

Inefficient provision of liquidity

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Abstract

We study an economy where the lack of a simultaneous double coincidence of wants creates the need for a relatively riskless claim (money). We show that, even in the absence of asymmetric information or an agency problem, the private provision of liquidity is inefficient. The reason is that liquidity affects prices and the welfare of others, and creators do not internalize this. This distortion is present even if we introduce lending and government money. In fact, we show that lending is complementary to, rather than a substitute for, inside and outside money. To eliminate the inefficiency the government has to restrict the creation of liquidity by the private sector; it cannot just introduce outside money.

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1. Introduction

Historically, one of the primary functions of the banking sector has been to create means of payment (“money like” liabilities). Yet most models of banking focus on other aspects, such as risk sharing or monitoring. Less attention has been paid to the role of banks in creating money and how this interacts with the government’s role in this area. Does a competitive banking sector generate the socially optimal amount of the means of payment (often referred to as liquidity or money)? How does the liquidity created by the banking sector affect the equilibrium level of asset prices? Does the degree of competition in the banking sector affect the amount of liquidity created and hence the level of asset prices?

In this paper we try to answer these questions. We show that, even in the absence of asymmetric information or an agency problem, the amount of liquidity generated by the banking sector is inefficient. A competitive banking sector generates an excessive quantity of means of payment, while a monopolistic one a scarcity. The distortion arises from two externalities the creation of money has in general equilibrium: more money increases the equilibrium prices of the goods that those with the money buy; but it also increases the wealth of the agents supplying these goods and so the prices of the goods they buy. (These externalities are pecuniary in nature since they operate through prices but unlike standard pecuniary externalities they have welfare consequences.) A competitive bank, which ignores the externality imposed on other buyers, will generate too much liquidity. A monopolistic bank that serves the buyers, and ignores the welfare of the sellers, generates too little liquidity. There is one intermediate level of competition that generates the efficient amount of liquidity but it would be fortuitous for the economy to end up there.

A key assumption that we make is that future labor income is not pledgeable (or contractible). When we relax this assumption, we show that there is an endogenous limit to the amount of liquidity that can be created without collateralized assets or government money. Thus, while the inefficiency is reduced when future labor income becomes contractible, it is not eliminated.

We analyze these issues in a simple economy. The economy consists of two groups of agents: doctors and builders. Doctors buy building services from builders and then builders buy doctors’ services from doctors (or the other way round). Agents are also endowed with wheat. There is a lack of a simultaneous double coincidence of wants: each builder requires a doctor at a

later date and typically one with different skills from the doctor he is building for. Since future labor income is not pledgeable this generates a need for money. Wheat is costly to carry and can easily rot, so banks arise in our model as depositary institutions, which store wheat and issue notes.

We start by studying the effect of this supply of notes on equilibrium prices and on social welfare when banks are purely passive institutions that “notify” all deposits, i.e., an agent who deposits a unit of wheat receives a note of equal value. In notifying the deposits of a doctor a bank imposes a negative externality on doctors at other banks: raising the amount of money increases the price of building services, which is bad for them since they consume these services. Because of this externality some wheat endowment is stored to create liquidity instead of being invested in more profitable opportunities.

Before the introduction of fiat money, the main commodity that was used as money, gold, was not particularly useful per se. Hence, as noted by Sargent and Wallace (1983), the inefficiency of commodity money was not the distortion in the use of gold, but its overproduction. Sargent and Wallace (1983) seem to suggest that the introduction of fiat money eliminates this problem. In contrast, our model indicates that the inefficiency in the provision of liquidity persists as long as investments differ in their collateralizability. If there are assets that can be collateralized and assets that cannot, it is not surprising that the assets that can be collateralized will be traded at a premium with respect to what their yield would imply. What our model also shows, however, is that a competitive equilibrium will lead to an excess investment in collateralizable assets.

This distortion is present even when banks can control the notification process, as long as they act in a competitive way. A monopolistic bank, instead, ends up under-producing money; this is the standard result that a profit maximizing monopolist restricts production. This distortion is present even when we introduce lending and government money. While the introduction of government money potentially crowds out the need for inside money, there is a limit to the amount of government money that can be injected in the system. This limit is determined by the ability of the government to tax.

We also show that the introduction of government money does not crowd out the need for bank lending. In fact, it is complementary to bank lending. Money is not neutral in our model:

an injection of government money has a positive effect on the level of economic activity and on welfare. Lending multiplies the effect that an injection of money has on the economy.

Our paper is related to two literatures: that on banking and that on money. Much of the banking literature is concerned with the role played by banks and the need for bank regulation (see, for example, Dewatripont and Tirole (1994)). One branch of the banking literature, starting with Diamond (1984) and continuing with Holmstrom and Tirole (1997), focuses on the asset side of banks: their role in monitoring loans. Another branch of the literature, starting with Diamond and Dybvig (1983), focuses on the liability side of banks: the ability of banks to provide risk sharing in the face of liquidity needs (Diamond and Dybvig (1983), Allen and Gale (1998) and (2007), and Bhattacharya and Gale (1987)), or to reduce future adverse selection (Gorton and Pennacchi (1990)). There are also some papers, such as Diamond and Rajan (2001), which try to integrate the two sides, showing how demand deposits are critical in making credible the ability of lenders to extract a repayment for their loans.

Holmstrom and Tirole (1998) and (2011) focus on moral hazard on the side of suppliers of liquidity, rather than on the side of users. They show that, in the presence of aggregate uncertainty, the state power to tax future income creates liquidity for the corporate sector, improving its ability to invest.

In all these papers the source of friction is either some informational asymmetry or some agency problem (or both). While these problems are important, they are not the only ones relevant for banks. One reason banks are unique is that they issue liabilities that are used as a means of payment. Our goal is to analyze the implications of this role. For this reason, we abstract from all the other frictions and focus on the externalities in the creation of money. A general theory of banking would bring all these frictions together.

A recent strand of the banking literature, which deals with the transactional role of deposits, introduces behavioral features. This strand derives the uniqueness of banks from the misperception by depositors that their claims are safe (Gennaioli et al. (2010), Rotemberg (2010)) or from the banks' ability to arbitrage irrationally exuberant markets and rationally priced deposits (Shleifer and Vishny (2010)). We do not introduce behavioral aspects here.

Our result that the creation of inside money is excessive is similar to Stein (2011). In his model, however, it is assumed that agents have a discontinuous demand for a riskless claim (money), while we do not make such an assumption. Similarly, his inefficiency arises from an

assumed friction in the financial markets (that patient investors cannot raise additional money), while ours arises endogenously. Stein's model, however, is richer in terms of implications for monetary policy. In this respect, the two models can be seen as complementary.

Our paper is also related to the huge literature on money. Much of this literature is concerned with the role money plays in general equilibrium (e.g., Hahn (1965)). To create such a role, one needs to dispense with the traditional Walrasian auctioneer and explicitly introduce an exchange process. Ostroy and Starr (1990) provide an excellent survey of attempts in this direction. As far as we can tell, none of these attempts analyze the externality we identify in our paper. The role money plays in our model (i.e., to address the lack of double coincidence of wants) is similar to that in Kyotaki and Wright (1989). Their focus, however, is on what goods can become money and how. Our focus is to what extent private banks can provide the efficient quantity of medium of exchange. Another large part of the literature on money analyzes the role of inside money on monetary policy, as in Brunnermaier and Sannikov (2010), Diamond and Rajan (2006), and Kashyap and Stein (2004). Our model is silent on this.

Our approach resembles that of Kiyotaki and Moore (1997) and (2002). Like us, they rely on some limited pledgeability of future income. Their main focus, however, is on the multiplier/contagion effect that the failure of one intermediary can have on the overall system. Our paper, instead, is concerned with externalities in the creation of inside money.

