with the purpose of quantifying the determinants of earnings persistence.

4 Model

Economic Environment

The economy is populated by single-individual dynasties, where each individual lives for at least 70 years and at most 100 years. A model period is five years. For the first four periods, or 20 years, of his life, an individual is part of the parents household and does not make any economic decisions. At age 20, a young individual moves out of his parent's house and forms his own household. At age 30, he has a child, and at age 65 he retires. The first decision a young adult must make is whether or not to enroll in college. All working age households, including college students, decide how much to work, consume, and save at a risk-free rate. College students also decide how much to invest in human capital production. There is a fixed time cost of attending college, and college students have to work at a low fixed wage, which is independent of their human capital. There is a probability of failing college, depending on the student's ability and prior level of human capital. Households are altruistic and care about their childrens utility. Households with a child, ages 5 to 19, decide how much to invest in the childs human capital. At the moment a child leaves home and begins his own household, the parent has the option of giving him a one-time gift of liquid assets to ensure that he gets a good start in life. This is, of course, a simplifying assumption, but it greatly reduces the complexity of the model. Empirically, the fact that the child receives a one-time gift at the beginning of his adult life can be motivated by the observation that many parents help their child

Figure 3: Household's Life Cycle



with paying for college or with buying a first home. Figure 3 illustrates the life cycle of a household.

Wages and Human Capital

Worker productivity in this economy depends on human capital, college completion, labor market experience, and labor market luck. Since there is no unemployment in the model, experience is equal to potential experience and is fully determined by age and whether a person attended college. Letting x denote the individual's experience level and h denote his level of human capital, his wage can be written:

$$w = h\gamma_0 e^{\gamma_1^j x + \gamma_2^j x^2 + \gamma_3^j x^3 + u} \tag{17}$$

$$u \sim N(0, \sigma_u^2) \tag{18}$$

Where u is an idiosyncratic productivity shock, and $j \in \{0, 1\}$ is an indicator for whether the individual is college educated. There are different age/experience paths for the wages of college- and high-school-educated workers. The human capital of a person must be built up during his childhood and during college. How much human

capital a person accumulates depends on his ability, A, and how much is invested in his human capital in each time period by the parents, I_p, by the individual himself in college, I_s, and by the government, I_g:

$$h' = h + A\big[h(I_p + I_g)\big]^{\psi_0}$$
$$h' = h + A\big[h(I_s + I_g)\big]^{\psi_1} \tag{19}$$

Here h' denotes human capital in the next time period. I follow the tradition in the literature on intergenerational earnings persistence (see Becker and Tomes (1979), Becker and Tomes (1986), and Solon (2004)) and think of human capital investments as investments of money or goods. However, while many definitions of what should be considered human capital investments have been suggested, I will think of it as investment in education. The ability or family endowment of the child is broadly defined to include things that do not have to be bought, like genetics, family culture, motivation, and knowledge acquired from the parents. Abilities are assumed to be log-normally distributed and imperfectly inherited from parent to child according to an AR(1) process:

$$\log(A_c) = \theta \log(A_p) + \nu, \qquad \nu \sim N(0, \sigma_\nu^2) \tag{20}$$

19 is the same functional form as in Ben-Porath (1967), except that Ben-Porath allowed for different exponentials on the human capital and goods inputs. The same production function has been used in some recent studies involving human capital accumulation; see, for instance, Huggett, Ventura, and Yaron (2007), or Ionescu (2009). These studies do, however, ignore the input of goods in the production of human capital and focus on the human capital input, which is modeled as the product of previous human capital and time. They are also different in that they focus on human capital accumulation during work-life and/or college. In my model

the input of time is kept constant, and human capital accumulation starts at age 5. It is known that the efficiency of human capital investments varies by age (see Cunha and Heckman (2007)), and this is the rationale for specifying different technologies before college and in college. One could have used a different technology at every age but this would complicate the model.

Preferences

The momentary utility is a function of consumption in adult equivalents, $\frac{c}{e(t)}$, where $e(t)$ varies depending on whether there is a child in the household, and work hours, n:

$$u(c,n) = \frac{\left(\frac{c}{e(t)}\right)^{1-\sigma}}{1-\sigma} - \chi \frac{n^{1+\eta}}{1+\eta} \tag{21}$$

A household discounts the future by a factor, δ. When the child leaves home, the parent cares about the child's utility, U^c, but discounts it by, α. Thus a household's lifetime utility, U, is given by:

$$U = \sum_{t=1}^{death} \delta^{t-1} u(c,n) + \delta^6 \alpha U^c \tag{22}$$

Borrowing for College and Probability of College Completion

Individuals who attend college are allowed to borrow up to an amount, z, while in college. I require that they do not retire in debt, and in subsequent periods, I let the borrowing constraint, $\phi(j,t)$, be linearly decreasing between college and retirement. High school graduates are not allowed to borrow:

$$\phi(j=1,t) = max\big(0, z(9-t)\big), \quad \phi(j=0,t) = 0 \tag{23}$$

However, if someone took out a loan for college and failed to complete college, they will also be subject to the borrowing constraint for college graduates. The probability of success in college, $\pi(Ah)$, is a function of ability and acquired pre-college human

capital:

$$\pi(Ah) = 1 - e^{\Omega Ah} \tag{24}$$

Recursive Formulation of the Household's Problem

A household can be in five different life stages, and therefore, there are five different household maximization problems. The first decision a young household must make is whether or not to go to college. This is done at age 20, or $t = 1$. In both cases he decides how much to consume, c, next period's capital, k', and how much to work, n. If he goes to college, he must also decide how much to invest in human capital, I_s. The state variables are age, t, capital, k, his level of human capital, h, his ability, A, and the productivity shock, u. In all time periods, experience, x, will be equal to the current model period minus 4 for high-school-educated workers and equal to the current model period minus 5 for college-educated workers. Formally, the individual solves the following Bellman problem:

$$\begin{aligned}
W(k,h,t=1,A,u) &= \max\{V(j=0,\cdot),V(j=1,\cdot)\}, \quad \textit{where}:\\
V(0,k,h,t,A,u) &= \max_{c,n,k'} u(c,n) + \delta E\big[V'(0,k',h,t',A,u')\big]\\
s.t.: \quad c(1+\tau_c)+k' &= k(1+r)+wn\big(1-\tau(wn)\big)\\
c>0,\ \ k' &\geq 0,\ \ 0\leq n\leq 1,\ \ t'=t+1\\
w &= h\gamma_0 e^{\gamma_1^0 x+\gamma_2^0 x^2+\gamma_3^0 x^3+u},\ \ u\sim N(0,\sigma_u^2)\\
V(1,k,h,t,A,u) &= \max_{c,n,k',I_s} u(c,n+\varpi)+\delta\pi(h,A)E\big[V'(1,k',h',t',A,u')\big]\\
&\quad +\delta\big(1-\pi(h,A)\big)E\big[V'(0,k',h,t',A,u')\big]\\
s.t.: \quad c(1+\tau_c)+k' &= k(1+r)+wn\big(1-\tau(wn)\big)-I_s\\
c>0,\ \ I_s\geq 0,\ \ h' &= h+A\big[h(I_s+I_g)\big]^{\psi_1},\ \ k'\geq\phi(1,1),\ \ 0\leq n\leq 1-\varpi\\
w=w_c,\ \ w' &= h\gamma_0 e^{\gamma_1^j x+\gamma_2^j x^2+\gamma_3^j x^3+u},\ \ u\sim N(0,\sigma_u^2),\ \ t'=t+1
\end{aligned} \tag{25}$$