Finally, our approach is close to that of Mattesini et al (2009). They study banking using the tools of mechanism design. They consider an economy with two groups of consumers who want to trade with each other. As in our model, there is a timing problem: the first group has to buy from the second group before they have sold their own output. Mattesini et al (2009) analyze how claims on deposits with third parties are a better means of exchange than claims on individual wealth. They study the social optimum but not the market equilibrium. In contrast, we take the superiority of third party deposits as given, and study the divergence between the market equilibrium and the social optimum.

The rest of the paper proceeds as follows. In Section 2 we lay out the framework and describe the Walrasian equilibrium. In Section 3 we analyze the effect of the introduction of banks as storage facilities. In Section 4 we introduce the possibility of bank lending. In Section 5 we study the interaction between private money and government money. In Section 6 we consider some extensions. Conclusions follow.

2. The Framework

We consider an economy that lasts three dates:

1 -----2-----3

There are two types of agents in equal numbers: doctors and builders. The doctors want to consume building services and the builders want to consume doctor services. Doctors and builders can consume wheat at any date and there is no discounting. Both doctors and builders have an endowment of wheat at date 1 equal to e . As will become clear we will assume that e is “large”.

We write agents’ utilities as:

$$\text{Doctors: } U_d = x_d + b_d - \frac{1}{2}l_d^2$$

$$\text{Builders: } U_b = x_b + d_b - \frac{1}{2}l_b^2$$

where x_i is the sum of the quantities of wheat consumed by individual $i = d, b$ at each date; b_d is the quantity of building services consumed by the doctors; l_d is the labor supplied by the doctors; d_b is the quantity of doctor services consumed by the builders; and l_b is the labor supplied by the builders. We assume constant returns to scale: one unit of builder labor yields one unit of building services and one unit of doctor labor yields one unit of doctor services. We normalize the price of wheat to be 1 at all dates. Let p_b and p_d be the price respectively of building and doctor services. In words, doctors and builders have a constant marginal utility of wheat, a constant utility of the service provided by the other group of agents, and a quadratic disutility of labor.

At date 1 each agent learns whether he will first buy or sell. Ex ante both events are equally likely. For convenience, we will assume that in the east side of town doctors will buy building services at date 2, while builders will buy doctors’ services at date 3. In the west side of town, the order is reversed. After each agent learns whether he will first buy or sell (equivalently, whether he is in the east or west side of town), he can invest part of his wheat endowment in a

risky project that pays off at date 3 (in the form of wheat), and whose expected return is $\bar{R} > 1$; the rest of the endowment is (costlessly) stored. This investment opportunity is specific to the agent. Each agent maximizes his expected utility; since utilities are linear in wheat consumption this means that agents are in effect risk neutral.

At dates 2 and 3 the market meets and the doctors and builders trade in the order determined at date 1. Throughout the paper we will analyze the east part of town, where doctors buy at date 2 and builders at date 3; the reverse case is completely symmetrical.

We assume there are many doctors and many builders, and so the prices for both services are determined competitively. It is crucial for our analysis that there is *no simultaneous* double coincidence of wants: builders and doctors are in either the market for buying or the market for selling; they cannot do both at the same time. Hence, even if the builder a doctor buys from wants the doctor services from his customer, he cannot buy them at the same time as he is selling building services.

2.1 A benchmark: the Walrasian equilibrium

In an ideal world the doctors could pledge to pay the builders out of income from supplying doctor services that they will earn at date 3 *and* from the return from their risky project. This is the assumption made in classic Walrasian or Arrow-Debreu theory and it is easy to compute the Walrasian equilibrium.

Let's start by analyzing the case where the project return is perfectly certain. In this case both the doctors and the builders invest all their endowment in the project since this maximizes their wealth; and under complete markets maximizing wealth is a necessary condition for maximizing utility. Thus, a doctor solves the following maximization problem:

$$(2.1) \quad \begin{aligned} \text{Max } & x_d + b_d - \frac{1}{2} l_d^2 \\ \text{S.T. } & x_d + p_b b_d \leq p_d l_d + e \bar{R}. \end{aligned}$$

The solution is

$$(2.2) \quad \begin{aligned} l_d &= p_d & \text{if } p_b \geq 1 \\ &= \frac{p_d}{p_b} & \text{if } p_b < 1 \end{aligned}$$

$$b_d = 0 \quad \text{if } p_b > 1$$

$$b_d = \frac{p_d^2}{p_b^2} + \frac{e\bar{R}}{p_b} \quad \text{if } p_b < 1$$

$$0 \leq b_d \leq \frac{p_d^2}{p_b^2} + \frac{e\bar{R}}{p_b} \quad \text{if } p_b = 1$$

The intuition is that, if $p_b > 1$, doctors prefer wheat to building services, if $p_b = 1$ they are indifferent, and, if $p_b < 1$, they prefer building services. The marginal utility of wealth for doctors is 1 if $p_b \geq 1$ and $\frac{1}{p_b}$ if $p_b < 1$, and this affects their labor supply decision. If $p_b \geq 1$, doctors

choose labor supply to maximize $p_d l_d - \frac{1}{2} l_d^2$; and, if $p_b < 1$, they maximize $\frac{p_d}{p_b} l_d - \frac{1}{2} l_d^2$.

Similarly, a builder solves:

$$(2.3) \quad \begin{aligned} &\text{Max } x_b + d_b - \frac{1}{2} l_b^2 \\ &\text{S.T. } x_b + p_d d_b \leq p_b l_b + e\bar{R}. \end{aligned}$$

The solution is

$$(2.4) \quad \begin{aligned} l_b &= p_b \quad \text{if } p_d \geq 1 \\ &= \frac{p_b}{p_d} \quad \text{if } p_d < 1 \end{aligned}$$

$$d_b = 0 \quad \text{if } p_d > 1$$

$$d_b = \frac{p_b^2}{p_d^2} + \frac{e\bar{R}}{p_d} \quad \text{if } p_d < 1$$

$$0 \leq d_b \leq \frac{p_b^2}{p_d^2} + \frac{e\bar{R}}{p_d} \quad \text{if } p_d = 1$$

Again the marginal utility of wealth for builders is 1 if $p_d \geq 1$ and $\frac{1}{p_d}$ if $p_d < 1$.

For markets to clear we must have

$$(2.5) \quad b_d = l_b$$

$$(2.6) \quad d_b = l_d$$

On the one hand, (2.5) and (2.6) cannot be satisfied if either $p_b > 1$ or $p_d > 1$ (demand will be less than supply for building/doctor services, respectively). On the other hand, we cannot have both $p_b < 1$ and $p_d < 1$ because then the demand for wheat would be zero, while the supply is $e\bar{R}$ ((2.5) and (2.6) imply that the wheat market clears). Hence, either $p_b < 1$ and $p_d = 1$, or $p_b = 1$, $p_d < 1$, or $p_b = p_d = 1$. It is easily seen that the first case is inconsistent with (2.5) and the second with (2.6).

We are left with $p_b = p_d = 1$. It is immediate that (2.2)-(2.6) hold if $b_d = l_b = d_b = l_d = 1$.

Hence, we have established

Proposition 1: In the unique Walrasian equilibrium all wheat is invested

and $p_b = p_d = b_d = l_b = d_b = l_d = 1$. The utilities of the doctors and builders are $U_d = e\bar{R} + \frac{1}{2}$, $U_b = e\bar{R} + \frac{1}{2}$, respectively, and total welfare (social surplus) equals $W \equiv U_d + U_b = 2e\bar{R} + 1$.

Note that the Walrasian allocation and prices are independent of the initial endowment e and \bar{R} (except for a doctor's consumption of wheat, which varies one to one with $e\bar{R}$). It follows from this that our analysis is unchanged if the project return is uncertain rather than certain—doctors and builders will still invest all their endowment in the risky project; and consume the (uncertain) return from the project at date 3. Finally, the Walrasian equilibrium achieves maximal social surplus, which is to be expected given the first theorem of welfare economics and the symmetry of the parties. In what follows we will refer to the Walrasian equilibrium allocation as the first best.