ϖ is here the time cost of attending college, τ_c is a flat consumption tax, and $\tau(wn)$ is a non-linear labor income tax rate. Also note that while in college, an individual must work at the fixed wage, w_c, which is independent of his level of human capital. The problem of a working household without a child and at age 30 when no human capital investments are made is:

$$
\begin{aligned}
V(j,k,h,t,A,u) &= \max_{c,n,k'} u(c,n) + \delta E\big[V'(j,k',h,t',A,u')\big] \qquad (26)\\
s.t.: \quad c(1+\tau_c) + k' &= k(1+r) + w(j,t,h,u)n\big(1-\tau(w(j,t,h,u)n)\big)\\
k' \geq \phi(j,t), \;\; 0 \leq n \leq 1, \;\; c &> 0, \;\; t' = t+1, \;\; for\; t = 2,3,8,9\; (age = 25,30,55,60)
\end{aligned}
$$

At age 30, (20) is also a constraint, as the ability of the child will be revealed in the next period, and the parent must have an expectation of his child's ability. Between ages 35 and 50 the parent must also decide on how much to invest in the child's human capital. He solves:

$$
\begin{aligned}
V(j,k,h_p,h_c,t,A,u) &= \max_{c,n,k',I_p} u(c,n) + \delta E\big[V'(j,k',h_p,h_c',t',A,u')\big]\\
s.t.: \quad c(1+\tau_c) + k' + I_p &= k(1+r) + w(j,t,h,u)n\big(1-\tau(w(j,t,h,u)n)\big)\\
I_p \geq 0, \;\; h_c' &= h_c + A\big[h_c(I_p+I_g)\big]^{\psi_0}, \;\; k' \geq \phi(j,t)\\
0 \leq n \leq 1, \;\; c > 0, \;\; t' &= t+1, \;\; for\; 4 \leq t \leq 6\; (35 \leq age \leq 50) \qquad (27)
\end{aligned}
$$

h_p here denotes the human capital of the parent, and h_c denotes the human capital of the child. The parent must keep track of both as state variables. A is now the ability of the child. There is no reason for the parent to know his own ability after the childs ability is revealed. When the parent is age 50 and the child is age 20, the child leaves the household and the parent has a one-time opportunity to give him a

gift or an inter vivos transfer, b. The parent's problem is:

$$
\begin{aligned}
V(j,k,h_p,h_c,t=7,A,u) &= \max_{c,n,k',b} u(c,n) + \delta E\big[V_p'(j,k',h_p,h_c',t=8,u_p')\big] \\
&\quad +\alpha E\big[W_c(b,h_c,t=1,A,u_c)\big] \\
s.t.: \quad c(1+\tau_c)+k'+b &= k(1+r)+w(j,t,h,u)n\big(1-\tau(w(j,t,h,u)n)\big) \\
k'\geq \phi(j,t), \quad c &> 0, \quad 0\leq n\leq 1, \quad b\geq 0, \quad t'=t+1 \qquad (28)
\end{aligned}
$$

α here controls the parent's degree of altruism. I assume that the parent does not observe the child's idiosyncratic shock before the size of the gift is decided. He must, therefore, take the expectation of the child's value function with respect to the idiosyncratic shock. A household in retirement simply solves:

$$
\begin{aligned}
V(j,k,h,t,A,u) &= \max_{c>0,k'\geq 0} u(c,n=0)+\delta\Gamma(t)E\big[V'(k',t')\big] \\
s.t.: \quad c(1+\tau_c)+k' &= k(1+r)+T \\
for\ 10\leq t &\leq 16 \ \ (65\leq age\leq 95) \qquad (29)
\end{aligned}
$$

T here is a constant amount of social security, and $\Gamma(t)$ is an age-dependent probability of survival to the next period.

5 Discussion of Modeling Choices

Life-Cycle Model with College Decision

Using a life-cycle model with college decision allows us to study government expenditure on different levels of education. We can separate the effects of spending on primary, secondary, and tertiary education. The cross-country variation in public education expenditure is largest for tertiary education. Another argument for using a life-cycle model is that when studying the impact of parents' earnings on the earnings of children, we are interested in the financial resources available to parents

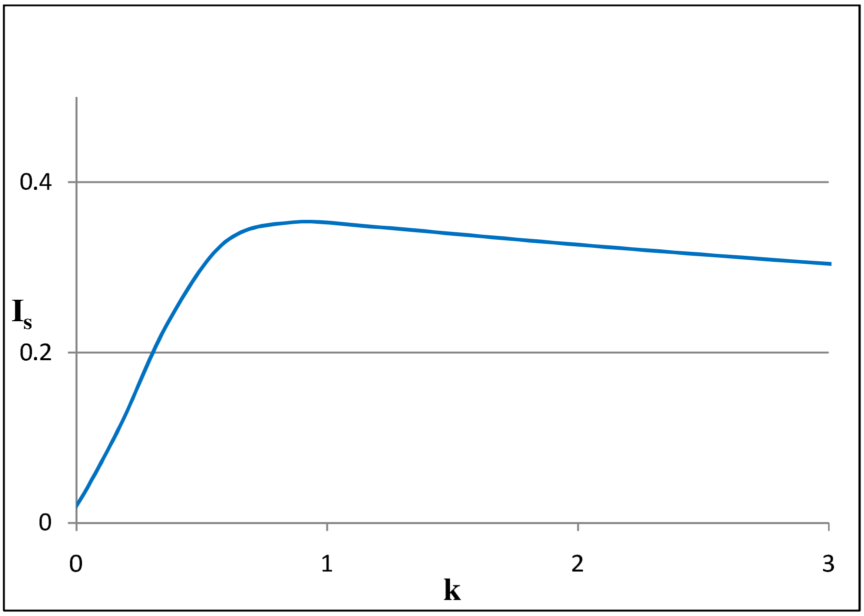
at the time when there are children in the household. There is a literature documenting that even after controlling for parents' lifetime income, the income of the parents during the childhood years matters for the children's income; see Cunha and Heckman (2007) for a survey.

Physical Capital, Inter Vivos Transfers, and Human Capital

I will argue that in a realistic quantitative model developed to study intergenerational earnings persistence, it is important to have financial assets and a mechanism for transfers from parent to child, in addition to human capital. The existence of physical capital in the model affects how much is invested in a child's human capital in various ways. In a model without financial assets, parents will divide their resources between their own consumption today and their children's future consumption or, equivalently, their children's human capital. This may create a too strong correlation between the earnings of the parent and the child's human capital, as the optimal investment in the child will always be increasing in the earnings of the parent. If there is physical capital and diminishing returns to human capital investments, there will be a point at which the return on capital is strictly higher than the return on human capital, and this will put a cap on human capital investments.

Children with low ability but rich parents will earn a lot more in a world with no financial assets, because the only way to help them is to invest in their human capital. With physical capital, their parents will rather give them some financial assets. Furthermore, since there is uncertainty in the model, parents would like to accumulate some physical capital to insure against negative shocks, even when the expected return on human capital investments is higher than the return on physical capital. This will take resources away from human capital investments. Finally, a popular explanation both for earnings persistence (see, for instance, Han and Mulligan (2001)), and for college enrollment in the literature is the existence of

Figure 4: Human Capital Investment for a Model College Student



borrowing constraints. To study the impact of borrowing constraints, it is crucial that the model have financial assets.

Labor Supply

Allowing agents in the model to choose their work hours affects the returns to human capital investments and will be important for the shape of the optimal investment policy as a function of physical capital. In Figure 4, I illustrate this point by plotting the optimal investment in human capital for an individual in college. As can be seen from the figure, the optimal investment peaks at some point and starts sloping downwards. This is because, as the agent becomes wealthier, he will enjoy more leisure in the future and the returns to investing in human capital are falling. Some families accumulate a lot of physical capital, but the fact that they enjoy leisure and can control their labor supply will affect the shape of their optimal human capital investments.