3. Introducing Banks

We now suppose that parties cannot pledge their future labor income or investment income—the returns from these activities can be diverted or hidden. We also assume that it is impossible for individuals to carry wheat around with them when they trade; it is too

cumbersome or the wheat would rot or be stolen. In the absence of any further assumption the model now becomes trivial. Trade between doctors and builders is impossible—they have nothing to trade with— and so each agent invests and eats his wheat at date 3. We have $U_d = e\bar{R}$ and $U_b = e\bar{R}$.

However, we now introduce storage facilities. These storage facilities are perfectly secure in the sense that wheat deposited at a facility at date 1 will remain there and be intact at the end of date 3.

Storage per se does not change anything since there is no advantage to doctors from storing wheat rather than consuming it right away. However, let us suppose that claims can be issued on the wheat deposited in a storage facility. In particular, if a doctor deposits f units of wheat he will receive f notes, where each note is a claim on a unit of wheat at the end of date 3; he can then use these notes to pay builders. The builders in turn can use these notes to pay doctors. At the end of date 3 the holders of the notes can go to the storage facility and redeem them for an equal number of units of wheat. We call these storage facilities banks. At the moment, these are completely passive institutions, which just store and issue notes on a one-for-one basis.

3.1 Individual optimization

Let's consider first the doctors. At date 1, a doctor invests $e - f_d$ units of wheat in the risky project and deposits f_d units of wheat in the bank, receiving f_d in notes. At date 2 he uses these notes to purchase building services. At date 3, he will choose l_d to maximize $p_d l_d - \frac{1}{2} l_d^2$, i.e., set $l_d = p_d$. Note that it is too late for the doctor to buy more building services and so his marginal return from work is p_d rather than $\frac{p_d}{p_b}$. A doctor's labor yields revenue p_d^2 in the form of notes, which he redeems for wheat at the end of date 3 ; in addition he incurs an effort cost of $\frac{1}{2} p_d^2$, and so his net utility is $\frac{1}{2} p_d^2$. Finally, he will also receive and consume the payoff from his investment.

Therefore, a doctor's (expected) utility when he buys first is

$$(3.1) \quad \frac{f_d}{p_b} + \frac{1}{2} p_d^2 + (e - f_d) \bar{R}$$

Each doctor chooses f_d to maximize (3.1), taking prices as given. The first order condition is

$$(3.2) \quad \frac{1}{p_b} = \bar{R}$$

Similarly, a builder's utility is

$$\frac{f_b}{p_d} + \frac{1}{2} \frac{p_b^2}{p_d^2} + (e - f_b) \bar{R}$$

We shall see shortly that builders will be a corner, so their first order condition is

$$(3.3) \quad \frac{1}{p_d} \leq \bar{R}$$

3.2 Market Equilibrium

We solve for the equilibrium under the conjecture that $f_b = 0$. Since only $f_d > 0$, we will drop the subscript and set $f_d \equiv f$. We also conjecture that p_d and p_b are less than one. In due course we will show that all these conjectures are correct.

We work backwards, starting with the market for doctors at date 3. After the doctors have bought building services, the builders find themselves with a quantity f of notes. Hence, their demand for doctor services will be given by $\frac{f}{p_d}$. The supply of doctor services is given by

$$(3.4) \quad l_d = p_d,$$

as in (2.2). Hence, market clearing requires

$$(3.5) \quad \frac{f}{p_d} = p_d.$$

Similarly, the market-clearing condition in the building services market is

$$(3.6) \quad \frac{f}{p_b} = \frac{p_b}{p_d}$$

since any income received by builders is spent on doctor services. Combining this with (3.5) yields

$$(3.7) \quad p_d = f^{\frac{1}{2}}, p_b = f^{\frac{3}{4}}.$$

Substituting the equilibrium price into (3.2) we have:

$$(3.8) \quad f^{\frac{3}{4}} = \bar{R}$$

or

$$f = \hat{f} = \frac{1}{\bar{R}^{\frac{4}{3}}}$$

where we assume that e is larger than \hat{f} , so the solution is not at the corner. Since $\bar{R} > 1$, $\hat{f} < 1$, which implies $1 > p_d > p_b$. Hence (3.3) holds and the builders will be at a corner solution with $f_b = 0$, as initially conjectured.

Note that the level of trade of doctors' services is $\frac{f}{p_d} = f^{\frac{1}{2}} < 1$ and builders' services is

$$\frac{f}{p_b} = f^{\frac{1}{4}} < 1.$$

3.3 Social Optimum

Obviously the market equilibrium is not first best optimal: as long as $\bar{R} > 1$, the economy will operate below the Walrasian equilibrium level of trade, regardless of the quantity of endowment e . (Recall that in the Walrasian equilibrium $p_d = p_b = 1$ and one unit of each service is traded.) We now show that the market equilibrium is also not second best optimal: a planner operating under the same constraints as the market can do better. Recall that ex ante it is not known who will buy first: doctors or builders. Thus the expected utility of each group is

$\frac{1}{2}U_d + \frac{1}{2}U_b$. The social optimum is obtained by maximizing $U_d + U_b$ taking into account the effect of f on prices.

That is, the planner maximizes

$$(3.9) \quad U_d + U_b = f^{\frac{1}{4}} + \frac{1}{2}f + \frac{1}{2}f^{\frac{1}{2}} + (e-f)\bar{R} + e\bar{R}$$

The first order condition is

$$(3.10) \quad \frac{1}{4}f^{-\frac{3}{4}} + \frac{1}{2} + \frac{1}{4}f^{-\frac{1}{2}} = \bar{R}$$

Comparing the left-hand side of (3.8) and (3.10) we have

Proposition 2: If $\bar{R} > 1$ and $e > \hat{f}$, the market equilibrium leads to an excessive amount of inside money f , and an excessive trade level, relative to the second best social optimum.

Proof: Since the l.h.s. of both (3.8) and (3.10) are decreasing in f , to prove that the solution of

(3.8) is large than the solution of (3.10) it is enough to show that $f^{-\frac{3}{4}} > \frac{1}{4}f^{-\frac{3}{4}} + \frac{1}{2} + \frac{1}{4}f^{-\frac{1}{2}}$. This

can be rewritten as $\frac{1}{2}(f^{-\frac{3}{4}} - 1) + \frac{1}{4}(f^{-\frac{3}{4}} - f^{-\frac{1}{2}}) > 0$, which is true as long as $f < 1$. When $f=1$,

the inequality becomes an equality. Trade levels are higher in the market equilibrium since equilibrium trade levels, given by $f^{\frac{1}{2}}$ for doctor services and $f^{\frac{1}{4}}$ for building services, are monotonically increasing in f .

The social and private returns from varying f are illustrated in Figure 1. There are two types of inefficiencies here. With respect to the first best Walrasian equilibrium, too much wheat is invested in unproductive storage instead of productive projects. (In the Walrasian equilibrium no wheat is invested in unproductive storage.) This inefficiency is due to the lack of pledgeability of future income. In addition, there is an inefficiency with respect to the second best. The creation of more means of payments imposes a positive externality on the builders (who see the price of their building services go up) and a negative externality on the other doctors (who see

the price of what they are buying go up). The social planner takes these externalities into consideration, while the market does not. In standard models the effect of these “pecuniary” externalities is second order and thus does not create a divergence between social and private optimality. Here, however, while the positive externality on other builders is second order, the negative externality on other doctors is first order, since the doctors are liquidity constrained.

Notice that if $\bar{R} = 1$, the social and private solutions do not differ: they are both $f = 1$ (as long as $e \geq 1$). In this case the first best is achieved. There is still a divergence between private and social incentives, but this divergence is infra-marginal.

As we have noted, as long as $\bar{R} > 1$, the economy will operate below the Walrasian equilibrium level of trade, regardless of the quantity of endowment e . Because risky projects cannot be collateralized, and the only investments that can be collateralized (wheat storage) have a lower yield, there is an opportunity cost of creating liquidity. Thus the optimal amount of liquidity produced is too little from a first best efficiency point of view: in the first best there would be enough liquidity to generate a trade of one unit of each service. This conclusion is reminiscent of Friedman’s famous result that with a non-negative rate of inflation people hold too little money. In Friedman, however, this inefficiency is in the form of a shoe-leather cost; here it is in the form of missed trading opportunities. The ultimate source of inefficiency in our paper is lack of pledgeability: the inefficiency would disappear if there was enough collateralizeable investment.

3.4 Overproduction of collateral

In our model there are three types of activities: non-collateralizeable investments (risky projects), collateralizeable investments (wheat), and future labor income. As long as the return from the collateralizeable investments is less than that from the non-collateralizeable investments, our result about the excess provision of liquidity holds. Interestingly, it would be natural to think that if future labor income is contractible, the problem would disappear. In Section 4 we show this is not the case.

Note that if some risky projects are (or become) collateralizeable, their prices will jump from $\frac{1}{\bar{R}}$ to $\left(\frac{1}{\bar{R}}\right)^2$, since they can be used both to produce \bar{R} at date 3 and to buy goods at date 1,

which has a return of $\frac{1}{p_b} = \bar{R}$. This conclusion can be generalized: assets that can be

collateralized to back credit will trade at a higher price than otherwise identical assets that cannot be collateralized to produce credit. As a result, if we add an early period where effort is exerted to produce various assets, there will be an overproduction of assets that can be collateralized to produce liquidity (Madrigan and Philippon, 2011). Therefore, there will be excess resources invested in collateralizeable assets and/or an overproduction of low-yielding assets that can be collateralized to produce liquidity, a result similar to the one Madrigan and Philippon (2011) obtains for houses.

3.5 Optimal Regulation

In this simple economy, it is straightforward for the government to restore the second best. Let f^* be the solution of (3.10). Then, the government can impose the requirement that no agent can deposit more than f^* . An alternative way is for the government to impose a tax on deposits above a certain level.¹

The optimal marginal tax τ can be derived by comparing the left hand side of (3.8) and (3.10):

$$\tau = f^{*\frac{3}{4}} - \frac{1}{4} f^{*\frac{3}{4}} - \frac{1}{2} - \frac{1}{4} f^{*\frac{1}{2}} = \frac{3}{4} \bar{R} - \frac{1}{2} - \frac{1}{4} \bar{R}^{\frac{2}{3}}$$

This explicit tax on deposits is similar to Regulation Q, introduced with the Glass–Steagall Act in 1933 and completely phased-out by 1986, which prohibited banks from paying interest on demand deposits (Gilbert, 1986). We are not trying to justify Regulation Q or say that it was introduced with this purpose, but simply to show that the type of regulation implied by the model is not so far-fetched.

4. Bank Lending

¹ This tax would work as long as the builders will have no incentive to deposit any wheat. For this to be the case, we need (3.3) to hold at $p_d = (f^*)^{\frac{1}{2}}$. Put differently, we need to show that the level of

p_d and hence f at which (3.3) holds as an equality is such that the l.h.s. of (3.10) is below the r.h.s. . Substituting $f^{*\frac{1}{2}} = \bar{R}$ into the l.h.s. of (3.10) we obtain $\frac{1}{4} \bar{R}^{\frac{3}{2}} + \frac{1}{2} + \frac{1}{4} \bar{R}$, which is below the r.h.s. as long as \bar{R} is below 7.

Let's now consider the case where banks have some ability to seize payments that go through the banking sector. Specifically, suppose that all payments for building and doctor services take place through check transfers and that the bank is able to seize these before these are cashed for consumption. In other words, labor income is now contractible. However, a bank cannot force anyone to work. That is, all a bank can do is to ensure that someone who defaults has zero consumption, apart from their investment income.

We continue to assume that investment income cannot be seized. In Section 6.2 we analyze what happens when we relax this assumption.

Given that doctors go first, only doctors will want to borrow. Builders, who buy second, will obtain no advantage from borrowing. A bank, knowing that it will have no power over builders at the end of date 3, will insist on being repaid before it approves the builders' payment to doctors. But then builders will have to repay their debt before they buy doctors' services, making their borrowing useless.

Let β be the amount borrowed by each doctor. By borrowing an amount β an individual doctor can consume $\frac{\beta}{p_b}$ of building services, which is attractive if he has to pay back only β (given $p_b < 1$). Of course, a bank needs to make sure that it will be repaid. At date 3, a doctor exerts $\frac{1}{2} p_d^2$ of effort, receiving in exchange p_d^2 in terms of payment. His net utility is $\frac{1}{2} p_d^2$. Thus, he cannot borrow more than $\frac{1}{2} p_d^2$: if he did he would prefer not to work at date 3, default, and consume nothing (except for his investment income).

A doctor's utility is now given by

$$(4.1) \quad \frac{f + \beta}{p_b} + \frac{1}{2} p_d^2 - \beta + (e - f)\bar{R}$$

Notice that given $p_b < 1$, a doctor's utility is increasing in β ; thus a doctor will borrow up to the constraint.

The equilibrium in the market for doctors' services will be given by

$$\frac{f + \beta}{p_d} = p_d$$

and the equilibrium in the market for building services will be given by

$$\frac{f + \beta}{p_b} = \frac{p_b}{p_d}.$$

Since doctors will borrow as much as possible we will have

$$\beta = \frac{1}{2} p_d^2 = \frac{1}{2} (f + \beta)$$

or

$$(4.2) \quad \beta = f$$

Hence,

$$p_d = (2f)^{\frac{1}{2}} \quad \text{and} \quad p_b = (2f)^{\frac{3}{4}}.$$

From (3.2) the FOC in a competitive market is

$$(4.3) \quad (2f)^{-\frac{3}{4}} = \bar{R}$$

or

$$\hat{f} = \frac{1}{2} \frac{1}{\bar{R}^{\frac{4}{3}}} = \frac{1}{2} \hat{f}$$

where \hat{f} is the solution of (3.8) when there was no borrowing and \hat{f} is the solution of (4.3)

when there is borrowing. Since $2\hat{f} = \hat{f}$, it follows that the presence of lending keeps prices constant. However, lending cuts in half the amount of wheat that is notified, reducing welfare losses. Without lending $e - \hat{f}$ is invested in the risky project, while with lending, the investment rises to $e - \hat{f}$.

It is easy to see that $U_d(\hat{f}) > U_d(\hat{f})$, since

$$U_d(\hat{f}) = \frac{\hat{f} + \beta}{p_b} + \frac{1}{2} p_d^2 + (e - \hat{f})\bar{R} - \beta$$

and

$$(2\hat{f})^{\frac{1}{4}} + (e - \hat{f})\bar{R} = (\hat{f})^{\frac{1}{4}} + (e - \frac{1}{2}\hat{f})\bar{R} > (\hat{f})^{\frac{1}{4}} + \frac{1}{2}\hat{f} + (e - \hat{f})\bar{R} = U_d(\hat{f})$$

Similarly, we have that

$$U_b(\hat{f}) = \frac{1}{2} \left(\frac{p_b}{p_d} \right)^2 + e\bar{R} = \frac{1}{2} (2\hat{f})^{\frac{1}{2}} + e\bar{R} = \frac{1}{2} \hat{f}^{\frac{1}{2}} + e\bar{R} = U_b(\hat{f}).$$

The planning solution is obtained by maximizing the sum of the utilities of a doctor and a builder (i.e., the expected utility of each):

$$W = \frac{f + \beta}{p_b} + \frac{1}{2} p_d^2 + (e - f)\bar{R} - \beta + \frac{1}{2} \left(\frac{p_b}{p_d} \right)^2 + e\bar{R}$$

Substituting the value of p_d and p_b , we obtain

$$W = (2f)^{\frac{1}{4}} + (e - f)\bar{R} + \frac{1}{2} (2f)^{\frac{1}{2}}$$

Thus, the FOC is

$$(4.4) \quad \frac{1}{2} (2f)^{-\frac{3}{4}} + \frac{1}{2} (2f)^{-\frac{1}{2}} = \bar{R}$$

By comparing (4.3) and (4.4) we have

Proposition 3: If $\bar{R} > 1$, the market equilibrium leads to an excessive amount of inside money even in the presence of lending. Lending, however, increases welfare.