Labor supply is also potentially important for college enrollment and for the importance of borrowing constraints with respect to human capital investments; see

Garriga and Keightley (2007), and Keane and Wolpin (2001). If a poor person cannot borrow to invest in his child, he may choose to compensate by working a bit more. Equivalently, if a college student cannot borrow, he may choose to take a part-time job. Having labor choice in the model reduces the importance of borrowing constraints. If a college student has no other way of raising money than borrowing, then borrowing constraints are more likely to be important.

6 Calibration

Many of the parameters can be obtained without solving the model. I calibrate 27 model parameters to their empirical counterparts. The remaining 11 parameters are estimated jointly using an exactly identified simulated method of moments approach. Tables 2 and 3 summarize the parameters calibrated outside and inside the model. The main source of data for the estimated parameters, 6 out of the 11 data moments, is employed males from the PSID (1999-2005). I use employed males because most of the literature on intergenerational earnings persistence is based on the relationship between father and son, and the analysis is carried out on working individuals. In addition there is no unemployment in my model. I use the years 1999-2005 because these are the years for which I also have data on education spending and taxes. Below I describe the data used in the calibration of each parameter as well as the estimation approach.

Preferences

The momentary utility function is the standard CRRA utility function in 21, with consumption measured in adult equivalents, $\left(\frac{c}{e(t)}\right)$. I use the so-called ÖECD-modifiedädult equivalence scale and set $e(t) = 1.3$ when there is a child in the household, and $e(t) = 1.0$ when there is not. Consistent with a survey of the empirical literature in Browning, Hansen, and Heckman (1999), I set the coefficient of

Table 2: Parameters Calibrated Outside of the Model

Parameter	Value	Description	Target
r	0.011	Risk free interest rate (annual)	3-mnth T-bill minus inflation (1947-2008)
σ	2	$u(c,n) = \frac{\left(\frac{c}{e(t)}\right)^{1-\sigma}}{1-\sigma} - \chi\frac{n^{1+\eta}}{1+\eta}$	Browning et al. (1999)
η	3		
e	1.0 or 1.3		OECD-modified equivalence scale.
γ_1^0	0.221	$w = h\gamma_0 e^{\gamma_1^j x + \gamma_2^j x^2 + \gamma_3^j x^3 + u}$	PSID (1968-1997)
γ_2^0	-0.029		
γ_3^0	0.001		
γ_1^1	0.295		
γ_2^1	-0.052		
γ_3^1	0.003		
τ_1	-0.573	$\tau(wn) = \tau_1(wn/AE)^{0.2}$	OECD tax data (2001-2005)
τ_2	1.706	$+\tau_2(wn/AE)^{0.4} + \tau_3(wn/AE)^{0.6}$	
τ_3	-1.096	$+\tau_4(wn/AE)^{0.8}$	
τ_4	0.221		
τ_c	0.084	Consumption tax	Vertex Inc. (2002)
ϖ	0.110	Time spent studying in college	American Time Use Survey
w_c	\$11.14/h	Wage rate in college	CPS (1999-2005)
$I_g(t)$	Primary: \$4522	Public spending per student	UNESCO (1999-2005)
	Secondary: \$5295		
	Tertiary: \$10672		
z	\$24856	College borrowing limit	Lochner and Monge-Narajano (2008)
T	\$13094	Old age Social Security	Social Security Administration (1999-2005)
$\Gamma(t)$	Varies	Death probabilities	NCHS (1991-2001)

Dollar amounts in annual 2005 dollars.

relative risk aversion, σ, equal to 2, and the inverse of the Frisch elasticity of labor supply, η, equal to 3. The elasticity of substitution between consumption and labor, χ, the time discount factor, δ, and the altruism parameter, α, are among the estimated parameters. The corresponding data moments are average hours worked for employed males 25-64, asset holdings of employed males 50-54, and asset holdings of employed males 25-29 in the PSID (1999-2005). Consistent with the American Time Use Survey (2003), I assume that the day has 15 hours not needed for personal care and normalize hours so that working 15 hours per day is equivalent to a labor supply of 1 in the model.

Risk-Free Interest Rate

Given the partial equilibrium nature of the model, I take the risk-free rate as fixed and calibrate it using data. I set the risk-free rate equal to the average of 3-month T-bill rates minus inflation over the period 1947-2008 based on data from the Federal Reserve Bank of St. Louis.[4]

Wages

I calibrate the life-cycle profile of wages exogenously, using the entire PSID from 1968-2005. I regress wages on model potential experience and control for the year of observation. I estimate different experience paths for college graduates and non-college graduates. For the data moments used in the structural estimation, I use only the years 1999-2005. I take the average wage of college graduates, the average wage of high school graduates, and the variance of log wages as the corresponding data moments to estimate the following parameters: the market return to human capital, γ_0, the starting level of human capital, h_0, and the standard deviation of the idiosyncratic earnings shock, σ_u. In the PSID, individuals are observed only every second year from 1999-2005, while they are observed every year until 1997. To get an estimate of the variance of five-year wages in the time period from 1999-2005, I

[4] Series TB3MS and GDPDEF

assume that the ratio between the variance of five-year and one-year wages in this time period is the same as it was in the period 1991-1997.

Production of Human Capital/Investment in Education

The corresponding data moments to the parameters of the human capital production function, ψ_0, and ψ_1, are private spending on elementary and college education. In addition I must know public spending per student at each level of education, $I_g(t)$. I follow Restuccia and Urrutia (2004) and think of education spending by local governments in primary and secondary education as private spending, while I take state and federal education spending as public spending. The rationale behind this is that local government spending is financed by local taxes and that parents, when they choose which neighborhood to live in, choose the level of local government education spending. Public schools receive both local and state/federal funding, and schools in wealthier neighborhoods have larger budgets due to more local funding; see also Fernandez and Rogerson (1996), Fernandez and Rogerson (1998). In one way, counting all local government spending as parental investment in education may be a strong assumption that leads to a high level of private education spending relative to public spending. On the other hand, defining education spending as the only form of monetary investment that parents make in human capital is very conservative. To construct the relevant calibration targets for each level of education under the above assumption, I use data on public expenditure per student as fraction of GDP per capita from the UNESCO Institute for Statistics (1999-2005), and data on private expenditure as a fraction of total expendiure, as well as local government's share of public expenditure from the OECD (1999-2005).

Intergenerational Correlation of Ability

The intergenerational correlation of ability, θ, obviously has an impact on the intergenerational persistence of earnings, and I use that as the calibration target for this parameter. I obtain the value of 0.47 for the intergenerational earnings persistence

Table 3: Parameters Estimated Endogenously

Parameter	Value	Description	Data Moment
γ_0	0.372	$w = h\gamma_0 e^{\gamma_1^j x + \gamma_2^j x^2 + \gamma_3^j x^3 + u}$	$\bar{w}$, skilled workers
h_0	0.467	Starting level of human capital	$\bar{w}$, unskilled workers
ψ_0	0.300	$h' = h + A\big(hI\big)^{\psi_0}$, before college	$\bar{I}_p$, elementary school
ψ_1	0.881	$h' = h + A\big(hI\big)^{\psi_1}$, in college	$\bar{I}_s$, in college
σ_u	0.398	$u \sim N(0, \sigma_\nu^2)$	Std. dev. of $\log(w)$
θ	0.332	$log(A_c) = \theta \log(A_p) + \nu$	β
σ_v	0.259	$\nu \sim N(0, \sigma_\nu^2)$	College enrollment
Ω	-0.427	$pi(Ah) = 1 - e^{\Omega Ah}$	College failure rate
α	0.302	Parental altruism	$\bar{k}$, age 25-29
χ	171.2	$u(c,n) = \frac{\left(\frac{c}{e(t)}\right)^{1-\sigma}}{1-\sigma} - \chi\frac{n^{1+\eta}}{1+\eta}$	$\bar{n}$,
δ	1.016	Discount factor	$\bar{k}$, age 50-54

from a meta study by Corak (2006). This also happens to be the same value as found by Grawe (2004), the latest study, using data from the PSID.