Proof: Since the l.h.s. of both (4.3) and (4.4) are decreasing in f , to prove that the solution of

(4.3) is larger than the solution of (4.4) it is enough to show that $(2f)^{-\frac{3}{4}} > \frac{1}{2} (2f)^{-\frac{3}{4}} + \frac{1}{2} (2f)^{-\frac{1}{2}}$,

or $(2f)^{-\frac{3}{4}} > (2f)^{-\frac{1}{2}}$, which is always true since $2f < 1$ by (4.3).

Lending, thus, does not resolve the tension between private and social objectives. Nevertheless, lending does improve welfare since it increases the volume of trade without sacrificing the higher return of the alternative investment. Interestingly, lending does not substitute for the notification of wheat, it only complements it. To see why, look at equation (4.2). The amount of feasible lending is directly related to the amount of notes present in the system. The reason is that lending faces a repayment constraint. With no notes in the system, the amount borrowed by doctors equals the purchasing power in the hands of builders, which in turn

equals the revenue received by doctors for their services. But if the revenue equals the debt, it is not in the interest of the doctors to work, given that they have to exert costly effort. Hence, the doctors will default. To have a functioning lending market, we need a minimum amount of deposits.

Finally, it is important to emphasize that we have only scratched the surface of borrowing. If we introduce uncertainty in the proceeds from trade, then borrowing will be risky. (In principle it could be state-contingent.) Some loans may not be repaid, which might cause some banks not to be able to honor their claims. This may lead to contagion effects, as consumers cannot redeem claims and in turn default, leading other banks to default. (Contagion effects are analyzed in Kiyotaki and Moore (1997), (2002).) The analysis becomes much more complex, and richer, and we hope to explore the consequences in future work.

5. Interaction between private and public money

So far we have ignored any role of the government in providing liquidity. In this finite horizon economy, to introduce government money we need to specify why it is accepted. Following a long tradition (e.g., Cochrane (1998)), we assume that government money is valuable because one can pay taxes with it. We suppose that each agent receives an amount m of government money at date 2 after investments have been made but before any trading takes place, and will have to pay some taxes at date 3. We assume that the agent has an option to pay taxes either in dollars or an equal number of units of wheat. This anchors the value of money.

In this type of model, it is generally assumed that taxation takes the form of a non-distortionary lump sum tax. What is ignored is that even a lump sum tax has some potential distortions. What happens if an individual refuses to pay the lump sum tax? Presumably, he would be thrown in jail. To make this a credible threat, however, the government would have to build prisons in advance, which is in itself distortionary.

Rather than working with a lump sum tax and a jail threat (the results are similar but less elegant), we prefer to use a standard model of a distortionary tax, such as a consumption tax. Specifically, we assume that the government can impose a mill tax on those who turn wheat into flour.

Assume that each agent can obtain λ units of flour at date 3 at the cost of $\frac{1}{2}c\lambda^2$ units of wheat. This activity occurs at facilities that can easily be monitored by the government, and so the sales tax cannot be avoided. One unit of flour yields one unit of utility. An agent's utility is now:

$$\text{Doctors: } U_d = x_d + b_d - \frac{1}{2}l_d^2 + (1-t)\lambda_d - \frac{1}{2}c\lambda_d^2$$

$$\text{Builders: } U_b = x_b + d_b - \frac{1}{2}l_b^2 + (1-t)\lambda_b - \frac{1}{2}c\lambda_b^2$$

where t is the tax rate on flour.

We assume that this transformation from wheat into flour can be financed out of the return from the risky investment. We suppose that even in the worst state of the world this return is always high enough so that agents are not at a corner solution, and so λ_d, λ_b satisfy the first order condition

$$(5.1) \quad \lambda_d = \lambda_b = \frac{1-t}{c}.$$

Hence, budget balance for the government implies

$$(5.2) \quad m = \frac{t(1-t)}{c}.$$

5.1 No private borrowing

For simplicity let's start with the case where there is no private borrowing. Consider the market for doctors. The number of notes in the hands of builders will be $f+m$ (from trading with doctors) plus m of their own. Thus, market equilibrium is given by

$$(5.3) \quad \frac{f+2m}{p_d} = p_d$$

We assume that $f+2m < 1$; otherwise (5.3) would be replaced by $p_d = 1$.

On the other hand, in the market for builders the number of notes available to doctors is $f+m$, so the market equilibrium is given by

$$(5.4) \quad \frac{f+m}{p_b} = \frac{p_b}{p_d}$$

Solving (5.3) and (5.4) yields

$$(5.5) \quad p_d = (f + 2m)^{\frac{1}{2}} \quad \text{and} \quad p_b = (f + m)^{\frac{1}{2}} (f + 2m)^{\frac{1}{4}}.$$

We can write the utilities of doctors and builders as

$$(5.6) \quad U_d = \frac{f + m}{p_b} + \frac{1}{2} p_d^2 + (e - f) \bar{R} + \frac{1}{2c} (1 - t)^2$$

$$(5.7) \quad U_b = \frac{m}{p_d} + \frac{1}{2} \left(\frac{p_b}{p_d} \right)^2 + e \bar{R} + \frac{1}{2c} (1 - t)^2$$

where we use (5.1) to substitute for λ_d , λ_b .

The competitive equilibrium is characterized by the first order condition of (5.6) with respect to f , where prices are taken as given:

$$(5.7) \quad \frac{1}{p_b} = \bar{R}.$$

As noted we require the solution to (5.5) and (5.7) to satisfy $f + 2m < 1$ for these formulae to be correct. This will be true as long as m is not too large. We assume this in what follows.

In contrast, the planner maximizes $W = U_d + U_b$ taking into account the effects of f and m on prices. In other words, the planner maximizes

$$(5.9) \quad W = \frac{(f + m)^{\frac{1}{2}}}{(f + 2m)^{\frac{1}{4}}} + \frac{1}{2} f + m + \frac{1}{c} (1 - t)^2 - f \bar{R} + \frac{1}{2} \frac{f + m}{(f + 2m)^{\frac{1}{2}}} + \frac{m}{(f + 2m)^{\frac{1}{2}}} + 2e \bar{R}.$$

Several questions are worth asking. First, starting at the competitive equilibrium, does the planner want to introduce outside money, i.e., $m > 0$, given that the market will adjust to a new competitive equilibrium? The answer seems to be ambiguous, perhaps because the market provides too much of its own money (sets f too high).

A second question is: given that the planner can regulate f , does she want to set $m > 0$? Here the answer is affirmative. To see this, set f at the regulatory optimum, i.e., maximize W with respect to f when $m = 0$ (this is equivalent to maximizing (3.9)). Now consider a small change in m (or equivalently in t). By the envelope theorem,

$$(5.10) \quad \frac{dW}{dm} \Big|_{m=0} = \frac{\partial W}{\partial m} \Big|_{m=0} = \frac{f^{\frac{1}{4}} \frac{1}{2} f^{-\frac{1}{2}} - f^{\frac{1}{2}} \frac{1}{4} 2f^{-\frac{3}{4}}}{f^{\frac{1}{2}}} + 1 - \frac{2(1-t)}{c} \frac{dt}{dm} + \frac{1}{2} \frac{f^{\frac{1}{2}} - f \frac{1}{2} f^{-\frac{1}{2}} 2}{f} + \frac{f^{\frac{1}{2}}}{f}$$

From (5.2), $\frac{dt}{dm} = \frac{c}{1-2t} = c$ at $m=0$, and so

$$(5.11) \quad \frac{dW}{dm} \Big|_{m=0} = -1 + \frac{1}{f^{\frac{1}{2}}} > 0$$

since (5.5) and (5.8) imply $f < 1$ when $m=0$ (given $\bar{R} > 1$). It follows that it is always better for the planner to set $m > 0$ than $m=0$.