Time Spent Studying in College, College Enrolment, Failure, and Borrowing

To calibrate the fixed time cost of attending college, ϖ, I use data from the American Time Use Survey (2004-2008). College students spend, on average, 3.3 hours per day on educational activities on weekdays. I assume that they attend two 13-week semesters per year and that they also study 3.3 hours per day on weekends. While this may be a bit optimistsic, many students also attend summer school. I use college enrollment as the data target for the standard deviation of abilities, σ_ν, and the college failure rate as the target for the parameter Ω, which determines the probability of failing college. I compute these targets from the fraction of males with college degrees in the PSID (1999-2005), and data on college survival probability from the OECD (2000, 2004). I get the college borrowing limit from Lochner and Monge-Narajano (2008). This is the borrowing limit for the federal loan program called Stafford loans, which is what most students are eligible for. There is another loan program called Perkins loans, which can provide further loans to the students

with greatest financial need, but in practice, few students make use of this program. Below I study the effect of relaxing the borrowing constraint.

Taxes

The labor income tax schedule is a polynomial function of an individual's earnings relative to the average earnings, AE:

$$\tau(wn) = \tau_1 \left(\frac{wn}{AE}\right)^{0.2} + \tau_2 \left(\frac{wn}{AE}\right)^{0.4} + \tau_3 \left(\frac{wn}{AE}\right)^{0.6} + \tau_4 \left(\frac{wn}{AE}\right)^{0.8} \tag{30}$$

As described in more detail in Appendix 10.2 I fit this polynomial to labor income tax data from the OECD tax database (2001-2005). These data are constructed by the OECD based on tax laws from different countries. It is well suited for cross-country comparisons; see also Guvenen, Kuruscu, and Ozkan (2009). Coming up with an accurate estimate of consumption taxes in the US is complicated by the fact that there are local county-level taxes in addition to state taxes. Vertex Inc. (a consulting company) estimated that the average consumption tax in the US was 8.4% in 2002. I use that number. For simplicity, I abstract from capital taxes. I do this because different types of capital are taxed differently, and this also differs across countries. Households do, for instance, have about half of their wealth in their homes, wealth that may or may not be taxed. In the US, interest income is taxed as labor income, while dividends and capital gains are subject to capital gains tax. The return on capital is, however, set very conservatively in the calibration. It is set equal to the return on risk-free bonds, which was 1.1% over the past 60 years.

Death Probabilities and Social Security

I assume that all retirees receive the same constant Social Security benefit. I obtain the average benefit for males from the Annual Statistical Supplement to the Social Security Bulletin (1999-2005). I obtain the probability that a retiree will survive to the next period from the National Center for Health Statistics (1991-2001).

Table 4: Estimation Statistics

Statistic	Data	Model
Mean hours worked	0.417	0.417
Mean wages of workers without college degrees	1.000	1.002
Mean wages of workers with college degrees	1.757	1.757
Std. dev. of log(wage)	0.570	0.571
Investment in elementary school	0.038	0.037
Investment in college	0.121	0.120
Fraction of workers enrolling in college	0.588	0.590
Fraction failing college	0.400	0.399
Intergenerational earnings elasticity	0.470	0.470
Mean assets of people ages 25-29	0.092	0.092
Mean assets of people ages 50-54	0.525	0.525

Estimation Method

Eleven model parameters are calibrated using an exactly identified simulated method of moments approach. I minimize the squared percentage deviation of simulated model statistics from the eleven data moments in Table 4. Let $\Sigma = \{\gamma_0, h_0, \psi_0, \psi_1, \sigma_u, \theta, \sigma_\nu, \Omega, \alpha, \chi, \delta\}$ and let $g(\Sigma) = \big(g_1\big(\Sigma\big), ..., g_{11}(\Sigma)\big)'$ denote the vector where $g_i(\Sigma) = \frac{\bar{m}_i - \hat{m}_i(\Sigma)}{\bar{m}_i}$ is the percentage difference between empirical moments and simulated moments. Then:

$$\hat{\Sigma} = \min_{\Sigma} g(\Sigma)' g(\Sigma) \tag{31}$$

Table 3 summarizes the estimated parameter values. As can be seen from Table 4, I get close to matching all of the moments exactly.[5] Because five of the empirical moments have unknown variance, it is not possible to compute any standard errors in this exercise. I set the intergenerational persistence of earnings equal to 0.47 based

[5] The reason that the match is not exact is that the objective function which I minimize is not continuous. Following Tauchen (1986), I approximate the processes for the shocks to ability and productivity as finite state Markov processes. It turns out that the combination of ability and productivity shock has a non-negligible impact on the college decision. When the parameters are changed, almost everyone with the same combination of ability and productivity shock may change their college decision at the same time. As I increase the number of ability and shock states, the objective function becomes smoother and the estimation fit improves; however, the computational time also increases.

on the meta study by Corak (2006). The moments on investment in early and college education are based on aggregate data from the UNESCO Institute for Statistics.

7 Decomposing Earnings Persistence

There are four main model elements that govern earnings persistence: the process by which abilities are inherited, the variance of idiosyncratic productivity shocks, inter vivos transfers from parent to child, and investments in human capital. Human capital investments are made by parents (individuals in college) and the government. Parental/individual investments and inter vivos transfers will be affected by the size of the government investment, returns to human capital investments, taxation, and borrowing constraints. To quantify how the different model elements affect earnings persistence, I shut them down and reintroduce them in the model one by one. We cannot set human capital investments to zero because everyone would get a zero wage, so we will keep government investments constant, relative to average earnings in the economy, and set parental investments to zero, inter vivos transfers to zero, the correlation of abilities to zero, and the variance of the idiosyncratic shock to zero. Then we will start reintroducing these elements in the model; see Table 5. I also keep the variance of the shocks to the log of abilities, σ_{ν}, constant in this exercise.

The main conclusion from Table 5 is that both parental/individual investments and correlation of abilities make significant positive contributions to intergenerational earnings persistence. The link between earnings persistence and private human capital investments comes from the fact that the optimal parental/individual human capital investment policy functions are usually upward sloping in financial resources; the exception is for very wealthy individuals.[6] The intergenerational earnings elasticity falls to approximately zero when all four model elements are left out. The reason

[6]Figure 4 displays an example investment policy function for a model college student. In the simulated model, almost all individuals would be on the upward sloping part of the graph.

Table 5: Earnings Persistence with Different Model Elements Present

Earnings persistence	Correlated abilities	Idiosyncratic shocks	Private investments	Inter vivos transfers
0.002				
0.314	X			
0.000		X		
0.256			X	
-0.017				X
0.180	X	X		
0.510	X		X	
0.304	X			X
0.222		X	X	
-0.030		X		X
0.249			X	X
0.428	X	X	X	
0.180	X	X		X
0.544	X		X	X
0.215		X	X	X
0.470	X	X	X	X

it is not exactly zero is that I approximate the continuous AR(1)-process for abilities by finite state Markov processes, as proposed by Tauchen (1986), when simulating the model.[7] This leads to slight inaccuracies, which become smaller as one increases the number of states. Introducing correlated abilities leads to an intergenerational earnings elasticity of 0.314. One might have expected it to be equal to the correlation of the log of abilities, 0.332, but there is a nonlinear relationship between ability and earnings. Having parental/individual investments alone in the model gives an earnings elasticity of 0.256.