This result can also be interpreted as a proof of the non-neutrality of money: welfare goes up when m goes up. To be fair, our model is unable to distinguish between monetary policy and fiscal policy, since the helicopter drop of money could easily be replaced by a government purchase of goods financed with debt (government money and debt are equivalent in our model). If we were to take that interpretation, though, we could have an example in which fiscal policy is not neutral even in the presence of Ricardian equivalence.

A third question is: given that the regulator can choose m (and t) optimally, does she want to set $f=0$, i.e., does outside money crowd out inside money? Here the answer is: it depends. It

is clear from (5.10) that f is small when \bar{R} is large. In fact one can go further: $\frac{dW}{df} \Big|_{f=0} < 0$

when \bar{R} is large at the optimal m , and so the socially optimal f is zero for large enough \bar{R} . On the other hand, we have seen that when \bar{R} is close to 1 the private market equilibrium approximates the first best, and so inside money is more efficient than outside money. We can carry out a similar exercise for variations in c . If c is small the deadweight costs of taxation are small and so outside money is more efficient than inside money; the socially optimal f will be close to zero. On the other hand, if c is large, then the maximum possible value of m (given by $\frac{1}{4c}$) is small.² In other words, a pure outside money economy will achieve very little trade, and inside money by itself can do better. Thus, the optimal f is bigger than zero.

² This maximum value of m can be obtained from maximizing (5.2) with respect to t .

5.2 Case with private borrowing

We now show that our result that some outside money is optimal ($m > 0$) generalizes to the case of borrowing. In the presence of private borrowing (5.3) and (5.4) are replaced by

$$(5.12) \quad \frac{f + \beta + 2m}{p_d} = p_d$$

and the equilibrium in the market for building services will be given by

$$(5.13) \quad \frac{f + \beta + m}{p_b} = \frac{p_b}{p_d}.$$

For these formula to apply we need $f + \beta + 2m < 1$. We then have

$$(5.14) \quad p_d = (f + \beta + 2m)^{\frac{1}{2}} \quad \text{and} \quad p_b = (f + \beta + m)^{\frac{1}{2}} (f + \beta + 2m)^{\frac{1}{4}}.$$

We know that $\beta = \frac{1}{2} p_d^2$, from which it follows that

$$(5.15) \quad \beta = f + 2m$$

Hence, total welfare is given by

$$(5.16) \quad W = \frac{f + \beta + m}{p_b} + \frac{1}{2} p_d^2 - \beta + (e - f)\bar{R} + \frac{1}{2c}(1-t)^2 + \frac{m}{p_d} + \frac{1}{2} \left(\frac{p_b}{p_d}\right)^2 + e\bar{R} + \frac{1}{2c}(1-t)^2 =$$

$$= \frac{(2f + 3m)^{\frac{1}{2}}}{(2f + 4m)^{\frac{1}{4}}} + \frac{1}{c}(1-t)^2 - f\bar{R} + \frac{1}{2} \frac{2f + 3m}{(2f + 4m)^{\frac{1}{2}}} + \frac{m}{(4f + 4m)^{\frac{1}{2}}} + 2e\bar{R}$$

The planner will maximize (5.16) with respect to f and m (given that β adjusts to satisfy (5.15)).

Using the envelope theorem we have

$$(5.17) \quad \frac{dW}{dm} \Big|_{m=0} = \frac{\partial W}{\partial m} \Big|_{m=0} = \frac{1}{2}(2f)^{-\frac{1}{2}} - 2 + \frac{3}{2}(2f)^{-\frac{1}{2}} > 0$$

if $f < \frac{1}{2}$, which we know to be the case from (4.3). Hence, it is optimal to set $m > 0$.

Interestingly, the introduction of government money does not eliminate the role for bank lending. In fact, it is complementary to bank lending, as shown by equation (5.15). The reason is similar to the one highlighted in Section 4: With no notes in the system, the amount borrowed by

doctors equals the purchasing power in the hands of builders, which in turn equals the revenue received by doctors for their services. But if the revenue equals the debt, it is not in the interest of the doctors to work, given that they have to exert costly effort. To have a functioning lending market, thus, we need either inside or outside money. Lending multiplies the effect from the injection of money.

6 Extensions

6.1 Non-competitive banks

So far we have treated banks as purely passive institutions. In this subsection we explore how the results change once we allow banks to be of non-negligible size and to behave strategically. We start with the conjecture that, as in the competitive case, the only agents who want to deposit are doctors. We will then verify this conjecture.

Let us assume that there is a fixed number of banks, $\frac{1}{\alpha}$, where $0 < \alpha < 1$. Each bank serves a fraction α of the doctors. For simplicity we assume that each bank is a monopolist with respect to its constituency of doctors; however, we doubt that much would change if we allowed several banks to compete for the same constituency of doctors.

Note that the case $\alpha = 0$ can be interpreted as the (limiting) situation where every doctor can set up his own bank. In what follows we will report the results only for mutual (or cooperative) banks. Considering outside-owned banks will not ameliorate matters; in fact, it would make them worse.

We assume that each bank offers the doctors in its constituency the following service: a doctor can deposit an amount of wheat up to σ and receive notes (or checks) equal to σ . Hence, σ is a policy instrument of the bank, which is the same for all customers; moreover, we assume that the bank can announce and commit to it.

If $\sigma \geq \hat{f}$ obtained in (3.8), then each doctor will set $f = \hat{f}$ and the bank policy would be irrelevant. Hence, we focus on the possibility that $\sigma < \hat{f}$, where each doctor will deposit the full amount allowed, σ .

Consider a single bank's choice of σ given that the bank serves a fraction α of the population of doctors and that the average choice of other banks is $\hat{\sigma}$. We know from Section 2.2

that even if every doctor deposits as much as he wants, then $p_b < 1$ and $p_d < 1$. A fortiori this must be true when $\sigma < \hat{f}$. Thus we can focus on situations where $p_b < 1$ and $p_d < 1$.

The doctor's utility will be given by

$$(6.1) \quad \frac{\sigma}{p_b} + \frac{1}{2} p_d^2 + (e - \sigma) \bar{R}$$

The mutual bank chooses σ to maximize the utility of a representative member, given by (6.1), taking into account the effect of the bank's choice of σ on prices p_b and p_d .

Let us consider this price effect. Given σ the total value of notes in circulation will be $\alpha\sigma + (1 - \alpha)\hat{\sigma}$; the first term represents the contribution of this bank and the second term the contribution of the other banks. Since doctors use all their notes on building services, the demand for building services is

$$(6.2) \quad \frac{\alpha\sigma + (1 - \alpha)\hat{\sigma}}{p_b},$$

while the supply is, as in (2.4), $\frac{p_b}{p_d}$. (Builders will spend all the proceeds of their building services on doctor services.) Equating these yields

$$(6.3) \quad p_b^2 = (\alpha\sigma + (1 - \alpha)\hat{\sigma}) p_d.$$

In the market for doctors, demand is

$$(6.4) \quad \frac{\alpha\sigma + (1 - \alpha)\hat{\sigma}}{p_d},$$

since the builders use all the notes received from doctors to buy doctor services; and supply is p_d . Combining this with (6.3) yields

$$(6.5) \quad p_b = (\alpha\sigma + (1 - \alpha)\hat{\sigma})^{\frac{3}{4}},$$

$$p_d = (\alpha\sigma + (1 - \alpha)\hat{\sigma})^{\frac{1}{2}}.$$

Substituting into (6.2), we see that the utility of a representative doctor at the bank choosing σ is

$$(6.6) \quad \frac{\sigma}{(\alpha\sigma + (1-\alpha)\hat{\sigma})^{\frac{3}{4}}} + \frac{1}{2}(\alpha\sigma + (1-\alpha)\hat{\sigma}) + (e - \sigma)\bar{R}.$$

We study a Nash equilibrium in which each bank chooses σ to maximize (6.6), taking $\hat{\sigma}$ as given. Let $y = \alpha\sigma + (1-\alpha)\hat{\sigma}$ and $z = (1-\alpha)\hat{\sigma}$. Then, maximizing (6.6) is equivalent to maximizing

$$(6.7) \quad y^{\frac{1}{4}} - \frac{z}{y^{\frac{3}{4}}} - e^{\frac{3}{4}}(1 - \frac{1}{2}\alpha)(y - z)$$

with respect to y . It is easy to see that (6.7) is strictly concave in y . Thus, there is a unique maximizer y and hence a unique maximizer σ of (6.6), given $\hat{\sigma}$.