When all model elements are present, the effect of leaving out inter vivos transfers is to reduce the intergenerational earnings elasticity from 0.47 to 0.428. Inter vivos transfers affect intergenerational earnings persistence in three ways. The absence of transfers limits the ability of children with rich parents to invest in college education, and this would negatively impact earnings persistence. Another effect is that if there are no inter vivos transfers, the only way a wealthy parent with a low ability child

[7]See Appendix 10.4 for details on computation.

can help the child is to invest more in human capital. This will reduce the dispersion of investments and reduce earnings persistence. However, introducing inter vivos transfers alone in the model yields a negative intergenerational earnings elasticity. This is because of the negative income effect on labor supply. Children of high earners get larger transfers and work less, which causes a negative correlation between the earnings of parents and children.

With all elements present in the model, removing the idiosyncratic shocks causes the intergenerational earnings elasticity to increase from 0.47 to 0.544. The effect of introducing idiosyncratic wage shocks in the model is generally to reduce earnings persistence. This is because the shocks are random and not correlated across generations, like abilities and investments in human capital. However, there is an exception when only inter vivos transfers are present in the model. Introducing shocks that are log-normally distributed around zero has the effect of making the society richer and causing parents to give larger transfers. In the case with only inter vivos transfers present, larger transfers lead to a stronger negative correlation between the earnings of parents and children.

The case when all model elements are present except private investments in human capital is particularly interesting. The intergenerational earnings elasticity is then 0.184, or about the same as in Scandinavian countries. In the context of the present model, we would need policy reforms that completely eliminate all private human capital investments to reach the same earnings persistence as in Scandinavia. This may imply that factors other than just policy impact cross-country differences in earnings persistence. Some of these factors may be captured by the correlation of abilities/family endowments. However, one shortcoming of the present model is that there is no explicit modeling of the supply of educational service. It may be realistic to assume that the human capital production function would change as the demand for education changes, and that this would impact the results.

8 Policy Experiments

In Section 2, I documented a strong cross-country correlation between intergenerational earnings persistence and tax progressivity and intergenerational earnings persistence and public spending on tertiary education. This motivates the study, in this section, of the contributions of differences in country policies to differences in earnings persistence. I also study the impact of relaxing and tightening the borrowing constraints. When I perform the policy experiments, I keep public education expenditure and taxes as functions of average earnings in the economy. In this way if the society becomes richer or poorer because of a policy change, education expenditure and taxes will adjust accordingly. Since there is no public good in the model, I do not keep a balanced government budget and excess tax revenues are assumed to finance bureaucracy.[8]

The Impact of Taxation and Public Education Expenditure

Out of the countries in Table 1, Denmark has the highest and most progressive taxes and they spend the most on tertiary education (see Figures 1 and 2). Denmark is also the country with the lowest earnings persistence. I therefore study how earnings persistence in my model economy, which is calibrated to US data, changes as I introduce Danish policies. I think of the change in earnings persistence due to the introduction of Danish policies as being in the upper range of how much of cross-country differences can be explained by policies, since the effect of introducing policies from any other country will be smaller. Table 6 displays how selected model statistics change in the policy experiments.

As can be seen from row 4 of Table 6, the greatest reduction in intergenerational earnings persistence comes from introducing a Danish tax system in the US.

[8] In Appendix 10.6, I relax this assumption in the sense that when I perform the policy experiments I redistribute the net change in tax revenues relative to the benchmark model evenly to all households. Redistribution does not have a large impact on the results with respect to intergenerational earnings persistence. It does, however, have a significant impact on labor supply and therefore on average earnings.

Table 6: Policy Experiments

Statistic	Bench-mark	Danish taxes	Danish educ. subsidies	Danish subsidies + taxes	Tax w. US level, Dan. prg.
Average hours worked	0.417	0.446	0.408	0.440	0.413
Std. dev. of log wages	0.571	0.499	0.632	0.550	0.520
Fraction enrolling in college	0.590	0.511	0.890	0.832	0.507
Intergen. earnings elasticity	**0.470**	**0.350**	**0.434**	**0.351**	**0.423**
Average human capital inv. age 5-9	$3998	$868	$5710	$1547	$2287
Average human capital inv. age 10-14	$5127	$1310	$7165	$2144	$3002
Average human capital inv. age 14-19	$5752	$1492	$5055	$864	$3337
Average human capital inv. in college	$14692	$1780	$13513	$1881	$6070
Average human capital inv. (all ages)	$5016	$1041	$6200	$1288	$2596
Average gift from parent to child	$78714	$10333	$128269	$21193	$26581
$\frac{tax\ per\ worker - educ.\ expenditure}{benchmark\ average\ earnings}$	0.343	0.546	0.379	0.599	0.318
Average Earnings	$61111	$53489	$71474	$60883	$53539
$\frac{\bar{I}_{private}}{\bar{I}_{total}}$	0.525	0.215	0.417	0.156	0.403
$Stdev\left(\frac{I_{private}-\bar{I}_{private}}{\bar{I}_{private}}\right)$	2.240	2.033	2.250	2.234	2.033
$Corr\big(college, \log\left(y_{parent}\right)\big)$	0.1939	0.1367	0.1572	0.1412	0.1705

Column 2 displays the results when introducing a Danish tax system into the model. Column 3 shows the resultswhen introducing Danish public education expenditure policies. Column 4 shows the results when introducing Danish taxes and education spending at the same time. Column 5 displays the results from introducing a tax system with the US average tax rate but with Danish progressivity. The dollar amounts are in annual 2005 dollars.

Introducing a Danish tax system in the US reduces the intergenerational earnings elasticity by 12 percentage points, to 0.35, or about 40% of the difference between the US and the Scandinavian countries; see Table 1. The higher and more progressive taxes greatly reduce the incentives for private investment in education, and this leads to lower earnings persistence. We observe that higher and more progressive taxes also lead to lower college enrollment and less cross-sectional inequality. A higher tax level has the effect of reducing the levels of private investments and private investments' share of total investments falls. Thus for a given percentage increase in private investments, the percentage increase in total investments is smaller. This weakens the relationship between the parents financial resources and the childs earnings and leads to lower earnings persistence. The effect of more progressive taxes is to disproportionally reduce the incentives for human capital investments for wealthy and/or high-ability individuals. This compresses the distribution of private human

capital investments and leads to lower intergenerational earnings persistence.

To investigate the quantitative impact of tax progressivity versus tax levels on earnings persistence, I impose a tax system with the same average labor income tax rate as in the US but with the same progressivity as in Denmark, as measured by 2.[9] The right column of Table 6 displays the results from this experiment. The intergenerational persistence of earnings is now 0.423. We can interpret this as if about 40% of the difference in earnings persistence between the benchmark economy and the economy with a Danish tax system is due to increased tax progressivity and about 60% is due to the increased tax level. We observe that the percentage variation in private human capital investments is the same in the experiment with Danish taxes and in the experiment with US tax levels and Danish tax progressivity. The difference in earnings persistence between the two experiments is due to the level of private investments relative to public investments.

Introducing a Danish public education expenditure scheme lowers the intergenerational earnings elasticity by 3.6 percentage points, to 0.434. This is explained by increased public expenditure reducing the incentives for parental/individual expenditure on education in relative terms. Total private education expenditure actually increases in absolute terms but this is because the society has became richer, and average earnings have increased by about 17%. Private education expenditure's share of total education expenditure does, however, fall from 53% to 42%.

Secondary and tertiary private education spending decreases with Danish public expenditure, while private spending on elementary education increases. This is because the Danish public investments are very large for tertiary and secondary education (see Table 7) and at about the same level as in the US for elementary education. Therefore, parents move their investments from late to early education. Not surprisingly, greatly increasing public expenditure in tertiary education increases

[9]See Appendix 10.5 for details.

Table 7: Public Education Expenditure Per Student as % of GDP Per Capita

Education level	US	Denmark
Primary	11.1	9.6
Secondary	13.0	19.5
Tertiary	26.3	0.671

Based on data from UNESCO (1999-2005) and OECD (1999-2005)

college enrollment. The correlation between college completion and parental earnings decreases.

Introducing both Danish public education expenditure and taxation at the same time actually increases earnings persistence by 0.01 percentage point, to 0.351, relative to the case with just a Danish tax. There are several competing effects here. On the one hand, private investment in education has become smaller relative to public investment and this should lead to lower earnings persistence, all else being equal. On the other hand, we observe that there has been an increase in the percentage variation in private investments in education. When public education spending increases, more people go to college and the private investment pattern changes. Another effect pointing in the direction of higher earnings persistence is that the society has become richer, and therefore, people invest more in human capital, in addition to the government investing more. When total human capital investments increase, human capital becomes more important for the log of earnings relative to the idiosyncratic shocks.

It is interesting to note that in the experiment with Danish taxes and public education expenditure, average earnings are approximately the same as in the benchmark economy, as public human capital investments have taken on the role of private investments. The average gift from parent to child does, however, drop from $78,714 in the benchmark model to $21,193 in the model with Danish taxes and public education expenditure. A large part of the incentives to give the child a transfer lies in the increased earnings from investment in college. When the incentives for investing

in college are reduced, the transfers from parents to children are also reduced.

We conclude that tax and education spending policies significantly impact earnings persistence. Taxation is quantitatively most important. Whether having low earnings persistence in the society is good or bad is naturally a different question. More high/progressive taxation as a stand-alone policy reduces human capital accumulation and leads to a poorer society, while increased public education expenditure has the opposite effect. Higher taxes may, however, be needed to finance education expenditure. When I introduced Danish education spending, the net change in tax revenues was actually positive. However, the society became richer, and the government only increased its spending on education. I did, for instance, let the Social Security payments stay at their old level. Yet another issue is, of course, general equilibrium effects. I will leave the study of optimal policies to future research.

The Impact of Borrowing Constraints

The importance of borrowing constraints both for intergenerational earnings persistence and college enrollment has received much attention in the literature. In this section, I study the effect of tightening and relaxing the college borrowing constraint, as well as relaxing the assumption that borrowing is allowed only if one attends college. Finally, I allow for negative inter vivos transfers; that is, the parents can pass on debt to their children. Table 8 displays the results from these experiments.

As can be seen from Table 8, relatively large changes to the borrowing constraint have relatively little impact on intergenerational earnings persistence. Completely eliminating borrowing for college reduces college enrollment by 18% and college completion by 14%; however, it is those who have the least to gain from college who drop out. Average earnings in the economy fall only by 1.7%, and intergenerational earnings persistence rises only by 0.4 percentage point. Letting people borrow more has little impact both on earnings persistence and on college enrollment. Human capital investments in college increase slightly and average earnings increase slightly when

Table 8: The Impact of Borrowing Constraints

Statistic	Bench-mark	0X BC	2X BC	2X BC w. o. college	Negative transfers
Fraction enrolling in college	0.590	0.483	0.619	0.611	0.418
Fraction completing college	0.355	0.306	0.371	0.368	0.262
Intergen. earnings elasticity	**0.470**	**0.474**	**0.466**	**0.468**	**0.472**
Average human capital inv. in college	$14692	$14914	$16874	$16715	$17819
Average gift from parent to child	$78714	$77168	$86709	$84367	$31390
Average Earnings	$61111	$60068	$63143	$62916	$60962

Columns 2 and 3 display the results when setting the college borrowing constraint to 0 and doubling the college borrowing constraint, to $49,712. The college borrowing constraint is linearly decreasing between college and retirement. Column 4 displays the results when people that do not attend college are also allowed to borrow up to twice the original college borrowing constraint, or $49,712, in all time periods before retirement. Column 5 displays the results when the borrowing constraint is 2 times the original college borrowing constraint in all time periods prior to retirement and parents are allowed to pass on debt to their children.

more borrowing is allowed. The obvious reason that relaxing the borrowing constraint has little effect on earnings persistence is simply that most individuals are not borrowing constrained from investing in human capital. Most individuals begin to accumulate positive asset holdings at a young age to save for retirement and for their children's college education. Thus, there are no binding constraints stopping them from investing more in human capital. It does, however, turn out that in the benchmark economy, the college borrowing constraint binds for about 30% of those who complete college. However, because it is also possible to work in college, tightening the borrowing constraint will not necessarily lead to large changes in human capital investments. Individuals in college can compensate by working more.

Columns 4 and 5 of Table 8 display the results from experiments in which everyone, not just those who attend college, can borrow up to twice the original college borrowing constraint in all time periods prior to retirement. In column 5, parents are also allowed to give their children negative inter vivos transfers. Allowing for borrowing against children's earnings leads to a very slight increase in intergenerational earnings elasticity, from 0.468 to 0.472, relative to the experiment in column 4 with identical borrowing constraints for parents but only positive transfers to children

allowed. Allowing parents to pass on debt to their children is bad for children with poor parents. Many parents choose to borrow toward their children's earnings. The loan is not used for human capital investments but is rather added to the parents' retirement savings. This leads to a society in which the average holdings of capital are lower and the average transfer from parent to child falls by about $53,000 relative to the experiment in column 4 with identical borrowing constraints for parents but only positive transfers to children allowed. There is a significant drop in college enrollment; however, average earnings decrease only slightly. It is those who would get marginal gains from college who drop out, and those who have large gains from college are able to invest almost the same amount as before. The average human capital investment in college actually increases, but this is because college completion is lower and those who drop out were investing little.

9 Conclusion

In this paper I develop an intergenerational life-cycle model of human capital accumulation and earnings, which features taxation, public education expenditure, borrowing constraints, partially inheritable abilities, inter vivos transfers from parent to child, and idiosyncratic wage shocks as determinants of intergenerational earnings persistence. I calibrate the model to US data and use it to quantify how earnings persistence in the US changes as I introduce policies from Denmark. I find that taxation and public education expenditure have a significant impact on earnings persistence and are likely contributors to the cross-country patterns that empirical researchers have found. Taxation is quantitatively most important. As I introduce a Danish tax system in the US, intergenerational earnings elasticity falls from 0.47 to 0.35, or about 40% of the difference between the US and the Scandinavian countries, which have the lowest earnings persistence among the countries in my sample. I also find

that borrowing constraints have a very limited impact on earnings persistence.

Future research in this area may include the study of optimal education expenditure and tax policies within an intergenerational general equilibrium framework. An extension is also to explicitly model the supply of educational services. In this paper I have assumed that the technology for human capital production stays the same as the demand for education changes.

10 Appendix

10.1 Discussion of Difficulties with Comparing Different Studies of Earnings Persistence

There are some difficulties related to comparing different studies of intergenerational earnings persistence. Solon (1992) and Blanden (2009) provide more in-depth discussions of some of the methodological issues. One problem in the estimation of 1 is the measure of earnings. Ideally the measure of earnings used in 1 should be permanent or lifetime earnings. Since this measure is rarely available, the econometrician will either use earnings observed in a single year or preferably take the average of several years of earnings. This will generally be an inaccurate measure of permanent earnings. It is easy to show that an inaccurate measure of the father's earnings in 1 will lead the estimate of β to be biased downward. A first step toward reducing this measurement error is controlling for age in 1, and this is done in pretty much every study. However, if more years of earnings are averaged, the measurement error is reduced, and this is a source of discrepancies between different studies. Another obvious source of discrepancies between studies is the quality of the data. If the sample is too homogeneous, i.e., the variance of earnings is too small, as is typical for unrepresentative data samples, the problem with measurement error is compounded; see Solon (1992).