Moreover, the optimal y is strictly increasing in z . It follows that, if two different banks choose different values of sigma in equilibrium, i.e., they face different values of z , then they will choose different values of y . But y equals the average value of sigma over all banks, and must therefore be the same for each bank. It follows that the equilibrium sigma is the same for all banks, i.e., any Nash equilibrium is symmetric.

Differentiating (6.6) and setting $\sigma = \hat{\sigma}$, we may conclude that the equilibrium level of σ , if $0 < \sigma < 1$, satisfies

$$(6.8) \quad \sigma^{-\frac{3}{4}} + \frac{1}{2}\alpha[1 - \frac{3}{2}\sigma^{-\frac{3}{4}}] = \bar{R}.$$

Let's now verify the conjecture that builders do not want to deposit any of their wheat. Define σ_d^* to be the solution of (6.8). This is the equilibrium level if and only if the builders do not want to deposit any more wheat. The utility of a builder who deposits σ_b is given by

$$\frac{\sigma_b}{p_d} + \frac{1}{2} \frac{p_b^2}{p_d^2} + (e - \sigma_b)\bar{R}$$

Thus, no builder wants to deposit if

$$\frac{1}{p_d} \leq \bar{R}$$

Substituting the equilibrium level of $p_d = \sigma_d^{*\frac{1}{2}}$ we have

$$(6.9) \quad \left[\frac{1 - \frac{3}{4}\alpha}{\bar{R} - \frac{1}{2}\alpha} \right]^{\frac{2}{3}} \geq \frac{1 - \frac{1}{4}\alpha}{\bar{R}}.$$

As long as (6.9) is satisfied (which is certainly true for a big enough \bar{R}), the solution determined by (6.8) is an equilibrium. As is easy to see, the level of \bar{R} for which (6.9) is satisfied depends on α . For $\alpha = 0$, (6.9) is satisfied for any $\bar{R} > 1$. For $\alpha = 0.25$, (6.9) is satisfied for $\bar{R} > 1.23$.

Having established that (6.8) characterizes an equilibrium if (6.9) is satisfied, we can compare it with (3.10) to obtain:

Proposition 4. In a competitive market (α close to 0) banks choose too high a level of deposits with respect to what is socially efficient. In a monopolistic market ($\alpha = 1$) banks choose too low a level of deposits with respect to what is socially efficient.

Proof: For $\alpha = 0$, (6.8) becomes (3.8) and the previous result applies. For $\alpha = 1$ the l.h.s. of (6.8) becomes $\sigma^{-\frac{3}{4}} + \frac{1}{2} - \frac{3}{4}\sigma^{-\frac{3}{4}} = \bar{R}$. Since both (6.8) and (3.10) are decreasing in their arguments, to prove that the solution of (6.8) is smaller than the solution of (3.10) it is enough to show that $\frac{1}{4}f^{-\frac{3}{4}} + \frac{1}{2} + \frac{1}{4}f^{-\frac{1}{2}} > f^{-\frac{3}{4}} + \frac{1}{2} - \frac{3}{4}f^{-\frac{3}{4}}$, or $\frac{1}{4}f^{-\frac{1}{2}} > 0$, which is always true.

The intuition for the competition result is as before. The intuition for the monopolist one is simple. Large mutual banks restrict σ , i.e., issue too few notes, to lower the price of building services; this helps their members since their members consume these services. In doing this, however, large banks ignore the positive externality they impose on builders, who gain from high prices since this allows them to buy more doctor services. Small banks choose a high σ because their impact on prices is limited.

When it comes to builders, they cannot create liquidity to increase the price of their own services; hence, they face a trade-off similar to before: increasing their own purchasing power at the expense of the return on their investments.

Since a competitive banking sector generates too much liquidity and a monopolistic one too little, by continuity there exists a level of oligopoly α that delivers the efficient level of

inside money. Note, however, that this level is contingent upon \bar{R} ; thus if the level of \bar{R} changes with the business cycle, so does the level of competition that delivers the first best. In other words, unless the government can somehow fine tune the level of competition over time, this does not seem a very reliable method to achieve the first best level of money.

6.2 Banks investing in risky projects

We have assumed that the risky projects can be undertaken only by the agents and not by the banks, because the projects require some unique human skills that the agents have. What would happen if the banks could undertake the risky projects too? In this case each doctor would deposit all his endowment in the bank in exchange for shares in the bank's assets (in proportion to his fraction of the total funds deposited). These shares could then be used for trading. The doctors would endorse these shares to the builders in exchange for building services and the builders would further endorse them to doctors when they buy their services.

In analyzing this case the two key assumptions are when the uncertainty about the risky projects gets resolved and what the risk-return characteristics of the projects undertaken by the banks are. If we take the extreme case where all the uncertainty about the risky projects is resolved after the last round of trading at date 3 and banks can undertake the same risky projects as the individuals, then the risk characteristics of the projects are irrelevant. The claims backed by risky projects, rather than the wheat, will be valued at the expected value of the risky projects and for a bank investing in the risky projects strictly dominates storing wheat. Hence, if the banks' risky projects have the same return as those of the individual agents, we achieve the second best, with all the wheat deposited and invested in risky projects. If $e\bar{R} \geq 1$, then we also achieve the first best (the doctors have enough collateralizable wealth to purchase one unit of building services at a price of 1).

If we maintain the same assumption about the resolution of uncertainty, but the return of the risky projects available to banks is inferior to the return available to individuals, then we have the same model as in Section 3 where the storage at the bank will be replaced by the investment in risky projects and the previous \bar{R} is replaced by the difference between the return on private investments and the return on banks' investment.

The more interesting case is where some uncertainty gets resolved before the two rounds of trading, so that the value of the notes backed by risky investments changes. While in this

economy individual agents are risk neutral, the transactional role played by money makes them risk-averse vis-à-vis fluctuations in the value of notes.

Agents dislike the uncertainty about the value of their claims because if the value of their claims drops below 1, the economy operates below full potential, while if the value of the claims is above 1, no extra benefits are generated.

Let's consider the case where banks can choose how to divide their deposits between a risky project and a safe project, and this choice is perfectly contractible, so there is no moral hazard involved. Let's also assume that banks can undertake the same projects as the individuals and uncertainty gets fully resolved before the first round of trading. Finally, we assume that the risky projects are perfectly correlated and have the following payoff per dollar (constant return to scale):

$$(6.12) \quad \begin{aligned} \tilde{R} &= \frac{\bar{R}}{\varepsilon} \text{ with probability } \varepsilon \\ &= 0 \quad \text{with probability } 1 - \varepsilon \end{aligned}$$

The expected return of the investment is \bar{R} , which exceeds 1. In this section we will focus on the limiting case where $\varepsilon \rightarrow 0$ and the investment becomes infinitely risky.

In this set up, since banks can commit to the mix of safe and risky investments, it is easy to show that doctors will deposit all their wheat e in the banks. (Builders will continue to invest in their own projects.) Thus, the crucial variable is the proportion μ of deposits that a typical bank invests in the riskless asset (storage) (where $1 - \mu$ is invested in the risky project).

Since trading will take place with banks' shares, the equilibrium prices will differ according to the state that is realized before the beginning of trading. Denote prices in the bad state by p_b, p_d and in the good state by p_b^g, p_d^g .