A possible solution to the problem with measurement error in the father's earnings is the use of instrumental variables. The instruments must be uncorrelated with the measurement error and, in addition, uncorrelated to the son's earnings. The problem with the instrumental variable approach is that most variables related to father's earnings may also have an independent impact on the son's earnings. Solon (1992) shows that in this case, the estimate of β will be biased upward. The instrumental variables approach is nonetheless becoming more popular in the literature.

Finally, the age at which father's and son's earnings are observed may have a substantial impact on the estimates of β; see Haider and Solon (2006) and Grawe (2003). Controlling for age in the regression does not solve this problem, since high and low earners have different life-cycle earnings profiles. Often the earnings of young sons are regressed on the earnings of old fathers, which is found to cause a downward bias in the estimate of β. Haider and Solon (2006) find that the years around 40 will be the best proxies for lifetime earnings.

Corak (2006) provides a cross-country meta study of intergenerational earnings persistence that tries to take into account how many years of the father's earnings were used as a measure for permanent earnings, whether an IV approach was used, and the age of the father at the time of observation. Table 1 displays the results from this study supplemented with earnings persistence from Italy and Spain, which I take from Piraino (2007) and Pla (2009). I adjust the number for Italy using a formula provided in Corak (2006). I cannot do the same for Spain, because I do not know the average age of the fathers in that study. Given the many problems with comparing different studies of intergenerational earnings persistence, it is clear that Table 1 should be interpreted as a stylized fact.

10.2 Fitting Tax Functions Based on Data from the OECD

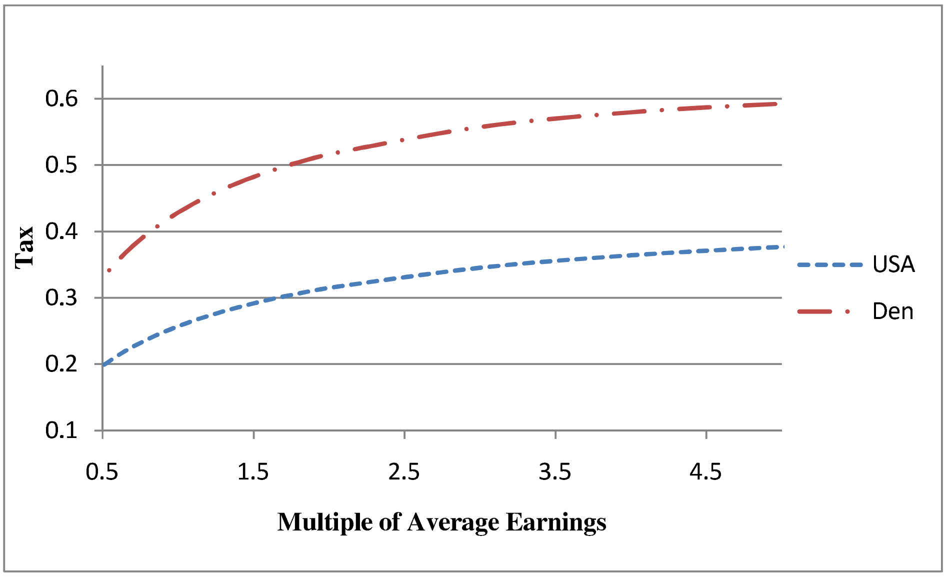
For every country in Table 1, I fit the polynomial in 30. I use this functional form because it generally gives me a very good fit, R^2 above 99.9%, and because I get functions that are strictly increasing and well behaved on a relatively wide range of labor income. I use labor income tax data from the OECD tax-benefit calculator[10] and the OECD tax database[11]. These data are constructed by the OECD based on tax laws from different countries.

The OECD tax-benefit calculator gives the gross- and net (after taxes and ben-

[10] Available at: `www.oecd.org/document/18/0,3343,en_2649_34637_39717906_1_1_1_1,00.html`.

[11] Available at: `www.oecd.org/document/60/0,3343,en_2649_34533_1942460_1_1_1_1,00&&en-USS_01DBC.html`.

Figure 5: Labor Income Tax Functions for the US and Denmark



efits) labor income at every percentage of average labor income on a range between 50% and 200% of average labor income, by year and family type starting in 2001. I use the data at every fifth percentage point for single individuals without children and take an average of the years 2001-2005. The OECD tax catabase provides the top marginal tax rate in each country and the starting point for this tax rate. To get the tax at earnings above 200% of average labor income, I use this information and compute the tax at every multiple of 0.5 times average earnings between 2.5 and 15 times average earnings. For most countries the top marginal tax rate kicks in before 200% of average labor income, but in the US, for instance, the top marginal tax rate starts at about 9 times average earnings. I then assume that the marginal tax rate increases linearly between 2 times average earnings and the point at which the top marginal tax rate becomes effective.

Since I only have tax data starting at 50% of average earnings, I add a random positive point of close to zero tax for close to zero earnings, to get my tax functions well behaved for very small earnings. This, however, has almost no impact on the fit with the real data points. The alternative would have have been to require all

Table 9: Country Tax Functions

Country	τ_1	τ_2	τ_3	τ_4	R^2
Denmark	-1.242825	3.603493	-2.456365	0.5239973	0.9997
Norway	-0.6488133	1.818972	-1.023706	0.1670745	0.9999
Finland	-0.71829	1.895892	-1.004558	0.1465101	0.9996
Canada	-0.3056732	0.8059581	-0.2546371	-0.0145851	0.9997
Sweden	-0.6629891	1.966373	-1.183786	0.2152142	0.9997
Germany	-1.329006	4.017692	-2.947534	0.6809511	0.9998
Spain	-0.2001187	0.3728243	0.1407691	-0.1200151	0.9994
France	-0.5460613	1.651868	-1.011427	0.1903222	0.9998
Italy	-0.7060691	1.782236	-0.9431628	0.137171	0.9989
USA	-0.5730303	1.705866	-1.096482	0.2207298	0.9998
UK	-0.5907906	1.778369	-1.163281	0.2362276	0.9998

people to work enough to make a certain amount of income. I fit the tax functions by running OLS regressions. Table 9 displays the country tax functions, while Figure 5 plots the tax functions for the US and Denmark.

10.3 Proof of Propositions 3.1 and 3.1

Proposition 3.1

Since $\log(y_t)$ is a monotonic transformation of y_t, it will be sufficient to take the derivatives of the top part of 11 with respect to y_t. We have:

$$\frac{\partial \log(y_{t+1})}{\partial y_t} = \frac{\psi(1-\tau)}{y_t(1-\tau)+I_g} > 0 \tag{32}$$

$$\frac{\partial^2 \log(y_{t+1})}{\partial y_t \partial \tau} = \frac{-\psi I_g}{\left(y_t(1-\tau)+I_g\right)^2} < 0 \tag{33}$$

$$\frac{\partial^2 \log(y_{t+1})}{\partial y_t \partial I_g} = \frac{-\psi(1-\tau)}{\left(y_t(1-\tau)+I_g\right)^2} < 0 \tag{34}$$

$$\frac{\partial^2 \log(y_{t+1})}{\partial y_t \partial \psi} = \frac{y_t(1-\tau)^2+I_g(1-\tau)}{\left(y_t(1-\tau)+I_g\right)^2} > 0 \tag{35}$$

■

Proposition 3.2

Differentiating 16, we obtain:

$$\frac{\partial\beta}{\partial\tau} = \frac{-\psi(1-\theta^2)\tilde{I}_g}{\left((1+\psi\theta)(1-\tau)+\tilde{I}_g\right)^2} < 0 \quad (36)$$

$$\frac{\partial\beta}{\partial\tilde{I}_g} = \frac{-\psi(1-\theta^2)(1-\tau)}{\left((1+\psi\theta)(1-\tau)+\tilde{I}_g\right)^2} < 0 \quad (37)$$

$$\frac{\partial\beta}{\partial\psi} = \frac{(1-\theta^2)(1-\tau)^2+(1-\psi\theta-\theta^2)(1-\tau)\tilde{I}_g}{\left((1+\psi\theta)(1-\tau)+\tilde{I}_g\right)^2} > 0 \quad (38)$$

$$\frac{\partial\beta}{\partial\theta} = \frac{(1-\psi^2)(1-\tau)^2+2(1-\tau)\tilde{I}_g+\tilde{I}_g^2}{\left((1+\psi\theta)(1-\tau)+\tilde{I}_g\right)^2} > 0 \quad (39)$$

■

10.4 Computational Details

Computation of Optimal Policies

I put boundaries on the capital and human capital space and pick a grid in each dimension. I pick 40 grid points in $K = [k^{min}, k^{max}]$ and 16 grid points in $H = [h^{min}, h^{max}]$. The grid points for capital are taken to be the scaled zeros of a 40th order Chebyshev polynomial ,while the grid points for human capital are taken to be the scaled zeros of a 16th order Chebyshev polynomial. Following the method outlined by Tauchen (1986), I approximate the processes for the idiosyncratic productivity shock, u, and ability, A, as finite state Markov processes. I use 7 equally spaced states for u in $U = [-2\sigma_u, 2\sigma_u]$, and 13 equally spaced states for A in $\bar{A} = [-3\sigma_A, 3\sigma_A]$. Let $J = \{0, 1\}$ be the state space for whether an individual is college educated. The maximum size of the state space occurs in periods 5-7, or ages 40-50, when there are 6 state variables apart from time. The state space is then $J \times K \times H \times H \times \bar{A} \times U$, or 1,863,680 grid points. I compute the household's optimal policies for each grid point in each time period by iterating backwards. I start from age 100, the last period of life. In that period, the next period's value function is 0, and the optimal policy

is to consume as much as possible. Knowing the value function at age 100, I can compute optimal policies and value functions for age 95, and so on. Reaching age 50, when the child leaves home, I need to know both the parent's value function at age 55 and the child's value function at age 20 to compute the optimal policies. The first time around, I use an educated guess for the child's value function at age 20. When I reach age 20, I get a new $V(age = 20, \cdot)$ and start over again from age 50. I continue this iteration until V converges.

To solve for the optimal policies in each time period, I use the routine called LCONF from the IMSL Fortran library. It is based on M. Powell's method for solving linearly constrained optimization problems; see IMSL documentation for details. To interpolate the value function outside of the grid, I use Chebyshev collocation; see Judd (1998), Heer and Maussner (2004). When there is a child in the household and the parent is investing in the child's human capital, the next period's value function must be interpolated in the $K \times H$-space. The value function is then represented as a polynomial with 40 X 16 = 640 coefficients. At one point in time, when the agent chooses whether or not to attend college, I am taking the max of two value functions. When these two value functions overlap, the value function considered by the parent, before the child makes the college decision, will generally not be concave. However, what the parent needs to consider is the expectation of the value function over the idiosyncratic shock. It turns out that the expectation of the value function is concave, although there is no theoretical guarantee for it. To be absolutely sure that I am finding a global max, I am multi starting the solver from points that are far apart.

Simulation

Knowing today's state, the policy functions, and drawing shocks, u and ν, I can find the next period's state. I make 200,000 draws from a random initial distribution of 20 year olds and run the simulation for 200 generations (enough to reach a stationary

distribution). In the simulation, the policy functions must be interpolated on the $K \times H \times H$-space as both the child's and the parent's human capital may be outside of the grid. I use linear interpolation.

Hardware and Software

I use Intel Fortran, version 11.1 and a computer with a 2.93 GHz Core-i7 processor. To speed up the computation, I use OpenMP to parallelize the code on the 8 threads.

10.5 Introducing a Tax System with US Level and Danish Progressivity

We want to introduce a new tax function, $\tilde{\tau}(y)$, which has the same average tax rate as in the US but where progressivity, as defined in 2, is the same as the tax system in Denmark, $\tau_D(y)$. We must have:

$$1 - \frac{1-\tilde{\tau}(y_2)}{1-\tilde{\tau}(y_1)} = 1 - \frac{1-\tau_D(y_2)}{1-\tau_D(y_1)} \Rightarrow \frac{1-\tilde{\tau}(y_2)}{1-\tau_D(y_2)} = \frac{1-\tilde{\tau}(y_1)}{1-\tau_D(y_1)} \tag{40}$$

for all levels of y_1 and y_2. Letting the fraction $\frac{1-\tilde{\tau}(y)}{1-\tau_D(y)}$ be equal to a constant, Λ, for all levels of y, we can obtain a new tax system with the desired properties as follows:

$$1 - \tilde{\tau}(y) = \Lambda\big(1 - \tau_D(y)\big) \Rightarrow \tilde{\tau}(y) = 1 - \Lambda + \Lambda\tau_D(y) \tag{41}$$

We must solve for Λ in the context of the model to obtain the same average tax level as in the US.

10.6 Policy Experiments With Redistribution of Net Changes in Tax Revenues

Below I reproduce the first 4 columns of Table 6 but this time I redistribute the net change in tax revenues relative to the benchmark model evenly to all households.

Table 10: Policy Experiments with Redistribution of Δ Tax Revenue

Statistic	Bench-mark	Danish taxes	Danish educ. subsidies	Danish subsidies + taxes
Average hours worked	0.417	0.391	0.403	0.383
Std. dev. of log wages	0.571	0.491	0.628	0.533
Fraction enrolling in college	0.590	0.418	0.889	0.739
Intergen. earnings elasticity	**0.470**	**0.347**	**0.437**	**0.357**
Average human capital inv. age 5-9	$3998	$631	$5501	$1072
Average human capital inv. age 10-14	$5127	$957	$6900	$1493
Average human capital inv. age 14-19	$5752	$1042	$4837	$533
Average human capital inv. in college	$14692	$1570	$12754	$987
Average human capital inv. (all ages)	$5016	$750	$5920	$869
Average gift from parent to child	$78714	$4409	$118617	$8052
$\frac{tax\ per\ worker - educ.\ expenditure}{benchmark\ average\ earnings}$	0.343	0.507	0.371	0.543
Average Earnings	$61111	$45265	$69554	$49448
$\frac{\bar{I}_{private}}{\bar{I}_{total}}$	0.525	0.193	0.413	0.138
$Stdev\left(\frac{I_{private}-\bar{I}_{private}}{\bar{I}_{private}}\right)$	2.240	2.118	2.263	2.202
$Corr\left(college, \log\left(y_{parent}\right)\right)$	0.1939	0.153	0.156	0.155

Column 2 displays the results when introducing a Danish tax system into the model. Column 3 shows the results when introducing Danish public education expenditure policies. Column 4 shows the results when introducing Danish taxes and education spending at the same time. The dollar amounts are in annual 2005 dollars.

Redistribution does not have a large impact on the results with respect to intergenerational earnings persistence. It does, however, greatly reduce labor supply and average earnings. The reduction in labor supply is the reason for why redistribution does not have a larger impact on earnings persistence. On one hand poorer households get more financial resources that could be invested in education but on the other hand poorer households tend to reduce their labor supply the most in response to the redistribution. this reduces their incentives to invest in education.

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