With probability $1 - \varepsilon$ the risky investment will yield zero, the riskless asset one, and the value of a doctor's claim will be μe . By contrast, with probability ε the risky investment will yield $\frac{\bar{R}}{\varepsilon}$ and the value of the doctor's claim will be $\mu e + (1 - \mu) \frac{e\bar{R}}{\varepsilon}$. In summary, if we let g be the value of the doctors' claim, then

$$(6.13) \quad g = g_1 = \mu e + (1 - \mu) \frac{e\bar{R}}{\varepsilon} \text{ in the good state}$$

$$= g_2 = \mu e \quad \text{in the bad state}$$

A doctor's expected utility is

$$(6.14) \quad (1 - \varepsilon) \left[\frac{\mu e}{p_b} + \frac{1}{2} p_d^2 \right] + \varepsilon \left[\frac{g_2}{p_b^2} + \frac{1}{2} p_d^{g_2} \right].$$

From the analysis of Sections 3 and 4,

$$p_b = (\mu e)^{\frac{3}{4}} \quad p_d = (\mu e)^{\frac{1}{2}}$$

$$p_b^g = \min \left\{ g_2^{\frac{3}{4}}, 1 \right\} \quad p_d^g = \min \left\{ g_2^{\frac{1}{2}}, 1 \right\}$$

If we keep $\mu < 1$ constant and let $\varepsilon \rightarrow 0$, p_b^g converges to one and (6.14) becomes

$$\frac{\mu e}{p_b} + \frac{1}{2} p_d^2 + (1 - \mu) e \bar{R}$$

which is identical to (6.1) with $\mu e = f_d$. Hence, all the solutions will be the same as in Section 3.

Each (competitive) bank will choose $\mu e = f_d$, where f_d solves (3.2). Also as $\varepsilon \rightarrow 0$ the welfare function will become

$$(6.15) \quad W \equiv U_d + U_b = (\mu e)^{\frac{1}{4}} + \frac{1}{2} (\mu e)^{\frac{1}{2}} + \frac{1}{2} \mu e + (1 - \mu) e \bar{R} + e \bar{R}$$

which is identical to (3.9) with $\mu e = f$. Thus we have

Proposition 5. When uncertainty is resolved before trading takes place, the model of this section with trading in bank shares is equivalent to the model of Section 3 with trading in fixed claims on deposits.

The intuition is that in the good state of the world the economy has plenty of liquidity and so the Walrasian equilibrium is realized. Since this state occurs with negligible probability the value of liquidity in this state is very small. Thus liquidity is provided by the safe investment, which pays off in the bad state. The risky investment still has value in expected terms but only because it provides a huge amount of extra wheat consumption in the good state. For these

reasons banks will hold a mix of safe and risky investments, just as individual doctors do in the model of Section 3. In the limit the two models are equivalent.

6.3 Contingent helicopter drops

So far we have considered a non-contingent injection of money at date 2. Yet, if some of the uncertainty is resolved at date 2, as in the previous section, and the government can observe it, it is optimal for the government to inject liquidity if and only if the bad realization occurs.

Let's consider again the example in Section 6.2 where banks can invest in the risky project and the uncertainty about this project gets realized before the beginning of trading. Unlike in the previous example, let's assume ε is small but not negligible. Then, if the risky project has the good realization, there is no need for any extra liquidity. But if the realization is zero, the system would benefit from extra liquidity. In this case, it would be optimal for the government to intervene, injecting liquidity in the system. From (5.11) we know that the social welfare function is increasing in m when m is zero. Thus, the government would like to carry out a helicopter drop, contingent on the realization of the risky asset. This can be interpreted as monetary policy.

In fact, the government can do better than a helicopter drop. By dropping money randomly the government will transfer liquidity both to doctors and builders. Yet only doctors use it. Given that liquidity is costly because it requires distortionary taxation, we would like to minimize the use of it. The government can do strictly better by injecting money into banks, where only doctors deposit.

Notice that the government cannot achieve the same goal with a reduction in taxes, at least not future taxes (as we have in this model). In fact, agents need liquidity at date 2 and not at the end of date 3, where there is plenty. In a multi-period model where taxes are assessed every period, a reduction in taxes would be equivalent to a helicopter drop of money, but it would still be more inefficient than an injection of liquidity in the banking system, which economizes on the amount of money introduced in the system and thus the amount of distortionary taxation.

If the government intervention is not unexpected, however, its ex post benefits will have to be traded off against some ex ante costs. In this respect a bailout of the banking sector will not necessarily be better than a helicopter drop of money. First, if the ex ante probability of a government injection of liquidity is large enough, then builders will want to deposit in the banks

to capture the gift. Thus, the amount of money to be injected will be the same as in a random drop. In addition, if the money deposited in the bank is less productive than funds invested in the private project, the prospect of the bailout will cause an ex ante distortion: too much wheat will be deposited in the relatively unproductive banks to take advantage of the bailout ex post.

Second, we have ignored any potential moral hazard. If agents anticipate that banks will be bailed out, then it is in the interest of even a mutually owned bank to invest all the deposits in the risky project.

7. Conclusions

We have built a simple framework to analyze the general equilibrium implications of the creation of liquidity by banks. To isolate these effects in this paper we consider either fully-backed money or lines of credit secured by a certain income stream. As a result there is no risk of banks' default. This risk is clearly a very important problem in reality, and we plan to consider it in future work.

In a world where some assets can be collateralized and other assets that cannot, our paper shows that the assets that can be collateralized will trade at a premium with respect to what their yield would imply. More surprisingly, our paper shows that the competitive equilibrium will lead to an excess of collateralizable assets. This distortion is present even if we introduce lending and government money. The source of this distortion is a (pecuniary) externality that arises from the creation of money. More money increases the equilibrium price of goods purchased by the agents who are liquidity constrained (think for example of the effect of the relaxed credit on the price of U.S. houses). This externality has welfare effects, because the buyers are liquidity constrained. The government can remove this distortion by imposing a tax on deposits.

Our model also shows that this effect can be attenuated or even eliminated in a less than competitive banking sector. Yet, we run the risk of the opposite problem: with too little competition we will have too little liquidity. Therefore, it is unlikely that restricting competition is the best way for the government to remove the externality we have identified. It would be better for the government to tackle the problem by directly targeting the creation of liquidity.

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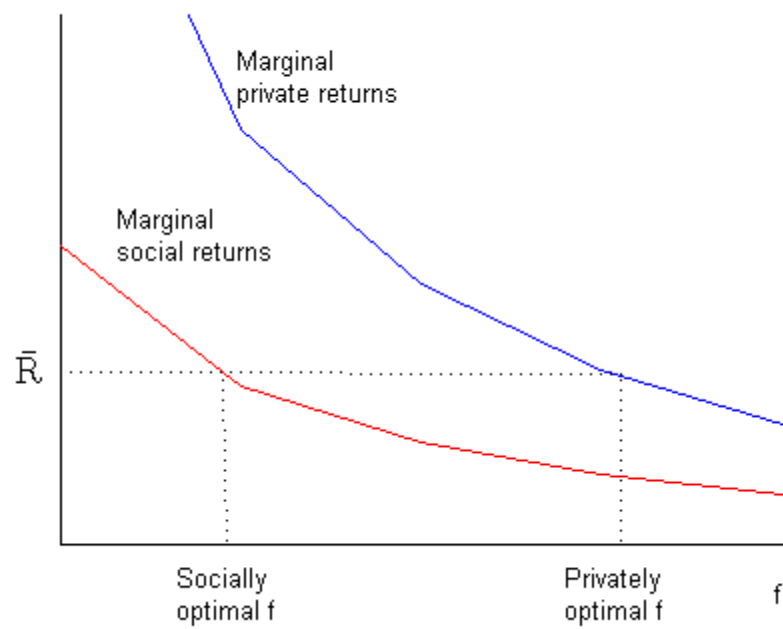


Figure 1 - Difference between private and social optimality